Analysis of hybrid adaptive/non-adaptive multi-user OFDMA systems with imperfect channel knowledge

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Kurzfassung

Die OFDMA (Orthogonal Frequency Division Multiple Access)-Übertragungstechnik ist ein viel versprechender Kandidat für zukünftige Mobilfunksysteme. Neben den günstigen Eigenschaften bezüglich der Implementierung und der Bekämpfung von Mehrwegeausbreitungseffekten ermöglicht OFDMA eine effiziente Anpassung an die Kanalbedingungen durch adaptive Zuweisung der verschiedenen Ressourcen an die verschiedenen Nutzer in Zeit und Frequenz. Im Falle der Abwärtsstrecke ist hierfür sendeseitige Kanalkenntnis über die einzelnen Verbindungen zwischen dem Sender und den Empfängern erforderlich, die in einem realistischen Szenario jedoch nicht als perfekt angenommen werden kann. Steht dem Sender perfekte Kenntnis über die Kanäle sämtlicher Nutzer zur Verfügung, so erbringen adaptive OFDMA-Verfahren sehr gute Performanzen durch die Ausnutzung von Mehrnutzerdiversität und die Anpassung an die momentanen Kanalbedingungen durch adaptive Wahl der Modulationsverfahren. Steht dem Sender dagegen keine Kanalkenntnis zur Verfügung, so ist die Verwendung nicht-adaptiver Verfahren, die keine Kanalkenntnis benötigen, jedoch Zeit-, Frequenz- oder räumliche Diversität ausnutzen, die beste Strategie. Hybride OFDMA-Verfahren ermöglichen es, beide Übertragungsstrategien zu nutzen. Hierbei stellt sich die Frage, welcher Nutzer adaptiv bzw. nicht-adaptiv bedient und welche Ressource welchem Nutzer zugewiesen werden soll, insbesondere dann, wenn die Güte der Kanalkenntnis für verschiedene Nutzer unterschiedlich ist, d.h. wenn für manche Nutzer die Kanalkenntnis nur geringfügig fehlerbehaftet ist, während sie für andere Nutzer völlig verfälscht ist. Hierbei ist zu beachten, dass dieses Problem nicht für jeden Nutzer unabhängig von den anderen Nutzern gelöst werden kann, weil die Performanz eines jeden Nutzers stark von der Mehrnutzerdiversität und damit der Anzahl der adaptiv bedienten Nutzer abhängt. Als Ziel wird die Maximierung der Systemdatenrate bei gleichzeitiger Einhaltung einer gegebenen Bitfehlerrate und Mindestnutzerdatenrate angestrebt. Dies soll für ein Mehrantennen-Einzellen-Szenario, bei dem Nutzer unterschiedliche Anforderungen bezüglich der Anzahl zugewiesener Ressourcen haben, realisiert werden, wobei sich auf Mehrantennen-Verfahren ohne räumliches Multiplexing beschränkt werden soll.

Hierzu werden zunächst für ein hybrides OFDMA-System mit Hilfe eines Weighted Proportional Fair Scheduling die unterschiedlichen Nutzeranforderungen für den adaptiven Übertragungsmodus realisiert. Als Kanalqualitätsinformation (CQI) am Sender wird das Signal-zu-Rausch-Verhältnis (SNR) verwendet, das entweder in kontinuierlicher oder in quantisierter Form vorliegt. Dazu wird für die hier betrachteten Mehrantennen-Verfahren Orthogonal Space Time Block Coding und Transmit Antenna Selection in Kombination mit Maximum Ratio Combining am Empfänger die unterschiedliche Gewichtung des WPFS entsprechend der angeforderten Ressourcenanzahl bestimmt. Für

den nicht-adaptiven Übertragungsmodus des hybriden OFDMA-Systems, der mit Hilfe einer Discrete Fourier Transformation-Vorkodierung Frequenzdiversität ausnutzt, erfolgt die Ressourcenzuweisung über einen Round Robin Ansatz. Bezüglich der Reihenfolge, in der die Ressourcen den adaptiven und nicht-adaptiven Nutzern zugeteilt werden, werden zwei unterschiedliche Ansätze betrachtet. Im ersten Verfahren werden zunächst die Ressourcen an die nicht-adaptiven Nutzer zugewiesen und anschließend werden die verbliebenen Ressourcen den adaptiven Nutzern zugeteilt. Im zweiten Verfahren erfolgt die Ressourcenzuteilung in umgekehrter Reihenfolge.

Um den Einfluß nicht perfekter Kanalkenntnis auf die Performanz des hybriden Systems berücksichtigen zu können, werden analytische Ausdrücke für die Nutzerdaten- und Bitfehlerrate als Funktion der Anzahl der adaptiv bedienten Nutzer, der angeforderten Ressourcenanzahl und der die Ungenauigkeit der Kanalkenntnis beschreibenden Parameter hergeleitet, wobei von vier in der Literatur bekannten Fehlerquellen für die CQI ausgegangen wird: Veralterung, Schätzfehler, Quantisierung und ein fehlerbehafteter Rückkanal. Hierbei werden alle Fehlerquellen gemeinsam und nicht, wie teilweise in der Literatur, separat betrachtet. Das Problem der Systemdatenratenmaximierung unter Einhaltung einer gegebenen Bitfehlerrate und Mindestnutzerdatenrate lässt sich nun in zwei kleinere Probleme aufteilen: erstens die Bestimmung optimaler SNR-Schwellwerte der angewandten Modulationsverfahren und zweitens die Bestimmung des passenden Übertragungsmodus, mit dem der jeweilige Nutzer bedient wird. Anhand der hergeleiteten analytischen Ausdrücke lassen sich nun die SNR-Schwellwerte so anpassen, dass eine geforderte Bitfehlerrate nicht überschritten wird und gleichzeitig die Nutzerdatenrate maximiert wird. Da hiermit für jede mögliche Kombination, Nutzer adaptiv oder nicht-adaptiv zu bedienen, die maximal erzielbaren Nutzerdatenraten unter Einhaltung der Bitfehlerraten-Anforderung bestimmbar sind, kann somit das kombinatorische Problem der Nutzerbedienung gelöst werden, wobei sich zeigt, dass nicht alle möglichen Bedienkombinationen ausprobiert werden müssen, um die beste Lösung zu finden, was durch eine Komplexitätsanalyse der vorgeschlagenen Lösungsalgorithmen veranschaulicht wird.

Für eine realistische Performanzabschätzung wird zusätzlich der in der Literatur häufig vernachlässigte Aufwand bezüglich Pilotübertragungen und Signalisierungen berücksichtigt, der in dem betrachteten hybriden System auftritt. Da die für die Abwärtsstrecke erforderlichen Signalisierungen und Pilotübertragungen in der Aufwärtsstrecke stattfinden und somit dort Ressourcen für die eigentliche Datenübertragung belegen, wird eine effektive Systemdatenrate definiert, die sowohl Abwärts- als auch Aufwärtsstrecke berücksichtigt. Dazu wird eine Frame-Struktur für ein hybrides OFDMA-System im Time Division und Frequency Division Duplex Modus erarbeitet, anhand dessen der Aufwand bezüglich Pilotübertragungen und Signalisierungen bestimmt wird.

Schließlich wird die Performanz des hybriden OFDMA-Systems in einem Szenario mit nutzerabhängiger nicht perfekter Kanalkenntnis evaluiert und mit der Performanz konventioneller rein adaptiver bzw. nicht-adaptiver OFDMA-Systeme verglichen. Hierbei zeigt sich, dass die Performanz hybrider Systeme bei einer geringen bis mittleren Anzahl an aktiven Nutzern in der Zelle der Performanz konventioneller Systeme für ein steigendes Mass an CQI Ungenauigkeit überlegen ist, selbst wenn der Aufwand bezüglich Pilotübertragungen und Signalisierungen berücksichtigt wird.

Abstract

The OFDMA (Orthogonal Frequency Division Multiple Access) transmission scheme is a promising candidate for future mobile radio networks. Besides the beneficial properties concerning implementation and combating the negative effects of multipath propagation, OFDMA provides an efficient adaptation towards the current channel conditions by adaptively allocating the different resources to the different users in time and frequency direction. In case of downlink transmission, transmitter sided channel knowledge of the individual lines between the transmitter and the receivers is required which cannot be assumed to be perfectly known in a realistic scenario. In case that perfect channel knowledge is available at the transmitter, the application of adaptive OFDMA schemes leads to very good performances by exploiting multiuser diversity and by adaptively selecting the applied modulation schemes with respect to the current channel conditions. In case that no channel knowledge is available at the transmitter, the use of non-adaptive schemes which do not rely on instantaneous channel knowledge but exploit frequency, time or spatial diversity is the best strategy. Hybrid OFDMA schemes offer the opportunity to use both transmission strategies. Using hybrid schemes, the question arises which users shall be served adaptively or non-adaptively and which resource shall be allocated to which user, especially in scenarios where the quality of the channel knowledge differs form user to user, i.e., for some users the transmitter has channel knowledge which is only slightly corrupted while for other users, the transmitter has only totally erroneous channel knowledge. In this regard, it has to be noted that the problem cannot be solved userwise independently from the other users as the performance of each user strongly depends on the exploited multiuser diversity and, thus, on the number of adaptively served users. The aim is to maximize the system data rate while fulfilling a given target Bit Error Rate (BER) and minimum user data rate requirement. This is accomplished in a single cell scenario with multiple antennas where different users have different demands regarding the number of allocated resources. Concerning multiple antenna schemes, only schemes without spatial multiplexing shall be considered.

At first, the different user demands for the adaptive transmission mode of the hybrid OFDMA system are realized by applying a Weighted Proportional Fair Scheduling. As Channel Quality Information (CQI), the Signal-to-Noise-Ratio (SNR) is applied where either continuous or quantized CQI values are assumed. To do so, the proper WPFS weights for the considered multiple antenna schemes, namely Orthogonal Space Time Block Coding and Transmit Antenna Selection in combination with Maximum Ratio Combining at the receiver, are determined with respect to the demanded number of resources. For the non-adaptive transmission mode which exploits frequency diversity

with the help of a Discrete Fourier Transform precoding, the resource allocation is done applying a round robin approach. Concerning the order of allocation in which the resources are allocated to the adaptive and non-adaptive users, two approaches are considered. Applying the first scheme, first the resources assigned for the non-adaptive users are allocated. Subsequently, the remaining resources are allocated to the adaptive users. Applying the second scheme, the order of allocation is vice-versa.

In order to take into account the impact of imperfect channel knowledge on the performance of the hybrid system, analytical closed form expressions for the user data rate and BER are derived as functions of the number of adaptively served users, the user demands and the CQI impairment parameters where four different sources of error for the CQI are assumed: time delays, estimation errors, quantization and an imperfect feedback link. In contrast to many contributions in the literature where only one of the sources of error is considered at the same time, all four sources of error are jointly considered in this work. For the mentioned errors, a modelling is developed. The problem of maximizing the system data rate subject to the target BER and the minimum rate requirement can be split up into two smaller problems: firstly, the determination of optimal SNR thresholds for the applied modulation schemes and secondly, the selection of the access scheme which serves a certain user . With the help of the derived analytical expressions, the SNR thresholds can be adjusted such that the target BER is fulfilled while the user data rate is maximized. Since the maximum achievable user data rates with respect to the target BER can be defined for any possible user serving combination, the combinatorial user serving problem can be solved. Furthermore, it can be shown that it is not necessary to test all possible user serving combinations to find the best solution. Moreover, a complexity analysis of the proposed solving algorithms is presented.

For a realistic performance evaluation of practical systems, also the effort in terms of pilot transmissions and signaling which occurs in the considered hybrid system and which is mostly neglected in the literature is taken in account. Since the signaling and the pilot transmissions, which are essential for the downlink, take place during uplink and, thus, occupy resources for the actual data transmission, an effective system data rate is defined which considers both up- and downlink. In order to identify the amount of overhead in terms of signaling and pilot transmissions, a frame structure for the hybrid OFDMA system is developed where both time division and frequency division duplex are considered.

Finally, the performance of hybrid OFDMA systems in a scenario with user-dependent imperfect CQI is evaluated and compared to the performance of conventional pure adaptive or non-adaptive OFDMA systems. It is shown that for a low to medium number of active users in the cell, hybrid systems outperform the conventional ones for increasing CQI inaccuracy even if the overhead due to pilot transmissions and signaling is considered.

Contents

Chapter 1

Introduction

1.1 Hybrid OFDMA systems

Orthogonal Frequency Division Multiple Access (OFDMA) [RTMG99] is regarded as a promising candidate for future mobile radio systems. Applying OFDMA, the available bandwidth is subdivided in overlapping but mutually orthogonal narrowband subcarriers which allows a spectrally efficient data transmission. The signal is transmitted in consecutive and mutually independent blocks which are separated by a guard interval. For this purpose, a Cyclic Prefix (CP) is typically used [WG00]. One advantageous property of the block transmission with CP is the fact that the subcarriers remain mutually orthogonal even for transmissions over frequency selective channels. This enables the use of simple receiver structures even for high data rates. Another advantage is the computationally efficient implementation of the OFDMA modulation and demodulation using the Fast Fourier Transform (FFT) algorithm [Ach78]. Furthermore, multiple antenna techniques which can enhance the system performance by exploiting spatial diversity to improve communication reliability and/or spatial multiplexing to improve throughput [PRG03] are applicable with OFDMA transmission schemes. Thus, OFDMA can be considered as a high data rate enabling transmission scheme at acceptable costs. Moreover, by assigning a variable number of subcarriers to a given radio link, the system can provide services with different rate requirements [LL05].

In general, one has to distinguish between downlink and uplink transmission. The uplink denotes the transmission from the Mobile Stations (MSs) to the central Base Station (BS) within a mobile radio cell which is connected to the communication network. The downlink denotes the transmission from the BS to the MSs. Concerning the multiple access scheme, there are different requirements for uplink and downlink, e.g., in the uplink the power efficiency is more critical as in the downlink, as the power supply of a MS is based on batteries.

Moreover, one has to distinguish between two scenarios concerning channel knowledge. In the first scenario, knowledge about the current channel conditions is available at the transmitter while in the second scenario, this is not the case. In case that reliable channel knowledge is available at the transmitter, good performances for transmissions can be accomplished by means of adaptation to the channel using, e.g., techniques like adaptive multi-user scheduling [MEV03], and adaptive power loading, modulation and coding [GC98]. However, the provision of accurate channel knowledge requires a considerable amount of overhead. In the following, OFDMA schemes applying these techniques are referred to as adaptive OFDMA. In case that no reliable channel knowledge is available, the use of diversity exploiting transmission schemes is the preferred strategy for provision of good performance [WIN05c]. In the following, OFDMA schemes applying this strategy are referred to as non-adaptive OFDMA. For both adaptive and non-adaptive OFDMA, different realizations for uplink and downlink are known in the literature.

Adaptive OFDMA with an adaptive subcarrier allocation based on user specific channel knowledge is intended as access scheme for adaptive downlink transmission in Worldwide interoperability for Microwave Access (WiMAX) [IEE04] as well as in Third Generation Partnership Project Long Term Evolution (3GPP LTE) [3GP08]. Moreover, adaptive OFDMA is intended as access scheme for both adaptive downlink and uplink in the European Wireless World Initiative New Radio (WINNER) system concept [WIN06].

For non-adaptive transmissions in the downlink, an OFDMA scheme which is able to exploit diversity shall be applied. In WiMAX and WINNER, OFDMA with an equidistant subcarrier distribution over the available bandwidth to exploit frequency diversity is considered, also known as Block Equidistant Frequency Division Multiple Access (B-EFMDA) [IEE04], [3GP08], [WIN06], [WIN07].

For non-adaptive transmissions in the uplink, it is desirable to apply a multiple access scheme which provides low fluctuations of the signal envelope as high fluctuations require a power back-off that reduces the power efficiency of the power amplifer [RAC+03]. Since OFDMA is known to suffer from high envelope fluctuations [NP00], a more suitable multiple access scheme is desired. By introducing a Discrete Fourier Transform (DFT) precoding of the data symbols, the OFDMA signal properties are changed resulting in considerablely lower signal envelope fluctuations [XZG03]. Localized Single Carrier Frequency Division Multiple Access (SC-FDMA) is an example of a DFT precoded OFDMA scheme [3GP06], [3GP08]. Combining DFT precoding with an equidistant subcarrier allocation leads to Interleaved Frequency Division Multiple Access (IFDMA) [SBS97]. IFDMA provides low signal envelope fluctuations while exploiting frequency diversity due to the spreading of the data over the whole bandwidth [Fra10]. As IFDMA is known to be sensitive to frequency offsets caused by the Doppler effect or oscillator imperfections [DLF04], Block Interleaved Frequency Division Multiple Access (B-IFDMA) is another promising scheme which overcomes this disadvantage of IFDMA. B-IFDMA is based on OFDMA with an equidistant distribution of blocks of adjacent subcarriers over the whole bandwidth in combination with a DFT precoding of the data symbols [SFF+07], [Fra10], [Soh11] and is intended for uplink transmissions in the WINNER system concept. Note that there are also other access schemes which are intended for non-adaptive uplink that do accept the low power efficiency and high cost of the power amplifier applying OFDMA. The desired frequency diversity is either introduced by frequency hopping or an equidistant subcarrier allocation [IEE04], [3GP08].

Summing up, adaptive OFDMA schemes require accurate channel knowledge at the transmitter and a considerable amount of signaling which limits the range of applications to scenarios with rather slowly changing channels, e.g., slowly moving MSs. In these scenarios, however, adaptive access scheme outperform non-adaptive access schemes [WIN06]. Nevertheless, non-adaptive OFDMA schemes are more suitable in scenarios with fast changing channels due to the use of diversity combining techniques which do not need transmitter sided channel knowledge resulting in marginal overhead. As in a realistic scenario, both situations are present, i.e., static up to semi-static users and fast moving users exist, it is beneficial to combine both multiple access schemes in a hybrid OFDMA scheme to serve all users with respect to the given conditions.

In general, there are three multiplexing strategies for the multiple access schemes [WIN06]. Firstly, the adaptive and non-adaptive transmissions may be multiplexed in time. In each time slot, all resources are either used for adaptive or non-adaptive transmissions. The second possibility is to multiplex the adaptive and non-adaptive transmissions in frequency. In this case, different resources in frequency direction are either reserved for non-adaptive or adaptive transmission over several time slots. Finally, the adaptive and non-adaptive transmissions may be multiplexed in space. Note that also combinations are possible. Fig. 1.1 illustrates the multiplexing in time and frequency where each rectangle represents a resource in time and frequency.

Time multiplexing is beneficial for systems with access to a rather narrow bandwidth where a reasonable frequency multiplexing is not applicable. Furthermore, time multiplexing offers the possibility to limit the power consumption of the mobile terminals in the uplink by entering a sleep mode in times where the terminal has no data to transmit. One drawback is the limited granularity compared to frequency multiplexing and a larger time delay between hops [WIN06].

Frequency multiplexing is beneficial for systems with access to a large bandwidth such that each user can exploit enough frequency diversity for both multiple access schemes. Also, low delay services can be supported. However, due to the high bandwidth, the

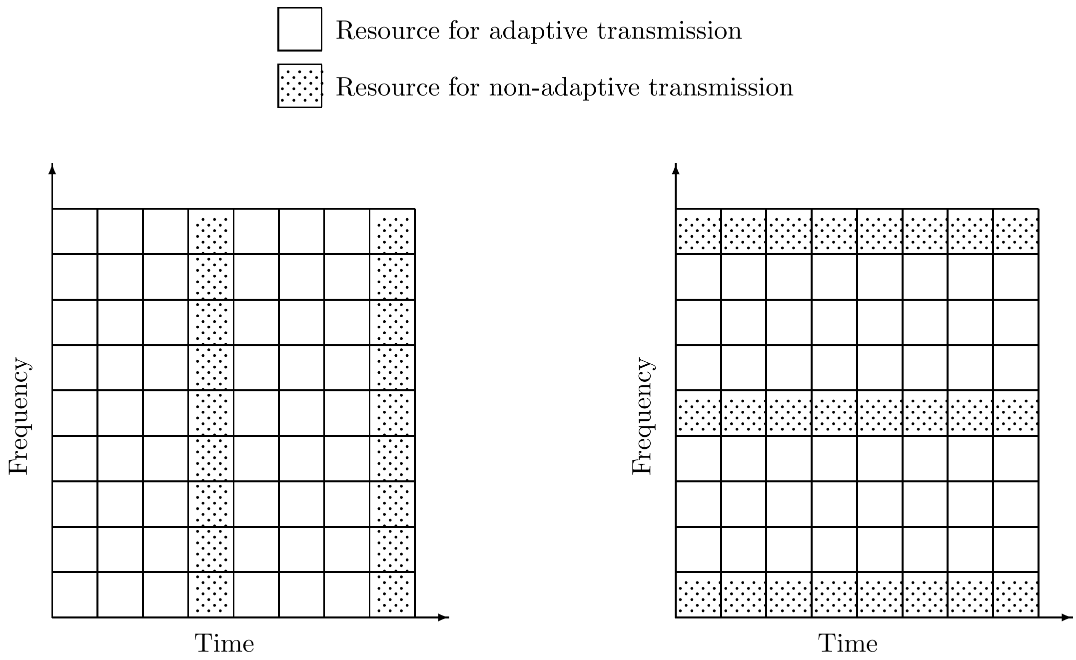


Figure 1.1. Time and frequency multiplexing of resources for adaptive and non-adaptive transmissions

time diversity between consecutive time slots is rather small decreasing coding gains when coding over several time slots [WIN06].

The usefulness of spatial multiplexing in general Multiple Input Multiple Output (MIMO) scenarios is less clear compared to time and frequency multiplexing as the spatial channels may lose their orthogonality over time. However, in certain grid of beams scenarios, spatial multiplexing may be applicable and useful [WIN06]. For example, one beam could serve rather static users which are spatially close to each other like in a stadium or a shopping mall while another beam serves the more dynamic users which enter or leave the stadium or shopping mall, respectively. This, however, requires specific environmental knowledge.

As future mobile radio systems are supposed to have access to a large bandwidth, a hybrid OFDMA scheme applying frequency multiplexing is considered throughout this work.

The key question concerning a hybrid OFDMA system is how to select the adequate access scheme to serve the different users such that the system throughput is maximized while fulfilling certain quality of service requirements especially when taking into account different data rate requirements, imperfect transmitter sided channel knowledge and the amount of signaling and pilot overhead which is needed by both access schemes to operate efficiently.

1.2 State-of-the-art

This section presents a review of the state of the art with regard to the application of hybrid OFDMA in the presence of imperfect channel knowledge.

Hybrid OFDMA systems which allow the co-existence or the switching between adaptive OFDMA transmission and non-adaptive OFDMA schemes, respectively, have already been introduced in the literature. In [DMO09] and [WIN06], the co-existence and adaptive selection of multiple access schemes in a hybrid OFDMA system with frequency multiplexing of the resources for frequency adaptive and frequency non-adaptive transmission schemes is discussed. As frequency adaptive scheme, adaptive chunk-based Time Division Multiple Access (TDMA)/OFDMA is applied both in downlink and uplink. As non-adaptive scheme in the downlink, Block Equidistant Frequency Division Multiple Access (B-EFMDA) is applied, where the subcarriers of a given user are blockwise equidistantly distributed over the bandwidth to exploit frequency diversity. In the uplink, Block Interleaved Frequency Division Multiple Access (B-IFMDA) is applied which is similar to B-EFMDA except for a Discrete Fourier Transform (DFT) precoding of the data. This DFT-precoding leads to lower envelope fluctuations which is beneficial for low cost amplifier in mobile terminals. Furthermore, additional frequency diversity is introduced as the data is spread over the total bandwidth. Within a so called super-frame, chunks of subcarriers are pre-allocated for the two modes. Between super-frames, the allocation of the subcarrier can change. The preselection of the applied access scheme mode for the different users is amongst others based on the type of service, the channel quality of the downlink and the Signal-to-Interference-and-Noise-Ratio (SINR). During operation, the access schemes can dynamically be changed based on the switching criteria which are, e.g., the CQI quality and the terminal velocity, i.e., a two step mechanism is applied.

Another OFDM-based hybrid multiple access scheme has been presented in [LLLB05]. Here, only the downlink was considered where adaptive OFDMA is employed as adaptive transmission and Frequency Hopping (FH)-OFDMA is employed as non-adaptive scheme which exploits frequency diversity. To select the applied access scheme, three classes are defined, namely the mobility class, the service class and the environment class. The mobility class are a) mobile users and b) nomadic users with a rather low terminal velocity. Concerning service, real-time and non-real time services are considered. The environment class are a) low and b) high intercell interference environments. According to the class affiliation of a given user, either adaptive OFDMA or FF-OFDMA is applied as multiple access scheme.

In both works, the decision whether a user is served by an adaptive or non-adaptive access scheme is not done based on analytical calculations. In [DMO09], e.g., the decision whether the CQI quality is good enough to apply the adaptive access scheme is based on simulative curves which are only valid for a certain set of simulation parameters. In [LLLB05], the expected throughput of either the adaptive or non-adaptive access scheme is used as criterion without considering the impact of imperfect CQI. Furthermore, concerning the mobility, only the coherence time of the channel of each user which has to be smaller than a given threshold to apply the adaptive scheme is used as criterion to select the access scheme. However, this approach totally disregards the impact of the number of users applying the adaptive access scheme on the multi-user diversity gains and, thus, the performance. Moreover, the determination of the threshold value is rather heuristic since the actual achievable data rate is not calculated.

From this, it follows that the proposed hybrid multiple access schemes cannot guarantee that certain quality of service requirements of each user are actually fulfilled as the multiple access scheme selection is not based on analytical calculations considering imperfect channel knowledge, pilot and signaling overhead but on heuristic approaches, especially concerning the terminal velocity which is the most crucial criterion assuming equal service classes.

Dealing with imperfect channel knowledge has been mainly discussed for conventional pure adaptive OFDM-based schemes in the literature. For the case of single user transmission with imperfect or partial Channel State Information (CSI) at the transmitter, OFDM transmission schemes have also been studied, see [YBC06], [SS01], [LC98], [RVG04], [SP01], [SZG02], [SH03], [YG05], [LRWH05], [MDG06] and references therein. In [YBC06], adaptive OFDM with imperfect CSI for uncoded variable bit rates is studied, where the imperfect CSI arises from noisy channel estimates and the time delay of getting the CSI to the transmitter. The authors propose the use of multiple estimates to improve the performance. In [SS01], the impact of imperfect CSI is investigated for an adaptive OFDM system using the bit and power loading algorithm of [FH96]. In [LC98], a subchannel loading algorithm is proposed combating the negative effects resulting from channel errors in coherent detection at the receiver. In [RVG04], the impact of imperfect one bit per subcarrier CSI feedback is studied. In [SP01], channel prediction is used to combat the impact of outdated CSI and in [SZG02], a statistical adaptive modulation scheme based on long-term statistics is proposed. In [SH03], the minimum feedback rate required to determine the set of active subchannels using an on-off power allocation in a multicarrier transmission scheme is studied. Optimizing the activation threshold results in an achievable data rate which is shown to be asymptotically equivalent to the channel capacity. In [YG05], an optimal power loading algorithm for OFDM based on average and outage capacity criteria is presented assuming imperfect CSI at the transmitter. In [LRWH05], a limited feedback OFDM power loading algorithm is proposed using a codebook of power loading vectors. In [MDG06], a loading algorithm is presented which aims at minimizing transmit power under rate and error probability constraints using quantized CSI. For single user OFDM systems with multiple antennas, the use of imperfect or partial CSI has been also investigated, e.g. in [XZG04] and [BM04].

All above mentioned references considered the single user case. For the case of multiuser transmission, adaptive schemes exploiting multi-user diversity based on imperfect or partial CSI have also been studied, for example, in [GA04], [HL04], [MT05], [VAH05] and [VAH06]. In [GA04], selective multi-user diversity is introduced, where only channel gains are fed back which are above a given threshold. In [HL04], the impact of partial CSI is studied in an OFDMA system, where each user only feeds back the CSI of the M best subcarriers. In [MT05], multi-user diversity with outdated channel information is studied. In [VAH05], combinations of frequency and space based diversity techniques for a multi-user scenario with limited feedback are discussed. In [VAH06], a multi-user scenario with either outdated or noisy CSI is analyzed.

Concerning the scheduling for adaptive OFDMA schemes, Proportional Fair Scheduling (PFS) approaches provide a good trade-off between system throughput and fairness. PFS in combination with OFDMA is well discussed in the literature, e.g., [MA06] and [RRSS05]. If, furthermore, different user priorities shall be considered, Weighted Proportional Fair Scheduling (WPFS) approaches can be applied, which are discussed, e.g., in [KKK06], [KKHL02] and [FKWD07]. These WPFS algorithms favor high priority users to get channel access even if their channel gain is low which leads to a degradation of the system throughput compared to PFS approaches. Both PFS and WPFS algorithms require channel knowledge at the transmitter. However, in a realistic scenario with imperfect channel knowledge, the performance also degrades compared to the case of perfect channel knowledge. The joint impact of imperfect channel knowledge and different user priorities on the performance of an adaptive OFDMA system has not been mentioned in the literature, especially not for a hybrid OFDMA system.

To the author's knowledge, an analytical assessment of a hybrid OFDMA scheme with different user demands taking into account imperfect CQI as well as pilot and signaling overhead has not been provided so far. Moreover, the problem of selecting the multiple access schemes based on analytical performance calculations to fulfill certain quality of service requirements as the target BER and minimum user data rates while maximizing the overall system performance has not been considered in the literature.

1.3 Open issues

In this section, open issues coming from the review of existing literature regarding hybrid OFDMA systems assuming imperfect channel knowledge are summarized.

Although pure adaptive OFDMA systems applying Weighted Proportional Fair Scheduling based on CQI to allocate the resources to the users considering the different channel access demand of the users have been studied, the determination of the weights to meet individual user demands has not been mentioned especially for quantized CQI and for different multiple antenna techniques like OSTBC and TAS in combination with MRC especially for hybrid OFDMA systems applying WPFS. Thus, the following questions arise:

1. How does the probability of getting access to the channel in hybrid multi-user OFDMA systems depend on the WPFS weighting factors for the different antenna techniques assuming continuous and quantized CQI?
2. How to adjust the weighting factors such that a given user demand in terms of allocated resources is fulfilled?

Dealing with imperfect transmitter sided channel knowledge has only been discussed in pure adaptive OFDM and OFDMA systems assuming outdated, noisy or quantized CQI. However, for a hybrid OFDMA system applying multiple antenna techniques such as OSTBC-MRC and TAS-MRC, an analytical description of the performance concerning achievable data rate and BER taking into account imperfect CQI such as outdated, noisy and quantized CQI with an imperfect feedback link has not been mentioned in the literature so far. Hereby, the following questions have to be answered:

3. How does the distribution of the SNR values of allocated resources in hybrid OFDMA systems look like taking into account the different user demands for both OSTBC-MRC and TAS-MRC assuming continuous and quantized CQI?

4. How to analytically determine the average user data rate and BER taking into account an imperfect CQI due to time delays, estimation errors and imperfect feedback link for both OSTBC-MRC and TAS-MRC assuming continuous and quantized CQI?

5. How to adjust the SNR thresholds for the adaptive modulation scheme selection such that a given target BER is met while maximizing the user data rate of each user?

As an analytical assessment of hybrid OFDMA schemes assuming imperfect CQI has not been discussed in the literature, also the multiple access scheme selection based on analytical expressions taking into account the individual user-dependent channel knowledge quality of the users is an open problem leading to the following questions:

6. How to decide in a hybrid OFDMA system whether a user shall be served adaptively or non-adaptively such that the total system data rate is maximized while fulfilling a minimum rate requirement for each user taking into account user-dependent CQI?

7. What is the complexity to solve this combinatorial problem?

Finally, an analytical consideration of signaling and pilot overhead has not been considered for hybrid OFDMA schemes so far, although the overhead is crucial when it comes to an reasonable and meaningful system performance evaluation and comparison with conventional pure adaptive or pure non-adaptive OFDMA systems resulting in the following questions:

8. What are the efforts in terms of pilot transmissions and signaling which have to be spent in a hybrid OFDMA system which operate either in a Time Division Duplex (TDD) mode or in a Frequency Division Duplex (FDD) mode?

9. How does this overhead effect the actual effective system data rate of a hybrid OFDMA system?

10. How does hybrid OFDMA systems perform compared to conventional pure adaptive or pure non-adaptive OFDMA systems in a scenario with user-dependent imperfect CQI considering overhead?

11. Up to which number of active users in the cell does the use of adaptive transmissions make sense in a hybrid OFDMA systems?

1.4 Contributions and thesis overview

This section gives an overview of the thesis and summarizes the main contributions addressing the open problems introduced in Section 1.3. In the following, the contents along with the main contributions of each chapter are briefly described.

In Chapter 2, the OFDMA system model together with the channel model and system assumptions is provided. Furthermore, the two transmission modes of the hybrid scheme, namely adaptive and non-adaptive OFDMA, are introduced. Finally, the modelling of imperfect channel knowledge is presented where four different sources of CQI impairments are assumed: time delays, estimation errors, quantization and an imperfect CQI feedback link.

In Chapter 3, the concept of a hybrid multi-user OFDMA system which is aware of imperfect user-dependent CQI is proposed. Hereby, two hybrid schemes are developed which differ in the resource allocation. Furthermore, the main problem formulation is introduced which aims at maximizing the system data rate while fulfilling a given BER and minimum data rate requirement for each user applying both the adaptive and non-adaptive OFDMA transmission modes. In the following, this optimization problem is solved by giving answers to the Questions 1 to 7 of the open issues by the following contributions:

1. For both continuous and quantized CQI, analytical expressions of the channel access probability for hybrid OFDMA systems applying either OSTBC or TAS at the transmitter and MRC at the receiver are derived as a function of the weighting factors used in the WPFS approach.
2. It is shown how to adjust the weighting factors of the WPFS to fulfill a certain user demand in terms of allocated resources by solving a constrained nonlinear optimization problem.
3. Analytical closed form expressions of the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of the SNR of allocated resources are derived for both continuous and quantized CQI in hybrid OFDMA systems applying either OSTBC-MRC or TAS-MRC considering different user demand.
4. For both continuous and quantized CQI analytical closed form expressions of the average user data rate and BER in hybrid OFDMA schemes applying either OSTBC-MRC or TAS-MRC are derived as a function of the user demand and the imperfect CQI.

5. For both continuous and quantized CQI, it is shown how to adjust the SNR thresholds for the modulation scheme selection in order to fulfill a given target BER while maximizing the user data rate.

6. For the combinatorial user serving problem, different algorithms are proposed where it can be shown that it is not necessary to check all possible 2^U user serving combinations in order to find the best solution.

7. For the proposed algorithms, a complexity analysis is provided.

Chapter 4 addresses the overhead in terms of pilot transmissions and signaling which occurs in hybrid OFDMA systems and gives answers to Questions 8 and 9 of the open problems:

8. For both TDD and FDD hybrid OFDMA systems, the effort in terms of pilot transmissions and signaling of side information is identified.

9. For both TDD and FDD hybrid OFDMA systems, the effective system data rate is derived considering both downlink and uplink and the pilot and signaling overhead involved. To do so, a time frame structure for the transmission in both downlink and uplink direction is introduced.

Chapter 5 presents performance evaluations for hybrid OFDMA systems assuming a scenario with user-dependent imperfect CQI and gives answers to Questions 10 and 11 of the open issues:

10. Performances evaluations for both TDD and FDD hybrid OFDMA systems are carried out and the results are compared with conventional pure adaptive and pure non-adaptive OFDMA systems in the presence of user-dependent imperfect CQI taking into account pilot and signaling overhead.

11. The impact of the number of users in the cell on the overhead and, thus, on the achievable system performance is investigated.

Finally, the main conclusions of the thesis are summarized in Chapter 6. Furthermore, a short outlook for future works is provided.

Chapter 2

OFDMA system model

2.1 Introduction

This chapter describes the system model for the considered multi-user OFDMA system. Moreover, two different multi-user OFDMA transmission modes are introduced considering different user demands in terms of channel access. The two schemes pursue different strategies. The first transmission scheme, referred to as non-adaptive transmission scheme, does not consider any instantaneous channel knowledge at the transmitter. The main objective is to increase the reliability of the transmission independent of any transmitter-side channel knowledge by exploiting frequency and spatial diversity. The second one, referred to as adaptive transmission scheme, uses transmitter-sided channel knowledge to adaptively allocate the resource units to the different users based on the channel quality of the different users. Thus, the transmission scheme is able to adjust to the current channel conditions exploiting so called multi-user diversity at the expense of requiring instantaneous channel knowledge to the transmitter [OR05]. Finally, the modelling of imperfect channel knowledge is introduced.

This chapter is organized as follows. In Section 2.2, the scenario under consideration in this thesis is presented. In Section 2.3, the minimum allocable resource unit in the considered OFDMA system is defined. Section 2.4 introduces the models to describe the mobile radio channel while Section 2.5 presents the two considered multiple antenna techniques. Section 2.6 introduces the concept of different user demands in terms of channel access. In Section 2.7, the non-adaptive multi-user OFDMA mode and in Section 2.8, the adaptive multi-user OFDMA mode are introduced and analyzed with regard to an analytical description of the resulting Signal to Noise Ratio (SNR) at the receiver. Section 2.9 presents the modeling of imperfect channel knowledge considering four different sources of errors.

2.2 Scenario Assumptions

In this section, the considered scenario is presented along with the main assumptions made in this work.

In this work, a single BS located in the middle of a hexagonal cell is considered with U MSs inside the cell (see Fig. 2.1) which are uniformly distributed where the shape of the cell is approximated by a circle.

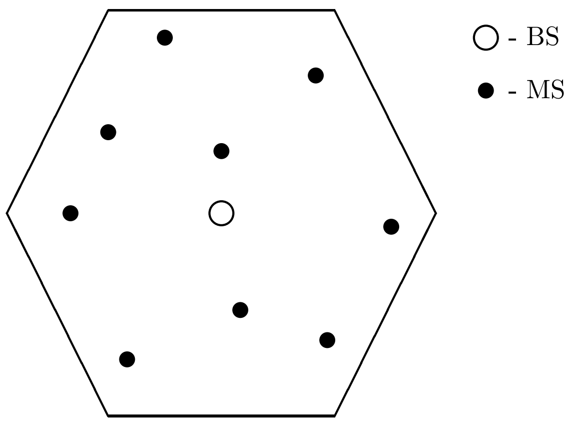


Figure 2.1. General scenario

The considered system shall work in a Time Division Duplex (TDD) scenario where the Downlink (DL) and Uplink (UL) transmission share the same frequency band and in a Frequency Division Duplex (FDD) scenario where the DL and UL transmission are performed using different frequency bands.

It is assumed that the BS is equipped with n_T transmit antennas and each MSs is equipped with n_R receive antennas. In this thesis, the n_T transmit antennas are used for either performing Orthogonal Space-Time Block Coding (OSTBC) or Transmit Antenna Selection (TAS) and the n_R receive antennas at each MS are used to perform Maximum Ratio Combining (MRC).

Further on, it is assumed that the BS has channel knowledge about the DL channels to the MSs even though the channel knowledge is not assumed to be perfect. Moreover, it is assumed that the BS and MSs have perfect Receive Channel State Information (R-CSI) to equalize the data, i.e., imperfect channel knowledge is only considered for the scheduling and modulation scheme selection. This assumption is reasonable considering the fact that the resource allocation and modulation scheme selection require a certain amount of computation time, i.e., the instantaneous channel knowledge has to be updated rapidly considering only a few pilot based channel estimations. For the equalization of the receive data, the duration of the whole frame can be utilized, i.e., advanced channel tracking algorithms can be used leading to almost perfect R-CSI.

Concerning the mobility of the MSs, there exist different models in the literature (see [NKK02] and references within) considering different levels-of-detail such as traffic and geographical information like streets maps. However, in order to keep the simulation simple to implement, no traffic modeling or geographical information are assumed in this work. Instead, it is assumed that each MS u with $u = 1, .., U$ has a different velocity $\mathbf{v}_u = [v_x, v_y]^{\mathrm{T}}$, where the x- and y-components of $\mathbf{v}_u$ are independent of each other and normally distributed with zero mean and variance σ_v. This approach is analogous to the Maxwell-Boltzmann distribution describing the particle speeds in gases [Lau05]. From this, it follows that the velocity component v_ϕ in any direction with angle ϕ is normally distributed with zero mean and variance σ_v^2. Hence, the radial component of the velocity $v_{\mathrm{rad},u}$ of user u is also normally distributed with zero mean and variance σ_v^2 ($\mathcal{N}(0, \sigma_v^2)$). The absolute value $|v_{\mathrm{rad},u}|$ is then half-normally distributed [Wei] with the PDF given by

$$p_{|v_{\mathrm{rad},u}|}(v_{\mathrm{rad},u}|) = \sqrt{\frac{2}{\pi \cdot \sigma_v^2}} \cdot \exp -\frac{|v_{\mathrm{rad},u}|^2}{2\sigma_v^2} \tag{2.1}$$

and with expectation value

$$\bar{v} = \sqrt{\frac{2}{\pi}} \cdot \sigma_v \tag{2.2}$$

In the following, the dynamics of the MSs mobility inside the cell is expressed by this average velocity $\bar{v}$.

Finally, it is assumed that the BS and each MS always has data to transmit, i.e., a full-buffer traffic model is assumed.

Note that more specific assumptions directly related to the topics discussed in the next sections will be introduced in the corresponding sections.

2.3 Resource Unit Definition

In this section, the resource units in time and frequency are defined.

The considered system employs OFDMA as multiple access scheme. In Fig. 2.2, the definition of a resource unit is shown in the time and frequency direction.

The system bandwidth B is subdivided into N perfectly orthogonal subcarriers, i.e., there is no Inter Carrier Interference (ICI) present in the system. It is assumed that $N \gg U$, meaning that channel access can be guaranteed for each user. The subcarrier

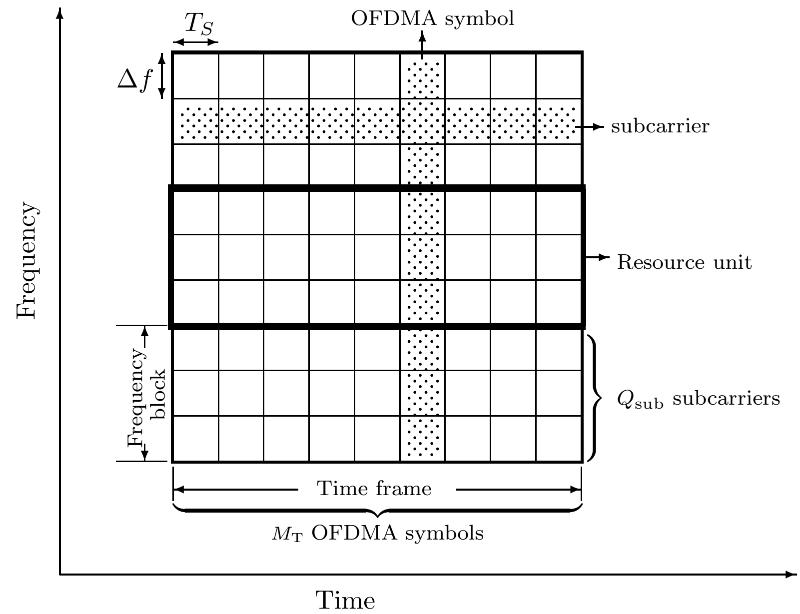


Figure 2.2. Resource units in time and frequency

bandwidth Δf is chosen such that the channel transfer function remains almost flat over a frequency block of Q_{sub} adjacent subcarriers, i.e., the channel coherence bandwidth B_{C} which denotes the bandwidth over which the channel transfer function remains almost constant [Pro95] is assumed to be much larger than the subcarrier bandwidth $\Delta f \ll B_{\mathrm{C}}$.

In the time domain, a time frame consists of M_T OFDMA symbols with a symbol duration T_{S} where perfect time synchronization is assumed. Moreover, a Guard Interval (GI) of adequate length is introduced to eliminate Inter Symbol Interference (ISI). As GI, a Cyclic Prefix (CP) is applied, i.e., a repetition of the end of the OFDMA symbol is prefixed. Beside eliminating ISI, the use of CP also allows to model the linear convolution as a circular convolution which enables the use of simple frequency domain signal processing such as channel estimation and equalization [NP00]. The symbol duration T_S is chosen such that the channel transfer function over the whole time frame is almost flat, i.e., the channel coherence time T_{C} which denotes the time during which the channel transfer function remains almost constant [Pro95] is assumed to be much larger than the symbol duration $T_{\mathrm{S}} \ll T_{\mathrm{C}}$.

In this thesis, a radio resource is described as time-frequency resource unit defined by one frequency block and one time frame as shown in Fig. 2.2. This resource unit is the minimum allocable radio resource unit in the system. Thus, there are

$$N_{\mathrm{ru}} = \left\lfloor \frac{N}{Q_{\mathrm{sub}}} \right\rfloor \tag{2.3}$$

available resource units in the system with $\lfloor \cdot \rfloor$ denoting the nearest integer smaller than or equal to the argument. Note that in the literature there exist similar resource unit definition such as chunks [WIN05b], slots [IEE04] or Physical Resource Blocks (PRBs) [3GP06]. The main idea of employing such a Block OFDMA multiple access is the reduction of signaling needed to inform the MSs about the allocated resources compared to the case that each subcarrier in each OFDMA symbol could be allocated to a different user. Further on, in case of a FDD system, also the amount of information which has to be fed back from the MSs to the BS can be significantly reduced. Finally, the computational complexity for the scheduling can be decreased which is crucial especially for systems with large number of subcarriers and users.

2.4 Channel Model

In this section, the channel model applied in this thesis is presented. Note that in this work, the whole system is considered in the equivalent baseband [Pro95].

It is assumed that the BS transmits with power P_{T} where the transmit power is equally shared among the N subcarriers, i.e., the transmit power per subcarrier is given by

$$P_{\mathrm{T,sub}} = \frac{P_{\mathrm{T}}}{N}. \tag{2.4}$$

The assumption of subdividing the power equally over the subcarriers is justified by the fact that the achievable gains applying optimal power allocation are negligible compared to the increase in complexity as shown in [KHK05]. Furthermore, optimal power allocation requires accurate channel knowledge, i.e., optimal power allocation is also prone to imperfect channel knowledge.

Due to free space pathloss and attenuation caused by buildings and other objects in the environment, the receive power at each MS depends on the position of the MS. In the following, the pathloss L_{P} in linear scale in modeled by

$$L_{\mathrm{P}} = \left(\frac{d_u}{d_0}\right)^{-\alpha} \tag{2.5}$$

with α denoting the pathloss exponent, d_u the distance between the BS and the MS of user u and d_0 the minimum distance between any MS and the BS [Rap02]. Let

N_0 denote the one-sided power spectral density of Additive White Gaussian Noise (AWGN) in the system. Then, the average SNR $\bar{\gamma}_u$ per subcarrier at the MS of user u is given by

$$\bar{\gamma}_u = \frac{P_{\mathrm{T,sub}} \cdot L_{\mathrm{P}}}{\Delta f \cdot N_0} = \frac{P_{\mathrm{T,sub}}}{\sigma^2} \cdot \left(\frac{d_u}{d_0}\right)^{-\alpha} \tag{2.6}$$

with σ^2 denoting the average noise power per subcarrier. If the signal power is normalized to one, the noise variance $\sigma^2_{\mathrm{n},u}$ of user is given by

$$\sigma^2_{\mathrm{n},u} = \frac{1}{\bar{\gamma}_u}. \tag{2.7}$$

In case of an UL transmission from the MS of user u to the BS, an UL factor κ_{UL} is introduced in (2.6) to account for the different transmission conditions in the UL such as different transmit powers or different pathloss due to different frequency dependencies in an FDD system. In this case, the average SNR $\bar{\gamma}_{\mathrm{UL},u}$ per subcarrier at the BS in the UL for subcarriers which are allocated to user u is given by

$$\bar{\gamma}_{\mathrm{UL},u} = \kappa_{\mathrm{UL}} \cdot \frac{P_{T,\mathrm{sub}}}{\sigma^2} \cdot \left(\frac{d_u}{d_0}\right)^{-\alpha}. \tag{2.8}$$

Beside the pathloss, a phenomena called fast fading has to be considered when describing the mobile radio channel. In general, the radio channel is typical characterized by a large number of propagation paths due to scattering, reflections and diffractions which is also called multi-path propagation. Thus, the complex receive signal is a noncoherent superposition of different signals propagating on different paths each having a different phase, Doppler shift and time delay. As a result, the signal strength variates on very short distances in the region of half the wave length of the carrier frequency. Hence, this phenomenon is called fast fading. A deterministic definition of the receive signal and, thus, the channel transfer function would require precise knowledge about the microstructure of the environment concerning geometry and the physical properties of the materials which is not feasible. However, assuming a sufficiently large number of uncorrelated paths also called uncorrelated scattering, it is possible to apply the central limit theorem to statistically model the real part and imaginary part of the complex receive signal as statistical independent Gaussian distributed random variables [Mol05]. Assuming Non Line of Sight (NLOS), i.e., there exists no dominant path from the BS to the MS, the random variables can be assumed to be zero-mean. From this, it follows that the complex channel transfer function can be modeled as Gaussian distributed random variable with zero mean and variance σ^2.

As presented in Section 2.3, it is assumed that the channel transfer function of a frequency block n with $n = 1, .., N_{\mathrm{ru}}$ is flat. Moverover, it is assumed that the channel transfer factor of a frequency block n is uncorrelated to the channel transfer factor of an adjacent frequency block, i.e., the coherence bandwidth B_{C} is smaller than the bandwidth of two adjacent frequency blocks.

As stated in Section 2.3, the channel transfer function is assumed to be almost flat over a time frame of M_T OFDMA symbols. Further on, it is assumed that channel transfer function of the k time frame with $k \in \mathbb{N}$ is temporally correlated to the previous time frame $k-1$, i.e., temporally correlated block fading is assumed.

For the spacing between the antennas it is assumed that the spacing is larger than half the wavelength of the carrier frequency, i.e., the channels between the i-th transmit antenna with $i = 1, .., n_{\mathrm{T}}$ and the j-th receive antenna with $j = 1, .., n_{\mathrm{R}}$ can be assumed to be uncorrelated.

From this, it follows that the complex channel transfer function $H_u^{(i,j)}(n,k)$ of user u with $u = 1, .., U$ on resource unit n with $n = 1, .., N_{\mathrm{ru}}$ in time frame k from transmit antenna i to receive antenna j is modeled as Gaussian distributed random variable with zero mean. The variance of $H_u^{(i,j)}(n,k)$ is set to $\sigma^2 = 1$, i.e., the power of the channel is normalized to one. The SNR at the receiver of user u on resource unit n in time frame k from transmit antenna i to receive antenna j is then given by the

$$\gamma_u^{(i,j)}(n,k) = \bar{\gamma}_u \cdot |H_u^{(i,j)}(n,k)|^2, \tag{2.9}$$

i.e., the expectation value $\mathrm{E}\{\gamma_u^{(i,j)}(n,k)\}$ of the instantaneous SNR of user u is given by

$$\mathrm{E}\{\gamma_u^{(i,j)}(n,k)\} = \bar{\gamma}_u \cdot \mathrm{E}\{|H_u^{(i,j)}(n,k)|^2\} = \bar{\gamma}_u. \tag{2.10}$$

2.5 Considered multiple antenna techniques

2.5.1 Introduction

Multiple Input Multiple Output (MIMO) techniques which perform spatial multiplexing or beamforming require the complete Transmitter-sided Channel State Information (T-CSI) at the BS which results in a significant amount of information which has to fed back to the BS in case that channel reciprocity cannot be exploited. In case of imperfect channel knowledge, not only the resource allocation is affected but also the separation of the different data streams as the precoding relies on accurate CSI. Hence, only multiple antenna techniques which do *not* use T-CSI to precode or weight the signal in order to perform spatial multiplexing or beamforming are considered. In this thesis, the n_{T} transmit antennas are used for either performing Orthogonal Space-Time Block Coding (OSTBC) or Transmit Antenna Selection (TAS) and the n_{R} receive antennas at each MS are used to perform Maximum Ratio Combining (MRC). By doing so, no multiplexing gain can be exploited. However, when applying TAS, selection diversity can be exploited. Further on, the application of OSTBC and MRC leads to an exploitation of spatial diversity. Besides, applying MRC, array gains can be exploited. Another advantage is the low feedback in case of an FDD system where the BS cannot measure the DL channel exploiting the reciprocity of the DL and UL channel. In this case, only the scalar SNR values have to be fed back which is much less information compared to the complete T-CSI.

For a better comprehension, the principles of the two considered antenna techniques are presented considering only one single carrier in one time frame. Hence, the user, resource unit and time frame indices u, n and k are omitted in the notation of the channels in Section 2.5.2 and 2.5.3. In the following, OSTBC is introduced in Section 2.5.2 followed by TAS introduced in Section 2.5.3.

2.5.2 Orthogonal Space-Time Block Coding in combination with Maximum Ratio Combining (OSTBC-MRC)

In this section, the principle of OSTBC is presented together with the combination of OSTBC with MRC at the receiver.

For simplicity, $n_{\mathrm{T}} = 2$ transmit antennas are assumed, i.e., the well-known Alamouti STBC [Ala98] can be applied. Furthermore, one single receive antenna is assumed for the beginning. A requirement for the application of the Alamouti STBC is that the channel $H^{(i)}$ is invariant for $n_{\mathrm{T}} = 2$ consecutive time slots which is fulfilled by the assumption made in Section 2.4. With the symbols s_1 and s_2, the Alamouti STBC matrix $X(\mathbf{s})$ is given by

$$X(\mathbf{s}) = \frac{1}{\sqrt{2}} \begin{pmatrix} s_1 & s_2 \\ s_2^\star & -s_1^\star \end{pmatrix}. \tag{2.11}$$

In the first time slot s_1 is transmitted over antenna 1 and s_2 over antenna 2. In the second time slot, the conjugate of s_2 is transmitted over antenna 1 and the negative conjugate of s_1 over antenna 2. The factor $\frac{1}{\sqrt{2}}$ normalizes the total transmit power, i.e., the same transmit power is used compared to a single antenna case. Note that there are also space-time block codes $X(\mathbf{s})$ for $n_{\mathrm{T}} \geq 2$ following the same principles, i.e., the elements of matrix $X(\mathbf{s})$ are linear functions of the K complex variables s_1,..,s_K and

their complex conjugates. Furthermore, for any arbitrary $\mathbf{s}$, $X(s)^H \cdot X(s) = ||s||^2 \cdot I$ must hold with I the identity matrix [GS05].
Applying the Alamouti STBC, the resulting receive signals r_1 in the first time slot and r_2 in the second time slot are then given by

$$r_1 = \frac{1}{\sqrt{2}} \cdot H^{(1,1)} s_1 + \frac{1}{\sqrt{2}} \cdot H^{(2,1)} s_2 + n_1 \tag{2.12}$$

and

$$r_2 = \frac{1}{\sqrt{2}} \cdot H^{(1,1)} s_2^\star - \frac{1}{\sqrt{2}} \cdot H^{(2,1)} s_1^\star + n_2 \tag{2.13}$$

with the AWGN values n_1 and n_2 with variance σ_n^2. Conjugating r_2 and using a matrix-vector notation leads to

$$\mathbf{r} = \begin{pmatrix} r_1 \\ r_2^\star \end{pmatrix} = \frac{1}{\sqrt{2}} \underbrace{\begin{pmatrix} H^{(1,1)} & H^{(2,1)} \\ -H^{(2,1)\star} & H^{(1,1)\star} \end{pmatrix}}_{\mathbf{\Lambda}} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underbrace{\begin{pmatrix} n_1 \\ n_2^\star \end{pmatrix}}_{\mathbf{n}}. \tag{2.14}$$

Multiplying the receive vector with $\mathbf{\Lambda}^\mathrm{H}$ from the left leads to

$$\begin{aligned} \mathbf{z} &= \mathbf{\Lambda}^\mathrm{H} \cdot \mathbf{r} \\ &= \frac{1}{\sqrt{2}} \cdot \mathbf{\Lambda}^\mathrm{H} \cdot \mathbf{\Lambda} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \mathbf{\Lambda}^\mathrm{H} \cdot \mathbf{n} \\ &= \mathbf{\Omega} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \tilde{\mathrm{n}} \end{aligned} \tag{2.15}$$

with the diagonal matrix

$$\mathbf{\Omega} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} |H^{(1,1)}|^2 + |H^{(2,1)}|^2 & 0 \\ 0 & |H^{(1,1)}|^2 + |H^{(2,1)}|^2 \end{pmatrix} \tag{2.16}$$

and the noise vector $\tilde{\mathrm{n}}$ whose variance $\sigma_{\tilde{\mathrm{n}}}^2$ is given by

$$\begin{aligned} \sigma_{\tilde{\mathrm{n}}}^2 = E\left\{\tilde{\mathrm{n}}^H \tilde{\mathrm{n}}\right\} &= E\left\{\mathbf{n}^H \mathbf{\Lambda} \mathbf{\Lambda}^H \mathbf{n}\right\} \\ &= E\left\{\mathbf{n}^H \mathbf{\Omega} \mathbf{n}\right\} \\ &= \mathbf{\Omega} \cdot E\left\{\mathbf{n}^H \mathbf{n}\right\} \\ &= \sigma_n^2 \cdot (|H^{(1,1)}|^2 + |H^{(2,1)}|^2). \end{aligned} \tag{2.17}$$

Due to the fact that $\mathbf{\Omega}$ is diagonal, the two data symbols s_1 and s_2 are decoupled resulting in two separated orthogonal data streams. On the basis of the diagonal elements $(|H^{(1,1)}|^2 + |H^{(2,1)}|^2)$ of $\mathbf{\Omega}$, the principle of spatial diversity becomes apparent. In this case, the data is transmitted over two spatially separated transmit antennas providing two replicas at the receiver in the spatial domain which are combined in a SNR maximizing way. Hence, the reliability of the transmission is increased, since the probability that both channels are in bad condition is much smaller compared to the case with just one transmit antenna [Kam04].

In case that the receiver is equipped with n_R receive antennas, MRC can be applied in combination with STBC. In this case, there are n_R different receive vectors $\mathbf{r}_l$ with $l = 1, .., n_\mathrm{R}$ given by

$$\mathbf{r}_l = \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} H^{(1,l)} & H^{(2,l)} \\ -H^{(2,l)\star} & H^{(1,l)\star} \end{pmatrix}}_{\mathbf{\Lambda}_l} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \mathbf{n}_l. \tag{2.18}$$

Multiplying each receive vector $\mathbf{r}_l$ with $\mathbf{\Lambda}_l$ from the left results in

$$\begin{aligned} \mathbf{z}_l &= \mathbf{\Lambda}_l^\mathrm{H} \cdot \mathbf{r}_l \qquad (2.19) \\ &= \mathbf{\Omega}_l \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \tilde{\mathrm{n}}_l \end{aligned}$$

with

$$\mathbf{\Omega}_l = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} |H^{(1,l)}|^2 + |H^{(2,l)}|^2 & 0 \\ 0 & |H^{(1,l)}|^2 + |H^{(2,l)}|^2 \end{pmatrix} \tag{2.20}$$

and the noise vector $\tilde{\mathrm{n}}_l$ with variance

$$\sigma^2_{\tilde{\mathrm{n}}_l} = \sigma^2_n \cdot \left(|H^{(1,l)}|^2 + |H^{(2,l)}|^2\right). \tag{2.21}$$

Applying MRC, a linear combination of the receive signals $\mathbf{z}_l$ shall be built which maximizes the SNR written as

$$\mathbf{y} = \sum_{l=1}^{n_\mathrm{R}} c_l \mathbf{z}_l = \sum_{l=1}^{n_\mathrm{R}} c_l \cdot \mathbf{\Omega}_l \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \sum_{l=1}^{n_\mathrm{R}} c_l \cdot \tilde{\mathrm{n}}_l \tag{2.22}$$

with c_l the MRC coefficients. Without loss of generality, only the first element of the combined signal vector $\mathbf{y}$ is considered. Thus, (2.22) reduces to

$$y_1 = \sum_{l=1}^{n_\mathrm{R}} c_l \cdot \frac{1}{\sqrt{2}} \cdot (|H^{(1,l)}|^2 + |H^{(2,l)}|^2) \cdot s_1 + \sum_{l=1}^{n_\mathrm{R}} c_l \cdot (|H^{(1,l)}|^2 + |H^{(2,l)}|^2) \cdot \tilde{\mathrm{n}}_{1,l}. \tag{2.23}$$

As shown in [Kam04], the MRC coefficient which maximizes the SNR when there are different noise powers $\sigma^2_{\tilde{\mathrm{n}}_l}$ is given by

$$c_l = \frac{(|H^{(1,l)}|^2 + |H^{(2,l)}|^2)^\star}{\sigma^2_{\tilde{\mathrm{n}}_l}}. \tag{2.24}$$

Inserting (2.21) in (2.24) leads to

$$c_l = \frac{(|H^{(1,l)}|^2 + |H^{(2,l)}|^2)^\star}{\sigma^2_n \cdot (|H^{(1,l)}|^2 + |H^{(2,l)}|^2)}. \tag{2.25}$$

Since $(|H^{(1,l)}|^2 + |H^{(2,l)}|^2)$ is always positive real-valued, c_l is given by

$$c_l = \frac{1}{\sigma_\mathrm{n}^2} \tag{2.26}$$

which means c_l is always a constant factor. Without loss of generality, c_l can be set to $c_l = 1$ as a constant factor does not effect the SNR leading to

$$\mathbf{y} = \sum_{l=1}^{n_\mathrm{R}} \mathbf{z}_l = \underbrace{\left[\sum_{l=1}^{n_\mathrm{R}} \mathbf{\Omega}_l\right]}_{\mathbf{\Omega}_\mathrm{MRC}} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underbrace{\sum_{l=1}^{n_\mathrm{R}} \tilde{\mathrm{n}}_l}_{\tilde{\mathrm{n}}_\mathrm{MRC}} \tag{2.27}$$

with

$$\mathbf{\Omega}_\mathrm{MRC} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} \sum_{l=1}^{n_\mathrm{R}}(|H^{(1,l)}|^2 + |H^{(2,l)}|^2) & 0 \\ 0 & \sum_{l=1}^{n_\mathrm{R}}(|H^{(1,l)}|^2 + |H^{(2,l)}|^2) \end{pmatrix} \tag{2.28}$$

and the noise vector $\tilde{\mathrm{n}}_\mathrm{MRC}$ whose variance $\sigma^2_{\tilde{\mathrm{n}}_\mathrm{MRC}}$ is given by

$$\begin{aligned} \sigma^2_{\tilde{\mathrm{n}}_\mathrm{MRC}} = E\left\{\tilde{\mathrm{n}}_\mathrm{MRC}^\mathrm{H}\tilde{\mathrm{n}}_\mathrm{MRC}\right\} &= E\left\{\left(\sum_{l=1}^{n_\mathrm{R}} \tilde{\mathrm{n}}_l\right)^\mathrm{H} \cdot \left(\sum_{l=1}^{n_\mathrm{R}} \tilde{\mathrm{n}}_l\right)\right\} \\ &= \sum_{l=1}^{n_\mathrm{R}} E\left\{\tilde{\mathrm{n}}_l^\mathrm{H}\tilde{\mathrm{n}}_l\right\} \\ &= \sigma_n^2 \cdot \sum_{l=1}^{n_\mathrm{R}}(|H^{(1,l)}|^2 + |H^{(2,l)}|^2). \end{aligned} \tag{2.29}$$

Analogue to the single receive antenna case, $\mathbf{\Omega}_\mathrm{MRC}$ is a diagonal matrix, i.e., the two data symbols s_1 and s_2 are decoupled as well. For the more general case of an OSTBC with n_T transmit antennas, the diagonal elements Ω_MRC of $\mathbf{\Omega}_\mathrm{MRC}$ are given by

$$\Omega_\mathrm{MRC} = \frac{1}{\sqrt{n_\mathrm{T}}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2 \tag{2.30}$$

and the variance of the noise vector $\tilde{\mathrm{n}}_\mathrm{MRC}$ is given by

$$\sigma^2_{\tilde{\mathrm{n}}_\mathrm{MRC}} = \sigma_n^2 \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2. \tag{2.31}$$

From (2.30), one can see that additional spatial diversity is exploited, since in total $n_\mathrm{T} \cdot n_\mathrm{R}$ different replicas of the transmitted data symbols are provided to the receiver. Further on, the receive energy of the n_R different receive antennas is collected at the combiner leading to a gain in SNR (also called array gain) compared to the case with just one receive antenna.

2.5.3 Transmit Antenna Selection in combination with Maximum Ratio Combining (TAS-MRC)

In this section, the principles of TAS in combination with MRC are presented. Having n_{T} transmit antennas, one selects the transmit antenna which provides the highest SNR at the receiver for transmission. Hence, TAS requires information about the channel quality of different transmit antennas. This channel quality can be either determined at the transmitter side, e.g., in a TDD system exploiting the reciprocity of the channel, or at the receiver side. In this case, the information has to be fed back to the transmitter as in an FDD system where the channel reciprocity cannot be exploited. Note that in case of a TDD system, the transmit antenna selection is performed at the BS by selecting the best transmit antenna based on SNR values. In case of an FDD system, there are two possibilities. In the first case, the MSs feed back the antenna index of the best antenna in addition to the SNR value of the best antenna, i.e., the transmit antenna selection is performed at the MSs. In the following, this TAS scheme is referred to as Transmit Antenna Selection - Feedback Best (TAS-FB). In the second case, the MSs feed back the SNR values for *all* transmit antennas so that the BS can select the best transmit antenna referred to as Transmit Antenna Selection - Feedback All (TAS-FA). Assuming that transmit antenna i^{+} was chosen for transmission, the receive signal r_l at receive antenna l with $l = 1, .., n_{\mathrm{R}}$ is given by

$$r_l = H^{(i^+,l)} \cdot s + n_l \tag{2.32}$$

with s the transmitted data symbol and n_l AWGN value with noise power σ_{n}^2. According to [Kam04], the MRC coefficient c_l is then given by

$$c_l = H^{(i^+,l)\star} \tag{2.33}$$

resulting in

$$y \;=\; \sum_{l=1}^{n_r} c_l \cdot r_l = \left[\sum_{l=1}^{n_{\mathrm{R}}} |H^{(i^+,l)}|^2\right] \cdot s + \underbrace{\sum_{l=1}^{n_r} H^{(i^+,l)\star} \cdot n_l}_{\tilde{\mathrm{n}}_l}\,. \tag{2.34}$$

with the noise $\tilde{\mathrm{n}}_l$ whose variance $\sigma_{\tilde{\mathrm{n}}}^2$ is given by

$$\begin{aligned} \sigma_{\tilde{\mathrm{n}}}^2 = E\left\{n_l^H n_l\right\} &= E\left\{\left(\sum_{l=1}^{n_{\mathrm{R}}} H^{(i^+,l)\star} \cdot n_l\right)^H \cdot \left(\sum_{l=1}^{n_{\mathrm{R}}} H^{(i^+,l)\star} \cdot n_l\right)\right\} \\ &= \sigma_n^2 \cdot \sum_{l=1}^{n_{\mathrm{R}}} |H^{(i^+,l)}|^2. \end{aligned} \tag{2.35}$$

2.6 Different user channel access demands in multi-user OFDMA schemes

In multi-user communication systems, not every user has the same requirement in terms of data rate or channel access, respectively. There are users with high demands for example for video conferencing, online gaming or other applications which require high data rates and other users with only low data rate requirements such as voice transmission. One could also think of a system where resources are allocated to the different users according to their mobile phone contract, e.g., premium costumers which pay more money have a higher priority concerning channel access. Hence, it is reasonable to allocate the available resources according to the demands of the different users. For that purpose, the channel access demand vector is introduced. Furthermore, the number of supportable user demand realizations is analyzed. Note that in this work, it is assumed that the delay requirements are fulfilled as the number of resource units is much larger than the number of users, i.e., in each time frame k at least one resource unit is allocated to each user.

In the following, it is assumed that each user u has an individual channel demand D_u with D_u an integer number and $D_u \geq 1$, i.e., at least one resource unit is allocated to each user. From this, it follows that D_u is upper bounded by $D_{\max}$ with $D_{\max} = N_{\mathrm{ru}} - (U-1)$ assuming the extreme case where $U-1$ resource units are allocated $U-1$ users while the remaining resource units are allocated to only one user. Hence, the resulting demand vector is given by

$$\mathbf{D} = [D_1, D_2, .., D_U] \tag{2.36}$$

with $1 \leq D_u \leq D_{\max}$ where

$$\sum_{u=1}^{U} D_u = N_{\mathrm{ru}}. \tag{2.37}$$

In case that the user demands exceed the available number of resource units, the BS appoints the granted demand of each user such that (2.37) is fulfilled.

Users which have the same channel access demand D_u are arranged into demand groups $\mathcal{G}_i$ with $i = 1, .., G$ where G denotes the number of demand groups. As shown in Appendix A.3, G is upper bounded by

$$G_{\max} = \min \left\{ U, \left\lfloor \frac{1}{2} \cdot \left(1 + \sqrt{1 + 8 \cdot (N_{\mathrm{ru}} - U)} \right) \right\rfloor \right\}. \tag{2.38}$$

To clarify the definition of demand groups, a simple example is presented. Let's assume there are $U = 5$ users in a system with $N_{\mathrm{ru}} = 10$ resource units. Hence, $G_{\max} = 3$, i.e., there exist no user demand vector which contains more than three different demand values while fulfilling (2.37). In this example, the user demand vector shall be given by

$$\mathbf{D} = [4, 2, 2, 1, 1].$$

Thus, there are $G = 3$ different demand groups $\mathcal{G}_i$ with $i = 1, .., 3$ which are given by

$$\begin{aligned} \mathcal{G}_1 &= \{1\} \\ \mathcal{G}_2 &= \{2, 3\} \\ \mathcal{G}_3 &= \{4, 5\}. \end{aligned}$$

Next, it is analyzed how many supportable realizations of $\mathbf{D}$ exist for a given number N_{ru} of resource units and number U of users assuming a fully loaded system. This number is important for the providers in order to design the system parameters such that a variety of demand vectors is supportable in order to be flexible fulfilling different user demands.

The number of possible demand vector realizations disregarding order is equivalent to the number of partitions of the integer number N_{ru} into U positive non-zero summands. From number theory, it is known that the intermediate partition function $p(\eta, \kappa)$ [Coh78] represents the number of partitions of η into κ summands which can only be written in recursive form

$$\begin{aligned} p(\eta, \kappa) &= p(\eta - 1, \kappa - 1) + p(\eta - \kappa, \kappa) \\ \text{with } & p(0, 0) = 1, \\ & p(\eta, \kappa) = 0 \text{ for } \kappa = 0,\ \eta > 0 \text{ or } \eta < \kappa \end{aligned} \tag{2.39}$$

Hence, in a fully loaded system with N_{ru} resource units and U users, assuming that at least one resource unit is allocated to each user, there exist

$$Z = p(N_{\mathrm{ru}}, U) \tag{2.40}$$

possible demand vector realizations disregarding order with integer number of allocated resources.

2.7 Non-adaptive multi-user OFDMA transmission mode

2.7.1 Introduction

In the following, the non-adaptive multi-user OFDMA transmission mode is introduced. The non-adaptive transmission modes is characterized by the fact that the transmitter does not require any instantaneous CSI. On that account, an optimal adaptation to the current channel condition is not possible leading to inferior performances compared to adaptive transmission modes [WIN06]. However, by exploiting frequency diversity in combination with spatial diversity using multiple transmit and receive antennas, the reliability of the transmission can be improved making non-adaptive transmission schemes good candidates in systems without instantaneous T-CSI. In the following, Discrete Fourier Transform (DFT)-precoded OFDMA applying OSTBC and MRC is employed as non-adaptive transmission technique which exploits frequency and spatial diversity [Fra10].

The non-adaptive transmission mode is introduced in Section 2.7.2. In order to analytically describe the performance of the non-adaptive transmission mode, the resulting SNR at the receiver is derived in Section 2.7.3. Furthermore, the applied scheduling and modulation of the non-adaptive transmission mode are described in Section 2.7.4 and 2.7.5.

For detailed information regarding the implementation of DFT-precoded OFDMA in combination with OSTBC at the transmitter and MRC and the receiver, the reader is referred to [FKCK06] and [Fra10].

2.7.2 Transmission scheme

In general, applying the non-adaptive DFT-precoded OFDMA transmission mode, each user is allocated to D_u resource units according to the user demand vector $\mathbf{D}$. In contrast to conventional OFDMA, the data symbols of each user are DFT-precoded before transmission. Thus, each subcarrier carries a DFT element, i.e., a weighted sum of all data symbols. Hence, the application of the non-adaptive transmission mode leads to an averaging over the frequency variations of the channel, i.e., frequency diversity is exploited [Fra10]. By doing so, the data of user u is spread over the bandwidth covered by the D_u resource units. In case of deep fading on one of the subcarriers the data on this subcarrier is not necessarily lost but can possibly be recovered by the IDFT at the receiver since all other subcarriers allocated to user u carry parts of the data. To further improve the reliability of the transmission, OSTBC at the transmitter and MRC at the receiver is performed leading to an additional exploitation of spatial diversity. Note that also OSTBC does not require any instantaneous CSI at the transmitter.

Fig. 2.3 shows the transmission chain of the non-adaptive OFDMA transmission mode.

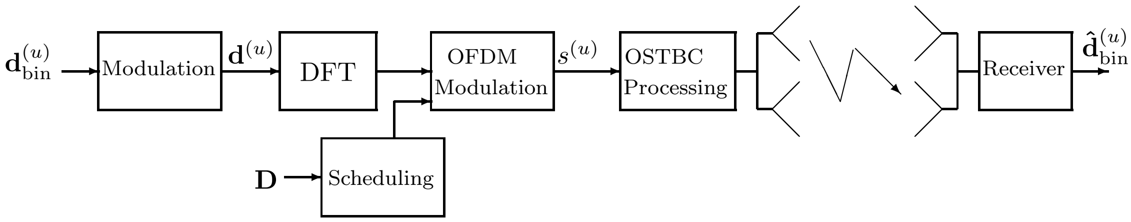


Figure 2.3. Transmission chain of non-adaptive OFDMA transmission mode

First, the binary data $\mathbf{d}_{\text{bin}}^{(u)}$ of user u is mapped on data symbols $\mathbf{d}^{(u)}$. Further on, these data symbols are DFT pre-coded before they are OFDM modulated according to the scheduling and the channel access user demands $\mathbf{D}$ resulting in the time domain signal $s^{(u)}$. Finally, OSTBC is applied at the transmit antennas. At the receiver, MRC is performed. Moreover, the impact of the channel is inverted as well as the Space-Time Coding, the OFDM modulation and the DFT precoding. Performing data estimation results in the estimated binary data $\hat{\mathbf{d}}_{\text{bin}}^{(u)}$ of user u.

2.7.3 Resulting SNR at the receiver

In this section, the derivation of the resulting SNR after the IDFT operation at the receiver which inverts the DFT precoding at the transmitter is presented. As derived in Section 2.5, applying OSTBC with n_{T} transmit antennas at the transmitter leads to an averaging over the n_{T} different SNR conditions of the subcarriers of a resource unit allocated to a given user at each receive antenna. With the application of MRC with n_{R} receive antennas at the receiver, these resulting SNRs are then superimposed, i.e., the SNR at the output of the MRC is a superposition of the SNR values at each receive antenna. Now, the effect of the IDFT operation performed at the receiver on the post-MRC SNR is discussed. To simplify the derivation, only one user is considered, and therefore, the user index u is omitted. Furthermore, these considerations are valid for each time frame, i.e., also the time frame index k is omitted. Finally, it is assumed that one resource unit consists of just one subcarrier in one OFDM symbol without loss of generality, since the channel within one resource unit is assumed to be constant. Thus, the channel transfer function of the channel from transmit antenna i to receive antenna j of resource unit n is given by $H^{(i,j)}(n)$. It is assumed that Q data symbols form a data vector $\mathbf{d}$. Applying DFT-precoded OFDM, data vector $\mathbf{d}$ is spread over Q

different resource units. With the $Q \times Q$ diagonally matrix

$$\mathbf{\Omega} = \frac{1}{\sqrt{n_\mathrm{T}}} \cdot \begin{pmatrix} \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(1)|^2 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \dots & \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(Q)|^2 \end{pmatrix}, \tag{2.41}$$

the $Q \times Q$ DFT matrix $\mathbf{F}_Q$ and the colored noise vector $\mathbf{v}$ where the q-th element v_q of $\mathbf{v}$ with $q = 1, .., Q$ has the variance

$$\sigma_{v_q}^2 = \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(q)|^2 \cdot \sigma_\mathrm{n}^2, \tag{2.42}$$

it was shown in [Fra10] that the receive signal vector $\mathbf{r}$ for such a DFT-precoded OFDM system which applies OSTBC in combination with MRC is given by

$$\mathbf{r} = \mathbf{\Omega} \cdot \mathbf{F}_Q \cdot \mathbf{d} + \mathbf{v}. \tag{2.43}$$

If Zero Forcing is applied for equalization, $\mathbf{r}$ is multiplied by the equalizer matrix $\mathbf{E} = \mathbf{\Omega}^{-1}$ followed by a multiplication with an IDFT matrix $\mathbf{F}_Q^H$ leading to the estimated data vector

$$\begin{aligned} \widehat{\mathbf{d}} &= \mathbf{F}_Q^H \cdot \mathbf{E} \cdot \mathbf{r} \\ &= \mathbf{F}_Q^H \cdot \mathbf{E} \cdot \mathbf{\Omega} \cdot \mathbf{F}_Q \cdot \mathbf{d} + \mathbf{F}_Q^H \cdot \underbrace{\mathbf{E} \cdot \mathbf{v}}_{\tilde{\mathbf{v}}} \\ &= \mathbf{d} + \underbrace{\mathbf{F}_Q^H \cdot \tilde{\mathbf{v}}}_{\mathbf{w}}. \end{aligned} \tag{2.44}$$

The variance of the noise vector $\tilde{\mathbf{v}}$ becomes

$$\sigma_{\tilde{v}}^2 = E\left\{\tilde{\mathbf{v}}^H \tilde{\mathbf{v}}\right\} = E\left\{\mathbf{v}^H \mathbf{E}\mathbf{E}^H \mathbf{v}\right\} = |\mathbf{E}|^2 \cdot E\left\{\mathbf{v}^H \mathbf{v}\right\}. \tag{2.45}$$

Thus, the variance of the q-th element of $\tilde{\mathbf{v}}$ is given by

$$\begin{aligned} \sigma_{\tilde{v}_q}^2 &= \sigma_{v_q}^2 \cdot \left[\frac{1}{\sqrt{n_\mathrm{T}}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(q)|^2\right]^{-2} \\ &= \sigma_\mathrm{n}^2 \cdot \left[\frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(q)|^2\right]^{-1}, \end{aligned} \tag{2.46}$$

inserting (2.42). To determine the variance of noise vector $\mathbf{w}$, it is enough to consider the q-th element w_q. In the following, the IDFT operation of matrix $\mathbf{F}_Q^H$ has to be considered. With the definition of the IDFT

$$x(\eta) = \frac{1}{\sqrt{Q}} \sum_{\kappa=1}^{Q} X(\kappa) \cdot \underbrace{\exp(j2\pi(\kappa-1)(\eta-1)/Q)}_{a_{\kappa,\eta}} \tag{2.47}$$

and the IDFT coefficient $a_{\kappa,\eta}$, w_q can be written as a function of $\tilde{\mathbf{v}}$ given by

$$w_q = \sum_{\kappa=1}^{Q} \frac{a_{\kappa,q}}{\sqrt{Q}} \cdot \tilde{v}_\kappa, \tag{2.48}$$

i.e., w_q is a weighted sum of Gaussian distributed random variables. Hence, the variance of w_q is determined by

$$\mathrm{Var}\{w_q\} = \sum_{\kappa=1}^{Q} \mathrm{Var}\left\{\frac{a_{\kappa,q}}{\sqrt{Q}} \cdot \tilde{v}_\kappa\right\} = \sigma_\mathrm{n}^2 \cdot \frac{1}{Q} \cdot \sum_{\kappa=1}^{Q} \left[\frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(\kappa)|^2\right]^{-1} \tag{2.49}$$

Hence, according to (2.44), the resulting SNR γ_IDFT after the IDFT is calculated by

$$\gamma_\mathrm{IDFT} = \frac{1}{\sigma_\mathrm{n}^2 \cdot \frac{1}{Q} \cdot \sum_{q=1}^{Q} \left[\frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}(q)|^2\right]^{-1}}. \tag{2.50}$$

With $\bar{\gamma} = \frac{1}{\sigma_\mathrm{n}^2}$ and $\gamma^{(i,l)}(q) = \bar{\gamma} \cdot |H^{(i,l)}(q)|^2$, (2.50) can be written as

$$\gamma_\mathrm{IDFT} = \frac{1}{\frac{1}{Q} \cdot \sum_{q=1}^{Q} \left[\frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} \gamma^{(i,l)}(q)\right]^{-1}}. \tag{2.51}$$

Thus, the application of a DFT precoding leads to an averaging over the Q reciprocal values of the resulting SNR values $\frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} \gamma^{(i,l)}(q)$ with $q = 1, .., Q$ obtained from OSTBC and MRC followed by an additional inversion. In general, the resulting SNR $\gamma_{\mathrm{IDFT},u}(k)$ of user u in time frame k whose data is spread over D_u resource units applying DFT-precoded OFDMA is given by

$$\gamma_{\mathrm{IDFT},u}(k) = \frac{1}{\frac{1}{D_u} \cdot \sum_{q=1}^{D_u} \left[\frac{1}{n_\mathrm{T}} \cdot \sum_{i'=1}^{n_\mathrm{T} \cdot n_\mathrm{R}} \gamma_u^{(i')}(q,k)\right]^{-1}}. \tag{2.52}$$

with $i' = 1, .., n_\mathrm{T} \cdot n_\mathrm{R}$ and $\gamma_u^{(i')}(q,k) = \mathrm{vec}\{\gamma_u^{(i,j)}(q,k)\}$ where the operation vec{} stacks the columns of a matrix on top of each other to form a vector.

2.7.4 Scheduling

Since the transmitter does not have any instantaneous information about the channel conditions of different users, scheduling has to be done non-adaptively fulfilling the channel access demands $\mathbf{D}$ of the different users. To do so, the scheduler follows a a round robin policy, i.e., the first D_1 resource units are allocated to user 1, the next D_2 resource units are allocated to user 2, and so on, as done in Localized Frequency Division Multiple Access (LFDMA) which is also known under the name of localized Single Carrier Frequency Division Multiple Access (SC-FDMA) [3GP06]. By doing so, it can be guaranteed that each of the $Z = p(N_{\mathrm{ru}}, U)$ possible user demand vectors can be realized. In the literature, there also exists other allocation pattern, e.g., an interleaved or block-interleaved resource allocation which equidistantly distributes the subcarriers over the total bandwidth where D_u needs to be an integer divisor multiple of the number N_{ru} of available subcarriers [FKCS05]. However, these allocation patterns are characterized by a limited flexibility concerning possible user demand vector realizations. Fig. 2.4 illustrates this for a system with $N_{\mathrm{ru}} = 8$ available subcarriers and $U = 4$ users.

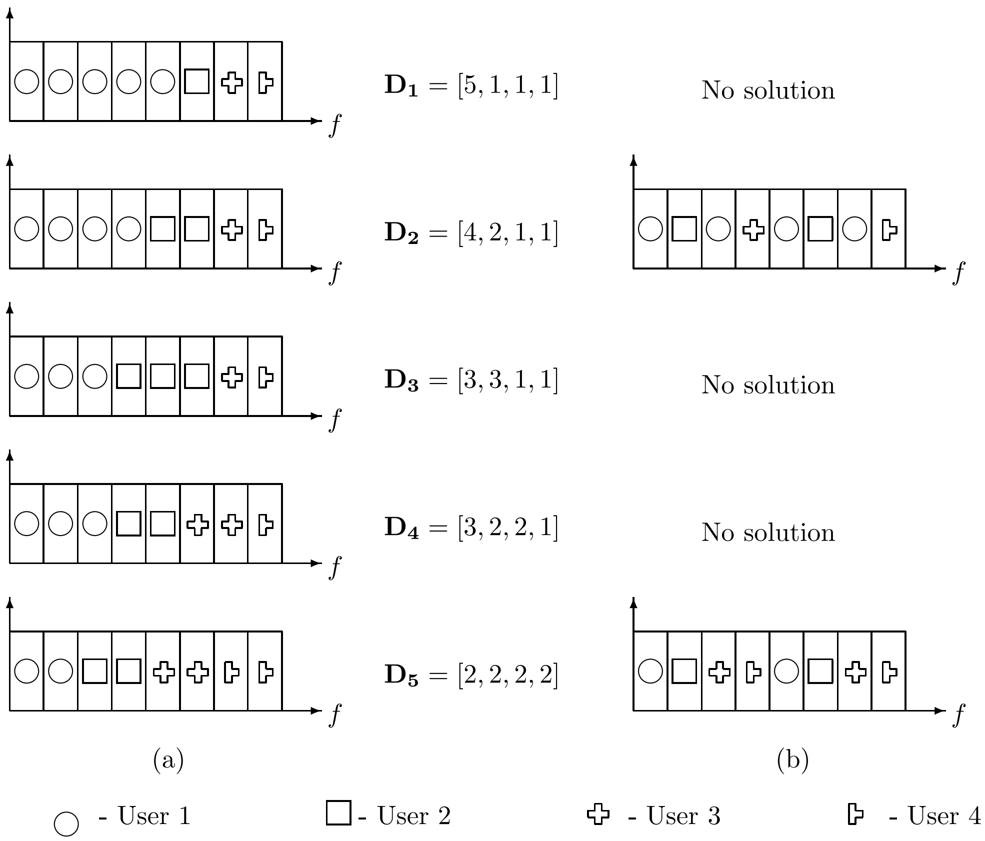


Figure 2.4. (a) Round robin subcarrier allocation and (b) interleaved subcarrier allocation considering different user demand vectors $\mathbf{D_i}$ with $i = 1, .., 5$

According to (2.40), there are $Z = p(8, 4) = 5$ supportable user demand vectors $\mathbf{D}$ disregarding order which are $\mathbf{D_1} = [5, 1, 1, 1]$, $\mathbf{D_2} = [4, 2, 1, 1]$, $\mathbf{D_3} = [3, 3, 1, 1]$, $\mathbf{D_4} = [3, 2, 2, 1]$ and $\mathbf{D_5} = [2, 2, 2, 2]$ with the maximum possible number $G_{\max}$ of different user demand groups $G_{\max} = \min\{4, \lfloor \frac{1}{2}(1 + \sqrt{33}) \rfloor\} = 3$ in $\mathbf{D_2}$ and $\mathbf{D_4}$. For all user demand vectors, the resource allocation of the considered round robin scheduler are depicted in Fig. 2.4(a). In case of an interleaved resource allocation with equidistant subcarrier spacing for each user, the resource allocations of the supportable user demand vectors are depicted in Fig. 2.4(b). As one can see, there are only two user demand vectors, $\mathbf{D_2} = [4, 2, 1, 1]$ and $\mathbf{D_5} = [2, 2, 2, 2]$, which fulfill the requirements.

2.7.5 Modulation

Assuming that the average SNR $\bar{\gamma}_u$ of each user is known to the BS, one fixed modulation scheme is selected for all resource units of one user, i.e., all subcarriers are allocated to one user apply the same modulation scheme. Thus, the modulation is only adapted to the pathloss and not to the fast fading. In this work, uncoded M-ary Quadrature Amplitude Modulation (M-QAM) and M-ary Phase Shift Keying (M-PSK) are considered.

2.8 Adaptive multi-user OFDMA transmission mode

2.8.1 Introduction

In the following section, the adaptive multi-user OFDMA transmission mode is introduced which exploits multi-user diversity, i.e., the resource units are only allocated to users which are in good channel conditions. Instead of combating the channel variations by applying some sort of averaging transmission scheme to exploit diversity in the time, frequency or spatial dimension, the variations in the channel of different users are capitalized to transmit data only on the strongest channels [OR05]. This leads to a superior performance compared to non-adaptive OFDMA schemes. However, the exploitation of multi-user diversity requires accurate channel knowledge at the transmitter to identify the channels of the best users. Because of this, adaptive transmission schemes are prone to imperfect channel knowledge. Hence, the application of adaptive transmission schemes is only reasonable in scenarios which allow the provision of accurate channel knowledge with feasible effort, e.g., in scenarios with slowly changing channel conditions. In the following, an adaptive multi-user transmission mode is introduced where the resource units are adaptively allocated to the users taking into account the current SNR conditions of the resource units and the different user demands applying a Weighted Proportional Fair Scheduling (WPFS) approach. As antenna techniques either OSTBC or Transmit Antenna Selection (TAS) is performed at the transmitter and MRC is performed at the receiver.

Section 2.8.2 presents an overview of the transmission chain of the adaptive scheme. For both multiple antenna techniques, the resulting SNR at the output of the maximum ratio combiner is derived in Section 2.8.3. Furthermore, Section 2.8.4 introduces WPFS applying continuous and quantized SNR values. Finally, Section 2.8.5 introduces the adaptive modulation.

2.8.2 Transmission scheme

Fig. 2.5 shows the transmission chain of the adaptive OFDMA transmission mode.

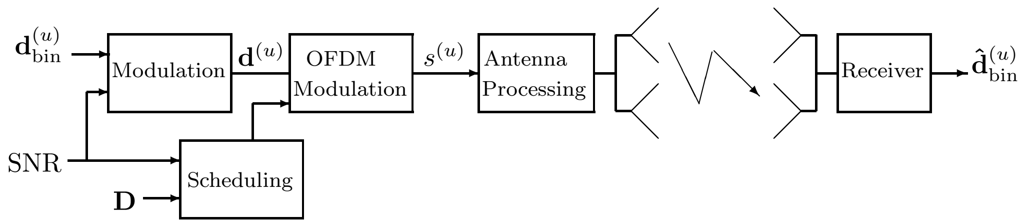


Figure 2.5. Transmission chain of adaptive OFDMA transmission mode

First, the binary data $\mathbf{d}_{\text{bin}}^{(u)}$ of user u is mapped on data symbols $\mathbf{d}^{(u)}$ taking into account the instantaneous SNR values of the resource units, i.e., depending on the current channel conditions, the applied modulation scheme is adapted. The higher the SNR, the higher the number of bits per data symbol. In contrast to the non-adaptive scheme, these data symbols are directly OFDM modulated according to the scheduling which depends on the channel access user demands $\mathbf{D}$ and the SNR values resulting in the time domain signal $s^{(u)}$. Finally, either OSTBC or TAS is applied at the transmit antennas. Similar to the non-adaptive transmission scheme, MRC is applied at the receiver followed by the inversion of the channel, the Space-Time Coding and the OFDM modulation.

2.8.3 Resulting SNR at receiver

In the following, the resulting SNR at the output of the combiner at the receiver is derived applying OSTBC-MRC and TAS-MRC. This resulting SNR can also be interpreted as the SNR of an equivalent Single Input Single Output (SISO) system. First, OSTBC at the transmitter using n_T transmit antennas and MRC using n_R receive antennas at each receiver is considered. Without loss of generality, only the first element of the receiving vector $\mathbf{y}$ is considered which according to (2.30) is given by

$$y_1 = \frac{1}{\sqrt{n_\mathrm{T}}} \left(\sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2 \right) \cdot s_1 + \tilde{\mathrm{n}}_\mathrm{MRC} \tag{2.53}$$

with $\sigma^2_{\tilde{\mathrm{n}}_\mathrm{MRC}} = \sigma_n^2 \cdot \left(\sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2 \right)$. Hence, the SNR γ at the output of the combiner is calculated by

$$\gamma = \frac{\frac{1}{n_\mathrm{T}} \cdot \left(\sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2 \right)^2}{\left(\sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} |H^{(i,l)}|^2 \right) \cdot \sigma_n^2}, \tag{2.54}$$

keeping in mind that the signal power is normalized to one as stated in Section 2.4. With $\bar{\gamma} = \frac{1}{\sigma_\mathrm{n}^2}$ denoting the average SNR and $\gamma^{(i,l)} = \bar{\gamma} \cdot |H^{(i,l)}|^2$ as shown in Section 2.4, γ is given by

$$\gamma = \frac{1}{n_\mathrm{T}} \cdot \sum_{i=1}^{n_\mathrm{T}} \sum_{l=1}^{n_\mathrm{R}} \gamma^{(i,l)}. \tag{2.55}$$

Hence, the resulting SNR of the equivalent SISO system in time frame k of resource unit n of user u is given by

$$\gamma_u(n,k) = \frac{1}{n_\mathrm{T}} \sum_{i=1}^{n_\mathrm{T}} \sum_{j=1}^{n_\mathrm{R}} \gamma_u^{(i,j)}(n,k). \tag{2.56}$$

which can be simplified to

$$\gamma_u(n,k) = \frac{1}{n_\mathrm{T}} \sum_{i'=1}^{n_\mathrm{T} \cdot n_\mathrm{R}} \gamma_u^{(i')}(n,k) \tag{2.57}$$

with $i' = 1, .., n_\mathrm{T} \cdot n_\mathrm{R}$ and $\gamma_u^{(i')}(n,k) = \mathrm{vec}\{\gamma_u^{(i,j)}(n,k)\}$ where the operation vec$\{\}$ stacks the columns of a matrix on top of each other to form a vector.

Second, TAS in combination with MRC at each receiver is considered. According to (2.34) and (2.35), the SNR γ at the at the output of the combiner is given by

$$\gamma = \frac{\left(\sum_{l=1}^{n_\mathrm{R}} |H^{(i^+,l)}|^2 \right)^2}{\left(\sum_{l=1}^{n_\mathrm{R}} |H^{(i^+,l)}|^2 \right) \cdot \sigma_n^2} \tag{2.58}$$

when transmit antenna i^+ is used for transmission. (2.58) can be rewritten to

$$\gamma = \sum_{l=1}^{n_\mathrm{R}} \gamma^{(i^+,l)}. \tag{2.59}$$

Since the best transmit antenna shall be selected for transmission, the resulting SNR of the equivalent SISO system applying TAS in time frame k of resource unit n of user u is given by

$$\gamma_u(n,k) = \max_i \sum_{j=1}^{n_\mathrm{R}} \gamma_u^{(i,j)}(n,k). \tag{2.60}$$

2.8.4 Scheduling

2.8.4.1 Scheduling algorithms

In the literature, there exits several scheduling algorithms with different objectives [Hah91, LBS99, CL01, Kol03, Hol01, LZ06, FKWD06, FKWD07, Fer10]. In general, scheduling algorithms are methods to share the available resources among different users. Depending on the scheduling algorithm, knowledge of the actual channel conditions and/or the throughput of different users are required. Scheduling algorithms always have to deal with a trade off between cell throughput and fairness. On the one hand, serving the users with the best channel conditions maximizes the cell throughput. On the other hand, each user wants to achieve at least a given minimum data rate. In the literature, there are four major strategies of adaptive scheduling approaches which are shortly summarized. The first strategy is so called Fair Resource Scheduling (FRS). With FRS, the available resources are allocated in equal share to the users, leading to a higher throughput for users in favorable channel conditions. One simple example of FRS is the Round Robin Scheduler which allocates resource to the users in a cyclic order without taking into account any channel knowledge [Hah91, LBS99]. The second strategy is so called Fair Throughput Scheduling (FTS) which aims at giving all users the same amount of throughput [Fer10]. This aim is achieved by giving more resources to users with bad channel conditions. Therefore, the scheduler requires knowledge of the average achieved throughput for each user. The third strategy is so called Proportional Fair Scheduling (PFS), which aims at increasing the cell throughput by considering channel conditions of the users while preserving a certain amount of fairness [Hol01, Kol03, FKWD07]. There are two PFS approaches. With PFS-SNR, a resource is allocated to the user with the highest ratio of current SNR to its average SNR. With PFS-TP, a resource is allocated to the user with the highest ratio of current achievable throughput to its average throughput. The first approach aims at scheduling the user if the channel conditions are good compared to the average conditions, which leads to a raise of the SNR in the cell since only users with good channel conditions are scheduled. The second approach considers the average throughput but also the actual achievable throughput, i.e., in contrast to the FTS algorithm, not the user with the lowest throughput is scheduled but the user with the best ratio between current and average throughput. For both approaches, the actual channel conditions have to be known by the scheduler. The last strategy is so called Max-SNR Scheduling which aims at maximizing the cell throughput by scheduling the user with the highest SNR [LZ06]. Max SNR Scheduling provides the highest cell throughput at the expense of fairness since users with bad SNR are hardly scheduled. In terms of throughput and fairness, FRS, FTS, PFS-SNR and PFS-TP provide a good trade off compared to Max-SNR Scheduling. Nevertheless, only PFS-SNR is employed as scheduling algorithm for the considered adaptive transmission scheme due to the fact that it provides a comparable throughput-fairness trade off as FRS, FTS and PFS-TP while having a much simpler resource-wise scheduling rule compared to the more complex FRS, FTS and PFS-TP algorithms. This also facilitate the mathematical traceability for analytical investigations. Further on, different user demands can easily be incorporated introducing a weighting factor resulting in Weighted Proportional Fair Scheduling (WPFS).

2.8.4.2 Weighted Proportional Fair Scheduling

In this section, WPFS applying continuous SNR values like they appear in TDD systems is presented. As stated before, WPFS requires information about the actual channel conditions, more precisely information about the SNR of the different resource units of different users. In a TDD system, the reciprocity of the up- and downlink channel can be exploited, i.e., the BS just has to measure the SNR $\gamma_u^{(i,j)}(n,k)$ of the channel from transmit antenna i to receive antenna j of user u in resource unit n in time frame k during an initial pilot phase in the uplink to get the SNR values for the downlink. With (2.57) and (2.60), the resulting SNR $\gamma_u(n,k)$ of the equivalent SISO channel can be calculated. In order to incorporate different user demands, a user specific weighting factor p_u with $p_u \leq 1 \; \forall \; u \in \{1,..,U\}$ is introduced. Based on that, the subcarriers of resource unit n in time frame k are allocated to the user $u^\star(n,k)$ with the highest ratio between the weighted instantaneous SNR and the average SNR $\bar{\gamma}_u$, leading to

$$u^\star(n,k) = \arg\max_u \left\{ \frac{p_u \cdot \gamma_u(n,k)}{\bar{\gamma}_u} \right\}. \tag{2.61}$$

By doing so, each resource unit is allocated to one user exclusively. The weighting can be interpreted as a virtual SNR boost, i.e., the higher the weighting factor, the higher the probability of getting access to the channel. In case that $p_u = 1 \; \forall \; u \in \{1,..,U\}$, all user have the same channel access probability as with conventional PFS.

2.8.4.3 Quantized Weighted Proportional Fair Scheduling

In this section, WPFS applying quantized SNR values like they appear in FDD systems is presented. In FDD systems, the uplink and downlink channels are different, i.e., it is not possible for the BS to measure the downlink channel during uplink. To overcome this problem, the MSs have to measure and calculate the resulting SNR $\gamma_u(n,k)$ of the equivalent SISO channel during the downlink phase. Furthermore, the MSs have to normalize the resulting SNR $\gamma_u(n,k)$ to the average SNR $\bar{\gamma}_u$ and feed back the normalized SNR $\frac{\gamma_u(n,k)}{\bar{\gamma}_u}$ in the next uplink phase plus the additional antenna index of the best transmit antenna in case of TAS-FB. The SNR values are quantized and digitized with N_Q bits to save uplink bandwidth, resulting in

$$\gamma_u^{\mathrm{q}}(n,k) = \mathcal{Q}_{u,N_Q}\left\{\frac{\gamma_u(n,k)}{\bar{\gamma}_u}\right\} \tag{2.62}$$

where the operation $\mathcal{Q}_{u,N_Q}\{x\}$ returns the quantization level index of x. Now, instead of having continuous SNR values as a TDD system, the signalled SNR values are discrete numbers representing the index of the quantization interval of the measured SNR value at the MS. Hence, the subcarriers of resource unit n in time frame k are allocated to user $u^\star(n,k)$ with the highest weighted normalized and quantized SNR value resulting in the following scheduling rule for Quantized Weighted Proportional Fair Scheduling (QWPFS):

$$u^\star(n,k) = \arg\max_u \left\{p_u \cdot \gamma_u^{\mathrm{q}}(n,k)\right\}. \tag{2.63}$$

In case that several users have the same weighted SNR value, one user is randomly selected.

In the literature, there exist approaches to obtain CSI of the downlink channel in the BS based on CSI of the uplink channel in multiple antenna FDD systems [PW10] to avoid CSI feedback which decreases the spectral efficiency.Note that in this work, only FDD systems which apply CQI feedback are considered.

2.8.5 Adaptive modulation

In the adaptive OFDMA transmission scheme, the modulation scheme is selected for each allocated resource unit based on the actual SNR values, i.e., for each subcarrier inside one resource unit the same modulation scheme is applied where the same transmit power per subcarrier is assumed. By doing so, the modulation is adapted to the pathloss and to the fast fading. In this work, uncoded M-ary Quadrature Amplitude Modulation (M-QAM) and M-ary Phase Shift Keying (M-PSK) are considered.

2.9 Modelling imperfect channel knowledge

2.9.1 Channel Quality Information (CQI)

In this section, the modelling and the parameters describing imperfect channel knowledge are introduced.

As seen in Section 2.8, adaptive transmission schemes require transmitter-sided channel knowledge. In general, channel knowledge at the transmitter is expressed by T-CSI which in the considered case denotes the complex channel transfer function $H_u^{(i,j)}(n,k)$ of the channel from transmit antenna i to receive antenna j of user u of resource unit n in time frame k, i.e., amplitude and phase in the equivalent base band. Another less complex metric is the so called Channel Quality Indicator or Channel Quality Information (CQI). Here, the quality of the channel is indicated only by a scalar value, for example the SNR. T-CSI is mainly required for Multiple Input Multiple Output (MIMO) transmission schemes which perform a precoding to spatially separate the signals intended for different users, so called spatial multiplexing. However, in this work the channel knowledge at the transmitter is only applied for scheduling and modulation scheme selection which can be done based on the instantaneous SNR values $\gamma_u^{(i,j)}(n,k)$, i.e., only CQI is used as channel knowledge. However, in a realistic scenario, the provision of CQI values at the BS can not be assumed to be error-free. In the following, four different sources of error for imperfect channel knowledge are considered:

- Measured CQI values are only estimates with a certain estimation error.
- The available CQI values are outdated due to time delays.
- In case of an FDD system, the CQI values are quantized and digitized before they are fed back to the BS over a feedback channel.
- When detecting the feedback bits at the BS, errors may occur due to a imperfect feedback link.

In the following sections 2.9.2 to 2.9.6, for each of the four sources of error, the model and the parameters describing the CQI imperfectness are presented. It is assumed that the BS is able to measure these parameters, i.e., the impairment parameters are assumed to be perfectly known at the BS. Note that the resource unit and antenna indices n, i and j are omitted for the sake of readability.

2.9.2 Noisy CQI

In a realistic scenario, the channel transfer function has to be measured applying for example Pilot Assisted Channel Estimation (PACE), i.e., the transmitter transmits a sequence of M_P pilot symbols $\mathbf{d}_p = [d_{p,1}, .., d_{p,M_P}]^{\mathrm{T}}$ with $\mathbf{d}_p^H \mathbf{d}_p = M_P$ which are known to the receiver. For user u in time frame k, the receive signal is given by

$$\mathbf{r}_u(k) = H_u(k) \cdot \mathbf{d}_p + \mathbf{n}_u, \tag{2.64}$$

with the additive white Gaussian noise vector $\mathbf{n}_u$ of user u with variance $\sigma^2_{n,u} = \frac{1}{\bar{\gamma}_u}$. Applying the Least Squares (LS) criterion given by

$$\arg \min_{\hat{H}_u(k)} ||\mathbf{r}_u(k) - \hat{H}_u(k) \cdot \mathbf{d}_p||^2, \tag{2.65}$$

the LS solution results in

$$\hat{H}_u(k) = (\mathbf{d}_p^H \mathbf{d}_p)^{-1} \cdot \mathbf{d}_p^{\mathrm{H}} \cdot \mathbf{r} \tag{2.66}$$

which can be written as

$$\begin{aligned} \hat{H}_u(k) &= (\mathbf{d}_p^H \mathbf{d}_p)^{-1} \cdot \mathbf{d}_p^{\mathrm{H}} \cdot (H_u(k) \cdot \mathbf{d}_p + \mathbf{n}_u) \\ &= H_u(k) + \underbrace{(\mathbf{d}_p^{\mathrm{H}} \mathbf{d}_p)^{-1} \cdot \mathbf{d}_p^{\mathrm{H}}}_{\mathbf{a}} \cdot \mathbf{n}_u. \end{aligned} \tag{2.67}$$

From this, it follows that the LS estimator is an unbiased linear estimator since the expectation value of $\hat{H}_u(k)$ is given by

$$\begin{aligned} E\{\hat{H}_u(k)\} &= E\{H_u(k) + \mathbf{a} \cdot \mathbf{n}_u\} \\ &= H_u(k) + \sum_{l=1}^{M_P} a_l \cdot E\{n_{u,l}\} = H_u(k) \end{aligned} \tag{2.68}$$

The variance of $\hat{H}_u(k)$ is calculated by

$$\begin{aligned} \mathrm{Var}\{\hat{H}_u(k)\} &= \mathrm{Var}\{H_u(k) + \mathbf{a} \cdot \mathbf{n}_u\} \\ &= \mathrm{Var}\{H_u(k)\} + \sum_{l=1}^{M_P} a_l^2 \cdot \mathrm{Var}\{n_{u,l}\} \\ &= \mathrm{Var}\{H_u(k)\} + \sigma^2_{\mathrm{n},u} \cdot \left((\mathbf{d}_p^H \mathbf{d}_p)^{-1}\right)^H \\ &= \mathrm{Var}\{H_u(k)\} + \frac{\sigma^2_{\mathrm{n},u}}{M_P}, \end{aligned} \tag{2.69}$$

i.e., the estimator is consistent as the variance of the estimation error converges to zero for increasing M_P [Hän01]. With (2.68) and (2.69), the LS estimate $\hat{H}_u(k)$ can be modeled by

$$\hat{H}_u(k) = H_u(k) + E_u, \tag{2.70}$$

where the estimation error E_u is a complex Gaussian distributed random variable with zero mean and variance $\sigma^2_{\mathrm{E},u}$ given by

$$\sigma^2_{\mathrm{E},u} = \frac{\sigma^2_{\mathrm{n},u}}{M_P} = \frac{1}{\bar{\gamma}_u \cdot M_P}. \tag{2.71}$$

Thus, assuming that the BS is able to perfectly measure the average SNR $\bar{\gamma}_u$ of user u, the error variance $\sigma^2_{\mathrm{E},u}$ can be perfectly determined as M_P is known to the BS.

2.9.3 Outdated CQI

Due to the time delay T between the time instant when measuring the SNR and the actual time of data transmissions, the CQI is outdated. In the following, the correlation coefficient ρ_u between the realization of the actual channel and the outdated channel is introduced as a figure of merit to determine the up-to-datedness of the CQI. From literature, e.g. [WJ94], it is known when the angles of arrival for the different propagation paths are assumed to be uniformly distributed and, thus, the distribution of the Doppler shifts corresponds to a Jake's spectrum, the correlation coefficient ρ_u only depends on the time delay T and the maximum Doppler shift $f_{D,u}$ of user u given by

$$\rho_u = J_0(2\pi f_{D,u} T) \tag{2.72}$$

with $J_0(x)$ denoting the 0th-order Bessel function of the first kind. With the carrier frequency f_0 and the speed of light c, $f_{D,u}$ is given by

$$f_{D,u} = \frac{f_0 \cdot |v_{\mathrm{rad},u}|}{c} \tag{2.73}$$

with $v_{\mathrm{rad},u}$ the radial component of the velocity of user u along a line from the user u to the BS. From this, it follows that the correlation coefficient ρ_u of the channel transfer factor of user u is given by

$$\rho_u = J_0\left(2\pi f_0 T c^{-1} \cdot |v_{\mathrm{rad},u}|\right). \tag{2.74}$$

To determine ρ_u, the BS has to observe and compare the values of the channel transfer function on subcarriers which are allocated to user u over a certain time span. With these values, the covariance and, thus, the correlation coefficient ρ_u can be determined numerically. Also, the MSs could determine ρ_u and then signal the information to the BS.

2.9.4 Outdated and noisy CQI

Since the effects of both time delays and noisy channel estimation are present in a real scenario, a model which combines both effects is presented. To model outdated CQI, a first order Markov model is applied. Thus, the channel $H_u(k-1)$ of user u in time frame $k-1$ is given by

$$H_u(k-1) = \sqrt{\alpha} \cdot H_u(k) + \sqrt{1-\alpha} \cdot X, \tag{2.75}$$

where X is a complex Gaussian distributed random variable with zero mean and variance one and independent from $H_u(k)$. According to [Hän01], the correlation coefficient $\rho(x, y)$ between two random variables x and y is defined by

$$\rho(x, y) = \frac{cov\{x, y\}}{\sqrt{\mathrm{Var}\{x\} \cdot \mathrm{Var}\{y\}}}, \tag{2.76}$$

where $cov(\cdot)$ denotes the covariance of two random variables. Thus, the correlation between $H_u(k)$ and $H_u(k-1)$ is given by

$$\begin{aligned} \rho(H_u(k), H_u(k-1)) &= \frac{cov\{H_u(k), H_u(k-1)\}}{\sqrt{\mathrm{Var}\{H_u(k)\} \cdot \mathrm{Var}\{H_u(k-1)\}}} \\ &= \frac{\sqrt{\alpha} \cdot 1 + \sqrt{1-\alpha} \cdot 0}{\sqrt{\alpha + (1-\alpha)}} = \sqrt{\alpha}. \end{aligned} \tag{2.77}$$

In order to have a correlation coefficient as given in (2.74), the factor α in (2.75) has to be set to

$$\alpha = \rho_u^2. \tag{2.78}$$

Combining (2.75) and (2.70), the relationship between the noisy and outdated channel $\hat{H}_u(k-1)$ and the actual channel $H_u(k)$ is modeled by

$$\begin{aligned} \hat{H}_u(k-1) &= H_u(k-1) + E_u \\ &= \rho_u \cdot H_u(k) + \sqrt{1-\rho_u^2} \cdot X + E_u \end{aligned} \tag{2.79}$$

In the following, the conditional Probability Density Function (PDF) $p_{H_u(k)|\hat{H}_u(k-1)}(H_u(k)|\hat{H}_u(k-1))$ of the actual channel $H_u(k)$ on condition that $\hat{H}_u(k-1)$ is measured is derived. Since the real part and imaginary part of $H_u(k)$ and $\hat{H}_u(k-1)$ are independent and have the same distribution, the derivation of the PDF of $H_u(k)|\hat{H}_u(k-1)$ is only done for the real part, i.e., the imaginary part of $H_u(k)|\hat{H}_u(k-1)$ has the same distribution as the real part. For a better readability, the user and time frame indices u and k will be omitted keeping in mind that $\hat{H}$

denotes the noisy *and* outdated channel. The conditional PDF $p_{H|\hat{H}}(H|\hat{H})$ can be determined applying Bayes' theorem [Hän01] for probability densities given by

$$p_{H|\hat{H}}(H|\hat{H}) = \frac{p_{H,\hat{H}}(H,\hat{H})}{p_{\hat{H}}(\hat{H})}. \tag{2.80}$$

As a first step, the joint PDF $p_{H,\hat{H}}(H,\hat{H})$ is derived by rewriting (2.79) to

$$\begin{pmatrix} H \\ \hat{H} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ \rho & 1 \end{pmatrix}}_{\mathbf{G}} \cdot \begin{pmatrix} X \\ Y \end{pmatrix}, \tag{2.81}$$

where X is $\mathcal{N}(0, \frac{1}{2})$ distributed as introduced in (2.75) and Y is $\mathcal{N}(0, \frac{1}{2}(\sigma_{\mathrm{E}}^2 + 1 - \rho^2))$ distributed as can be seen from (2.79. With the factor $\frac{1}{2}$, the fact that only the real part is considered is taken into account. The joint PDF $p_{H,\hat{H}}(H,\hat{H})$ is then given by

$$p_{H,\hat{H}}(H,\hat{H}) = p_{X,Y}\left(\mathbf{G}^{-1} \cdot \begin{pmatrix} H \\ \hat{H} \end{pmatrix}\right) \cdot |\det(\mathbf{G})^{-1}|. \tag{2.82}$$

Knowing that X and Y are independent and Gaussian distributed, the joint PDF $p_{H,\hat{H}}(H,\hat{H})$ is calculated by

$$\begin{aligned} p_{H,\hat{H}}(H,\hat{H}) &= \frac{1}{2\pi\sigma_x \cdot \sigma_y} \cdot e^{-\frac{H^2}{2\sigma_x^2}} \cdot e^{-\frac{(\hat{H}-\rho H)^2}{2\sigma_y^2}} \\ &= \frac{1}{2\pi\sigma_x \cdot \sigma_y} \cdot e^{-\frac{1}{2}\left(\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2 \cdot \sigma_y^2} \cdot H^2 - \frac{2\sigma_x^2 \rho H \hat{H}}{\sigma_x^2 \cdot \sigma_y^2} + \frac{\sigma_x^2}{\sigma_x^2 \cdot \sigma_y^2}\hat{H}^2\right)} \end{aligned} \tag{2.83}$$

with $\sigma_x^2 = \frac{1}{2}$ and $\sigma_y^2 = \frac{1}{2}(\sigma_E^2 + 1 - \rho^2)$. From (2.75), it is known that $\hat{H}$ is $\mathcal{N}\left(0, \frac{1}{2}(1+\sigma_e^2)\right)$ distributed, i.e.,

$$p_{\hat{H}}(\hat{H}) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{\hat{h}}^2} \cdot e^{-\frac{\hat{h}^2}{2\sigma_{\hat{h}}^2}} \tag{2.84}$$

with $\sigma_{\hat{h}}^2 = \frac{1}{2}(1+\sigma_e^2)$. Inserting (2.83) and (2.84) in (2.80) leads to

$$p_{H|\hat{H}}(H|\hat{H}) = \frac{1}{\sqrt{2\pi} \cdot \sqrt{\frac{\sigma_x^2 \cdot \sigma_y^2}{\sigma_{\hat{h}}^2}}} \cdot \exp\left(-\frac{\left(H - \frac{\rho\sigma_x^2}{\sigma_{\hat{h}}^2}\hat{H}\right)^2}{2\frac{\sigma_x^2 \cdot \sigma_y^2}{\sigma_{\hat{h}}^2}}\right). \tag{2.85}$$

Considering both real and imaginary part, $H_u(k)|\hat{H}_u(k-1)$ is a complex Gaussian distributed random variable with mean value

$$\mu_u = \frac{\rho_u}{1+\sigma_{\mathrm{E},u}^2} \cdot \hat{H}_u(k-1) \tag{2.86}$$

and variance

$$\sigma_{r,u}^2 = \frac{1-\rho_u^2+\sigma_{\mathrm{E},u}^2}{1+\sigma_{\mathrm{E},u}^2}. \tag{2.87}$$

For perfect CQI, i.e., $\rho_u = 1$ and $\sigma^2_{e,u} = 0$, (2.85) reduces to a Dirac function $\delta(H - \hat{H})$ meaning that the estimated channel $\hat{H}$ is equivalent to the actual channel H. For totally outdated or totally noisy CQI, i.e., $\rho_u = 0$ or $\sigma^2_{\mathrm{E},u} \to \infty$, respectively, the mean value of $H|\hat{H}$ becomes $\mu = 0$ and the variance becomes $\sigma^2_{r,u} = 1$ meaning that there is no information about the actual channel H except that its real and imaginary part are Gaussian distributed with zero mean and variance $\frac{1}{2}$ resulting in a total variance of one.

2.9.5 Quantized CQI

In case of an FDD system, the frequency band of the uplink and downlink channel are different, i.e., it is not possible for the BS to measure the downlink channel during the uplink frame as in TDD systems. Thus, the MSs have to feed back the SNR values to the BS on a special feedback channel during the uplink. In order to decrease the amount of feedback, the CQI of each resource unit n in each time frame k is digitized at each MS u. In this case, the scheduler at the BS can not distinguish between the channel qualities of different users as precisely as with continuous CQI values, since there is only a limited numbers of CQI levels. The quantized CQI is formed in two steps. First, each MS u quantizes the measured SNR value in $L = 2^{N_Q}$ quantization levels with $L+1$ quantization thresholds $\gamma^{(u)}_{\mathrm{th},l}$ with $l = 0, .., L$, where N_Q denotes the number of quantization bits per resource unit. In general, the quantization thresholds $\gamma^{(u)}_{\mathrm{th}} = [\gamma^{(u)}_{\mathrm{th},0}, .., \gamma^{(u)}_{\mathrm{th},L}]$ for each user u can be selected arbitrarily following a certain quantization function

$$\gamma^{(u)}_{\mathrm{th}} = f_Q(u, N_Q), \tag{2.88}$$

i.e., according to the user index u and the number N_Q of quantization bits, the quantization function returns a SNR threshold vector $\gamma^{(u)}_{\mathrm{th}}$. For example, the SNR thresholds could be equidistantly distributed over a given SNR range. Second, the quantized CQI feedback is digitized according to a certain bit coding scheme. In this work, two coding schemes are considered, namely binary coding and binary-reflected Gray coding [Wil89]. With binary coding, the integer quantization level index X_{int} is translated into its N_Q bit binary representation X_{bin}. The translation from a binary value X_{bin} to the corresponding binary reflected Gray code X_{gray} is given by

$$X_{\mathrm{gray}} = X_{\mathrm{bin}} \oplus X_{\mathrm{bin}/2} \tag{2.89}$$

where $X_{\mathrm{bin}/2}$ denotes the 1-bit shifted version of X_{bin} to the right and $\oplus$ denotes the exclusive OR (XOR) operation [Wil89].

Both coding schemes can be characterized by an $L \times L$ Hamming distance matrix $\mathbf{B}$. This Hamming distance matrix is necessary to determine the probability of an erroneous feedback bit detection in case of an imperfect feedback link. The (x, y)-th element $b_{x,y}$ of matrix $\mathbf{B}$ with $x, y = 1, .., L$ contains the Hamming distance between the bit coding of the x-th quantization level and the bit coding of the y-th quantization level. As shown in Appendix A.4.1, the Hamming distance matrix $\mathbf{B}_{N_Q}$ for binary encoded quantization levels applying N_Q bits can be constructed iteratively according to

$$\mathbf{B}_{N_Q} = \begin{pmatrix} \mathbf{B}_{N_Q-1} & 1 + \mathbf{B}_{N_Q-1} \\ 1 + \mathbf{B}_{N_Q-1} & \mathbf{B}_{N_Q-1} \end{pmatrix} \tag{2.90}$$

with $\mathbf{B}_0 = 0$ and $N_Q \geq 1$. For binary-reflected Gray encoded quantization levels, the Hamming distance matrix $\mathbf{B}_{N_Q}$ applying N_Q bits is given by

$$\mathbf{B}_{N_Q} = \begin{pmatrix} \mathbf{B}_{N_Q-1} & 2 \cdot \mathbf{I}_{\mathbf{B},N_Q-1} + \mathbf{B}_{N_Q-1} \\ 2 \cdot \mathbf{I}_{\mathbf{B},N_Q-1} + \mathbf{B}_{N_Q-1} & \mathbf{B}_{N_Q-1} \end{pmatrix} \tag{2.91}$$

with $\mathbf{B}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $N_Q \geq 2$ and the $2^{N_Q} \times 2^{N_Q}$ block identity matrix $\mathbf{I}_{\mathbf{B},N_Q}$ consisting of two $2^{N_Q-1} \times 2^{N_Q-1}$ one matrices and two $2^{N_Q-1} \times 2^{N_Q-1}$ zero matrices given by

$$\mathbf{I}_{\mathbf{B},N_Q} = \begin{pmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{pmatrix} \tag{2.92}$$

as shown in Appendix A.4.2.

Note that the quantization of the CQI also has an impact on the modulation scheme selection. For a TDD system with continuous CQI values where M different modulation schemes are available, there are $M+1$ different SNR thresholds $\gamma_{\mathrm{th},l}^{(u)}$ for each user u with $l = 0, .., M$, assuming that below the first threshold $\gamma_{\mathrm{th},1}^{(u)}$ no transmission is performed. However, in case of an FDD system with quantized CQI values, the SNR thresholds are preset by the quantization thresholds, i.e., if the fedback SNR values are quantized into $L = 2^{N_Q}$ quantization levels at the MSs, then at most L different modulation schemes can be applied for the L different quantization levels which leads to a loss in flexibility adapting to the current channel conditions.

2.9.6 Imperfect feedback link

In a realistic scenario, the transmission of the digital CQI over the feedback channel can not be assumed to be error-free. Depending on the quality of the feedback channel and the used modulation scheme, bit errors may occur when detecting the feedback bits with a bit error rate p_b. If an error occurs when detecting the feedback bits, an SNR value, which was measured to be in the x-th quantization level is now assumed to be in the y-th quantization level. To determine the probability of this event, the $L \times L$ error probability matrix $\mathbf{E}$ is introduced. The (x, y)-th element $e_{x,y}$ of $\mathbf{E}$ with $x, y = 1, \cdots, L$ denotes the probability that an SNR value which was measured at the MS to be in the y-th quantization level is assumed to be in the x-th quantization level at the BS. Matrix $\mathbf{E}$ is calculated using the hamming distance matrix $\mathbf{B}$ according to

$$e_{x,y} = (1 - p_b)^{N_Q - b_{x,y}} \cdot p_b^{b_{x,y}}, \tag{2.93}$$

where $(1-p_b)^{N_Q-b_{x,y}}$ determines the probability that $N_Q-b_{x,y}$ bits are received correctly and $p_b^{b_{x,y}}$ determines the probability that $b_{x,y}$ bits are received incorrectly. To determine p_b for the feedback channel, BER measurements can be done.

Chapter 3

Combining adaptive and non-adaptive transmission modes in the presence of imperfect CQI

3.1 Introduction

In this section, the combination of adaptive and non-adaptive multi-user OFDMA transmission modes in the presence of imperfect CQI is discussed.

As already mentioned in Chapter 2, the application of adaptive OFDMA transmission modes leads to very good performances by exploiting multi-user diversity in case of having perfect CQI for all users at the BS [OR05]. Having no CQI at all at the BS, the use of non-adaptive OFDMA modes exploiting frequency diversity [SBS97], [SFS+05] independent from any CQI is the best strategy, however, not achieving the performance of adaptive schemes with perfect CQI. But what should be done for imperfect CQI? In the literature, the problem of dealing with imperfect transmitter sided channel knowledge is mainly addressed in pure adaptive OFDM-based systems. However, future radio systems shall support both adaptive *and* non-adaptive transmission modes. The OFDM-based IEEE 802.16 WIMAX standard offers the opportunity of applying adaptive and diversity-driven transmission modes [IEE04]. Also, for fourth generation systems it is planed in the proposal of the European Wireless World Initiative New Radio (WINNER) project plans to support adaptive and non-adaptive transmission mode [WIN05a].

Assuming that each user suffers the same degree of CQI imperfectness, it is possible to consider a system which switches between an adaptive and non-adaptive mode depending on the current quality of the CQI or, in other words, the current CQI imperfectness as done in [KK08b]. In such a system, all users are served either adaptively or non-adaptively. As expected, it is beneficial for the overall system performance to switch from adaptive to non-adaptive transmission in case of decreasing quality of the CQI. However, in a realistic scenario, it is not reasonable to assume that the level of CQI imperfectness is equal for all users since each users has its own transmission conditions. Instead, it is much more reasonable to assume that the CQI quality differs from user to user, i.e., for some users, the CQI is only slightly corrupted, whereas for other users the CQI is totally inaccurate. For such a scenario, a hybrid transmission scheme which is able to support both transmission modes becomes eligible. In this context, two main question arises. First, how is the coexistent service of users applying an adaptive transmission mode and a non-adaptive transmission mode taking into account imperfect CQI put into practice. Second, how to decide which user is served adaptively or non-adaptively.

In this section, a hybrid multi-user OFDMA system is introduced where both adaptive and non-adaptive transmission modes are supported. Non-adaptive users are served by applying the non-adaptive OFDMA transmission mode presented in Section 2.7. Adaptive users apply the adaptive OFDMA transmission mode introduced in Section 2.8 which performs an adaptive resource allocation together with an adaptive modulation based on the instantaneous CQI while also taking into account the fact that the CQI is imperfect. The adaptive and non-adaptive transmissions are multiplexed in frequency, i.e., different resource units in frequency direction are either reserved for non-adaptive or adaptive transmission over several time slots. For the order of serving the users, two possibilities are considered. Firstly, the resource units are allocated to the non-adaptive users in a first step and the remaining resource units are then allocated to the adaptive users in a second step. Secondly, first the adaptive users are served followed by the non-adaptive users.

The overall goal of the considered hybrid system is to achieve a maximum system data rate under the constraint of a minimum user data rate and target Bit Error Rate (BER). In this context, it has to be resolved how to adaptively adjust the applied modulation schemes to the current channel conditions while taking into account imperfect CQI in order to maximize the system data rate considering the user requirements. This implies that the functional interrelation between the user data rate and BER and the parameters describing the impairments of the CQI has to be known which requires derivations of the user data rate and BER as function of the CQI impairment parameters.

Further on, the question of which user shall be served adaptively or non-adaptively taking into account user-dependent imperfect CQI has to be answered. Since the performance of an adaptive users depends on the total number of adaptive users in the system due to the selection process and the multi-user diversity involved, the decision whether a user is served adaptively or non-adaptively cannot be made userwise independent from the other users but has to be done jointly considering all users, resulting in a combinatorial problem.

The remainder of this section is organized as follows. In Section 3.2, the hybrid multi-user OFDMA scheme is introduced. Section 3.3 presents the two orders of allocating resources to users in a hybrid OFDMA system which are Non-Adaptive First resource allocation (Section 3.3.2) and Adaptive First resource allocation (Section 3.3.3). Section 3.4 provides the problem formulation. Section 3.5 shows that the problem can be split up into two smaller problems which are then discussed and solved in Sections 3.6 and 3.7. Several parts of this Chapter 3 have been originally published by the author in [KK07a, KK07b, KK08b, KK08a, KKWW08, KK09, KK10, KK11].

3.2 Hybrid transmission scheme

In the following, the hybrid transmission scheme is introduced. As mentioned in the Introduction, the BS has to perform several preprocessing before the actual data transmission is done. Fig. 3.1 illustrates the preprocessing which has to be done for each time frame k. At first, the system has to select the applied access scheme for each user u, i.e., it has to decide whether a user is served adaptively or non-adaptively. This decision is based on the System Parameters (SP) which are the number N_{ru} of available resource units, the number U of users to be served, the number N_Q of feedback bits, the feedback BER p_b, the target BER BER_T and the user-dependent average SNR $\bar{\gamma}_u$. Furthermore, the decision is based on the parameters describing the CQI imperfectness which are the correlation coefficients ρ_u stacked together in the vector

$$\boldsymbol{\Gamma} = [\rho_1,\ \rho_2, ...,\ \rho_U] \tag{3.1}$$

and the estimation error variances $\sigma^2_{\mathrm{E},u}$ given by

$$\boldsymbol{\Sigma} = [\sigma^2_{\mathrm{E},1},\ \sigma^2_{\mathrm{E},2}, ...,\ \sigma^2_{\mathrm{E},U}]. \tag{3.2}$$

Note that it is assumed that these impairment parameters are perfectly known at the BS.

Finally, the decision whether a user u is served adaptively or non-adaptively depends on the channel access demand vector $\mathbf{D}$ of (2.36) which is known at the BS. The outcome of the access scheme selection is the user serving vector

$$\vartheta = [\vartheta_1, ..., \vartheta_U]^{\mathrm{T}}, \tag{3.3}$$

where $\vartheta_u = 0$ if the user u is served non-adaptively and $\vartheta_u = 1$ if the user u is served adaptively. Together with the channel access demand vector $\mathbf{D}$ and the CQI values for each resource unit of each user, the user serving vector is used to perform the adaptive and non-adaptive resource allocation. The outcome of the resource allocation is the $U \times N_{\mathrm{ru}}$ allocation matrix $\mathbf{X}$. The elements $x_{u,n} \in \{0, 1\}$ of $\mathbf{X}$ denote whether the n-th resource unit is allocated to user u ($x_{u,n} = 1$) or not ($x_{u,n} = 0$).

Besides the resource allocation represented by matrix $\mathbf{X}$, the user serving vector ϑ and the system parameters, the impairment parameters $\mathbf{\Gamma}$ and $\mathbf{\Sigma}$ and the channel access demand vector $\mathbf{D}$ are used to determine the SNR threshold vector γ_{th} for the applied modulation schemes. Since the calculation of the SNR thresholds does not depend on the instantaneous CQI, the calculation can be performed in parallel to the resource allocation, i.e., both operations are independent from each other.

Finally, with the SNR threshold vector γ_{th}, the allocation matrix $\mathbf{X}$ and the CQI values for each resource unit of each user, the $U \times N_{\text{ru}}$ modulation scheme matrix $\mathbf{X}_{\text{M}}$ is computed where the elements $x_{\text{M},u,n}$ denote which modulation scheme is applied in the n-th resource unit allocated to user u in time frame k.

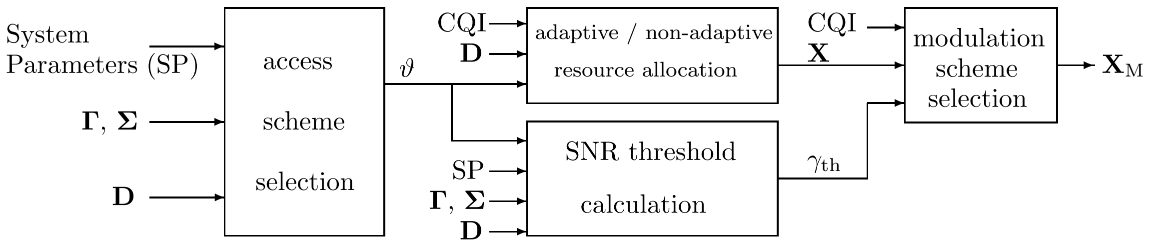


Figure 3.1. Preprocessing of the hybrid transmission scheme

Note that in a practical system, the SNR thresholds could be calculated off-line for certain values of ϑ, SP, $\mathbf{\Gamma}$, $\mathbf{\Sigma}$ and $\mathbf{D}$ and stored in a look-up table to reduce the computational complexity.

After the preprocessing is completed, ϑ, $\mathbf{X}$ and $\mathbf{X}_{\text{M}}$ are utilized for the actual data transmission applying the hybrid scheme. Fig. 3.2 shows the transmission chain of the hybrid scheme for a given user u. First, the binary data $\mathbf{d}_{\text{bin}}^{(u)}$ of user u is mapped on data symbols $\mathbf{d}^{(u)}$ utilizing the u-th row ${\mathbf{X}_{\text{M}}}^{(u)}$ of modulation scheme matrix $\mathbf{X}_{\text{M}}$. The resulting data symbols are then either directly OFDM modulated according to the u-th row $\mathbf{X}^{(u)}$ of allocation matrix $\mathbf{X}$ or DFT precoded followed by the OFDM modulation depending on the user serving vector element ϑ_u. The time domain signal at the output of the OFDM modulation is denoted by $s^{(u)}$. In case of an adaptive user, either OSTBC or TAS is applied at the transmit antennas while for a non-adaptive user, always OSTBC is applied. At the receiver, MRC is applied followed by the inversion of the channel, the Space-Time Coding, the OFDM modulation and the DFT precoding in case of a non-adaptively served user. Note that is assumed that each user is informed about whether it is served adaptively or non-adaptively. Applying data estimation results in the estimated binary data $\hat{\mathbf{d}}_{\text{bin}}^{(u)}$ of user u.

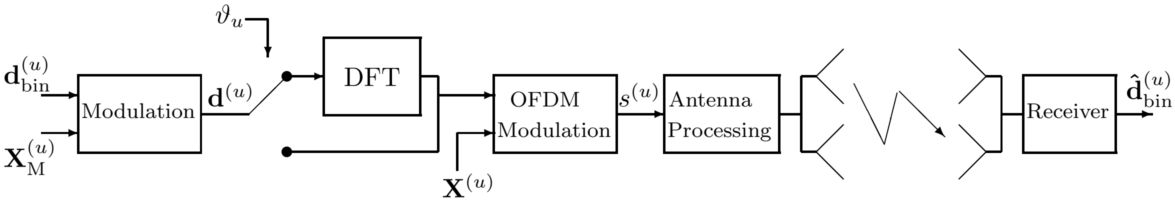


Figure 3.2. Transmission chain of hybrid transmission scheme

3.3 Order of resource allocation

3.3.1 Introduction

For the resource allocation of adaptive and non-adaptive users, two different resource allocation strategies are considered. For a given user serving vector ϑ, there are

$$U_{\mathrm{A}} = \sum_{u=1}^{U} \vartheta_u = \vartheta^{\mathrm{T}}\vartheta \tag{3.4}$$

adaptive user in the system. Each adaptive user u demands access to D_u resource units on average resulting in

$$W_{\mathrm{A}} = \sum_{u=1}^{U} \vartheta_u \cdot D_u \tag{3.5}$$

resource units dedicated to the U_{A} adaptively served users and

$$W_{\mathrm{NA}} = N_{\mathrm{ru}} - W_{\mathrm{A}} \tag{3.6}$$

resource units dedicated to the $U - U_{\mathrm{A}}$ non-adaptively served users.

3.3.2 Non-Adaptive First resource allocation

The first strategy referred to as Non-Adaptive First scheme is to allocate the W_{NA} resource units to the non-adaptively served users without using any CSI in a first step. As described in Section 2.7.4, this is done blockwise in a cyclic fashion. The remaining W_{A} resource units are then allocated to the adaptive users following the WPFS policy and QWPFS policy, respectively. By doing so, the non-adaptive users obtain their demanded resource units and the adaptive users can benefit from the multi-user diversity. However, since certain resource units are no longer available for adaptive users since they are given to non-adaptive users, possibly good channel conditions on these restricted resource units can not be exploited.

3.3.3 Adaptive First resource allocation

To overcome this drawback, the second resource allocation strategy, referred as to Adaptive First is introduced. Now, first the resource units of the adaptively served users are allocated, i.e., WPFS/QWPFS is applied over all N_{ru} resource units taking into account only the U_{A} adaptive users. By doing so, the variety of all N_{ru} resource units is exploited in the adaptive resource allocation process. However, the non-adaptively served users demand W_{NA} resource units which have to be re-allocated from the adaptive users. As for non-adaptive users it is not important which resource units are allocated to them since the non-adaptive mode works independent from any CQI, W_{NA} out of the N_{ru} selected resource units with the lowest ratio between weighted instantaneous SNR and average SNR are re-allocated from the adaptive users to the non-adaptive users. By doing so, the best W_{A} out of N_{ru} resource units are selected for the adaptive users while the non-adaptive users still obtain their demanded resource units.

In the following, the resource assignment of both resource allocation schemes is illustrated in Fig. 3.3 for a system with $U = 4$ users and $N_{\mathrm{ru}} = 8$ resource units assuming different numbers of adaptive and non-adaptive users and two consecutive time frames. Note that blank symbols represent adaptively served users while filled symbols represent non-adaptively served users. In Fig. 3.3 (a), all users are served non-adaptively ($U_{\mathrm{A}} = 0$, $U_{\mathrm{NA}} = 4$). In this case, there is no difference between Non-Adaptive First and Adaptive First. This case is equivalent to a conventional pure non-adaptive transmission. In Fig. 3.3 (b) and (c), only user $u = 4$ is served adaptively. In this case, there is a major difference between both schemes. With Non-Adaptive First shown in Fig. 3.3 (b), the adaptively served user actually does not have any choice in selecting a resource unit since only two given resource units remain. With Adaptive First shown in 3.3 (c), the adaptive user can select its two best out of all eight available resource units while the remaining 6 resource units are allocated to the 3 non-adaptive users. Furthermore, applying Non-Adaptive First, the allocation of the non-adaptively served users remains the same for consecutive time frames assuming that ϑ remains constant while with Adaptive First, the resource allocation for all users can be totally different for consecutive time frames as the position of the best resource units of user $u = 4$ can differ from frame to frame. This observation is also valid for all cases when $1 \leq U_{\mathrm{A}} \leq 4$ as seen in Fig. 3.3 (b) to 3.3 (f). Note that the resource allocation could be in any order for both Non-Adaptive First and Adaptive First. The chosen examples are just used to clarify the differences between both schemes. For the case of $U_{\mathrm{A}} = 4$ adaptive users, c.f. Fig. 3.3 (g), both schemes are again identical. This case is equivalent to a conventional pure adaptive transmission.

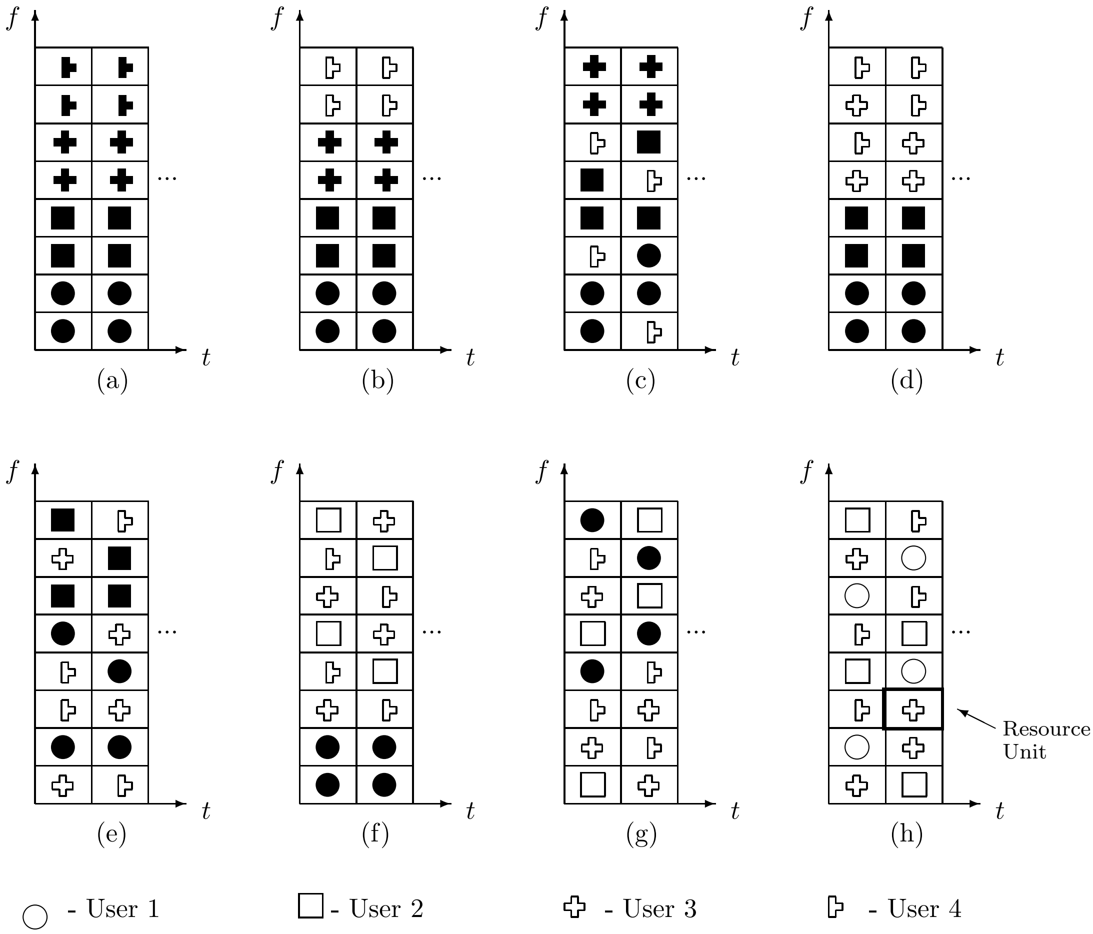


Figure 3.3. Hybrid adaptive - non-adaptive resource allocation with n adaptive users and $(4-n)$ non-adaptive users: (a) $n = 0$ (equivalent to pure non-adaptive), (b) $n = 1$ with NAF, (c) $n = 1$ with AF, (d) $n = 2$ with NAF, (e) $n = 2$ with AF, (f) $n = 3$ with NAF, (g) $n = 3$ with AF and (h) $n = 4$ (equivalent to pure adaptive); blank symbol: user is served adaptively, filled symbol: user is served non-adaptively

3.4 Problem Formulation

As stated in the introduction, the goal of the considered hybrid system is to achieve a maximum average system data rate under the constraint of a minimum user data rate and target Bit Error Rate (BER). The two parameters which are adjustable by the system to accomplish this task are the user serving vector ϑ and the SNR threshold vector $\gamma_{\text{th}}^{(u)}$ of each user u. In the following, the average system data rate $\bar{R}_{\text{sys}}$ is defined as the sum over the U different user data rates $\bar{R}_{\text{A}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)})$ and $\bar{R}_{\text{N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)})$ applying either the adaptive or non-adaptive transmission scheme divided by the U. This average system data rate shall be maximized over the vectors ϑ and $\gamma_{\text{th}}^{(u)}$ subject to a minimum user date rate $\bar{R}_{\min}^{(u)}$ and a target BER BER_{T}:

$$\begin{aligned}\bar{R}_{\mathrm{sys,opt}} &= \max_{\vartheta,\gamma_{\mathrm{th}}^{(u)}} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \left[\vartheta_u \bar{R}_{\mathrm{A}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) + (1-\vartheta_u)\cdot \bar{R}_{\mathrm{N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)})\right] \\ &\quad \text{subject to} \\ &\quad \vartheta_u \bar{R}_{\mathrm{A}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) + (1-\vartheta_u)\cdot \bar{R}_{\mathrm{N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) \geq \bar{R}_{\min}^{(u)} \\ &\quad \vartheta_u \overline{BER}_{\mathrm{A}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) + (1-\vartheta_u)\cdot \overline{BER}_{\mathrm{N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) \leq BER_{\mathrm{T}}.\end{aligned} \tag{3.7}$$

Note that the factor $\left(\frac{D_u}{N_{\mathrm{ru}}}\right)$ represents the probability of user u to get access to a given resource unit. Furthermore, it is assumed throughout this work that the required target BER is equal for all users. However, the problem can easily extended to different target BERs.

From (3.7), it follows that for each user u, the optimal SNR threshold vector $\gamma_{\mathrm{th,opt}}^{(u)}$ which maximizes the user data rate has to be found while fulfilling the BER requirement. Furthermore, the best user serving vector ϑ_{opt} out of 2^U possible realizations which maximizes the total system data rate has to be found, i.e., the best user serving combination searching from the one extreme case of serving all users adaptively to the other extreme case of serving all users non-adaptively has to be identified. Since the data rate $\bar{R}_{\mathrm{A}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)})$ of an adaptive user strongly depends on the number of adaptive users in the system due to the multi-user diversity, as also shown in the next sections, the decision whether a user is served adaptively or non-adaptively cannot be made userwise but has to be made jointly considering all users.

3.5 Splitting up the problem into two smaller problems

In order to solve the optimization problem (3.7), it can be split up into two smaller problems. For each possible serving vector realization ϑ, the user data rate $\bar{R}_{\mathrm{A}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)})$ applying the adaptive transmission scheme and the user data rate $\bar{R}_{\mathrm{N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)})$ applying the non-adaptive transmission scheme is optimized subject to a target BER, resulting in the following problem referred to as SNR threshold problem:

$$\begin{aligned}\bar{R}_{\mathrm{A/N,opt}}^{(u)}(\vartheta) &= \max_{\gamma_{\mathrm{th}}^{(u)}} \left(\bar{R}_{\mathrm{A/N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)})\right) \\ &\quad \text{subject to} \\ &\quad \overline{BER}_{\mathrm{A/N}}^{(u)}(\vartheta,\gamma_{\mathrm{th}}^{(u)}) \leq BER_{\mathrm{T}}.\end{aligned} \tag{3.8}$$

Being able to solve (3.8) for each possible ϑ, the optimal user serving vector ϑ_{opt} can be found by solving the second problem referred to as user serving problem:

$$\begin{aligned}\bar{R}_{\mathrm{sys,opt}} &= \max_{\vartheta} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \left[\vartheta_u \bar{R}_{\mathrm{A,opt}}^{(u)}(\vartheta) + (1-\vartheta_u)\cdot \bar{R}_{\mathrm{N,opt}}^{(u)}(\vartheta)\right] \\ &\quad \text{subject to} \\ &\quad \vartheta_u \bar{R}_{\mathrm{A,opt}}^{(u)}(\vartheta) + (1-\vartheta_u)\cdot \bar{R}_{\mathrm{N,opt}}^{(u)}(\vartheta) \geq \bar{R}_{\mathrm{min}}^{(u)}.\end{aligned} \tag{3.9}$$

By doing so, the problem of (3.7) is not simplified, i.e., (3.7) describes the same problem as (3.8) and 3.9). Instead of jointly searching for the optimal user serving vector ϑ and SNR thresholds $\gamma_{\mathrm{th}}^{(u)}$ in (3.7), one looks for the optimal SNR thresholds $\gamma_{\mathrm{th}}^{(u)}(\vartheta)$ as a function of the user serving vector ϑ in (3.8) and then optimizes ϑ in (3.9).

In the following two sections, solutions for the two problems (3.8) and (3.9) are presented. In Section 3.6, it is assumed that there exist a given user serving vector ϑ. For this ϑ, the optimal SNR thresholds are then determined solving (3.8). In Section 3.7, it is then shown how to solve (3.9), i.e., how to find the optimal user serving vector ϑ.

3.6 The SNR threshold problem

3.6.1 Introduction

In the following, the SNR threshold problem of (3.8) is addressed for both TDD and FDD systems. The reason for considering TDD systems and FDD systems separately is the difference in acquiring transmitter sided channel knowledge and the resultant different properties of the channel knowledge for both systems. These differences result in a different processing of the channel knowledge in the scheduling process for both systems which will be explained in details in Section 3.6.2 for TDD systems and in Section 3.6.3 for FDD systems.

3.6.2 TDD systems

3.6.2.1 Non-Adaptive First

3.6.2.1.1 Introduction In this section, the Non-Adaptive First resource allocation scheme is analyzed concerning the channel access and resulting SNR distribution of the adaptively and non-adaptively allocated resource units assuming that the user serving vector ϑ is given.

3.6.2.1.2 Channel access

3.6.2.1.2.1 Introduction As shown in Section 2.7, each non-adaptive user u with $\vartheta_u = 0$ gets access to D_u resource units, i.e. the channel access demand is fulfilled for the non-adaptive users. The remaining

$$W_{\mathrm{A}} = N_{\mathrm{ru}} - \sum_{\substack{u=1 \\ \vartheta_u \neq 1}}^{U} D_u \tag{3.10}$$

resource units are then allocated to the $U_{\mathrm{A}} = \vartheta^T \vartheta$ adaptive users following the WPFS policy. As depicted in Section 2.8.4, WPFS employs a user-dependent weighting factor p_u to adjust the probability of getting access to the channel. Since U_{A} different users are competing for the resource units, the channel access probability $P_A(u, \mathbf{p})$ of user u depends on the weighting factors p_u with $\vartheta = 1$ of all U_{A} adaptive users which are represented by the vector $\mathbf{p}$.

In the following, it is shown in Section 3.6.2.1.2.2 and 3.6.2.1.2.3 how to compute the channel access probability $P_{\mathrm{A}}^{(u)}(\mathbf{p})$ for a given adaptive user u as a function of a given weighting vector $\mathbf{p}$ and how to determine the average number of resource units allocated to user u applying both OSTBC and TAS, respectively. Having derived the interdependency between weighting factor p_u and the average number $E\{N_{\mathrm{ru},u}\}$ of resource units allocated to user u, it is shown in Section 3.6.2.1.2.4 how to adjust the weighting factors $\mathbf{p}$ such that the number of expected resource units allocated to each user u equals a given user demand D_u, i.e., the weighting vector $\mathbf{p}$ becomes a function of the user demand vector $\mathbf{D}$. Fig. 3.4 illustrates this interrelationship.

$$\mathbf{p} \longrightarrow \boxed{\text{WPFS}} \longrightarrow P_{\mathrm{A}}^{(u)}(\mathbf{p}) \longrightarrow E\{N_{\mathrm{ru},u}\}(\mathbf{p}) \overset{!}{=} D_u$$

Figure 3.4. Interrelationship between weighting vector $\mathbf{p}$ and user demand D_u of user u

3.6.2.1.2.2 Calculation of the channel access probability for adaptive users applying OSTBC-MRC The probability $P_{\mathrm{STC-NAF}}^{(u)}(\mathbf{p})$ of the adaptive user u to get access to a resource unit in a system applying OSTBC at the transmitter and

MRC at the receiver is now derived as a function of the weighting vector $\mathbf{p}$. With the WPFS policy of (2.61) from Section 2.8, it is can be seen that only the user $u^{\star}(n,k)$ with the highest normalized and weighted SNR value gets access to resource unit n in time frame k. From Section 2.8 it is also known that the resulting SNR $\gamma_u(n,k)$ after OSTBC and MRC is given by (2.57). In the following, the indices n and k are omitted since the calculations are valid for each resource unit and time frame. From (2.57) it can be shown that the Probability Density Function (PDF) of the resulting SNR γ_u is a chi-square distribution with $2n_{\mathrm{T}} \cdot n_{\mathrm{R}}$ degrees of freedom [Pro95] given by

$$p_{\gamma_u}(\gamma_u) = \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_u}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{\gamma_u^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}}\gamma_u}{\bar{\gamma}_u}\right). \tag{3.11}$$

Hence, the PDF of the weighted and normalized SNR $\gamma_{\mathrm{w}} = \frac{p_u \cdot \gamma_u}{\bar{\gamma}_u}$ is given by

$$p_{\gamma_{\mathrm{w}}}(\gamma_{\mathrm{w}}) = \left(\frac{n_{\mathrm{T}}}{p_u}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{\gamma_{\mathrm{w}}^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}}\gamma_{\mathrm{w}}}{p_u}\right). \tag{3.12}$$

In order to determine $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p})$, the probability that user u successfully competes against the other $U_{\mathrm{A}}-1$ adaptive users has to be calculated as

$$\begin{aligned} P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}) &= \int_{y_1=0}^{\infty}\int_{y_2=0}^{y_1}\cdots\int_{y_{U_{\mathrm{A}}}=0}^{y_1} p_{\gamma_{\mathrm{w}}}(y_1)\cdot p_{\gamma_{\mathrm{w}}}(y_2)\ldots p_{\gamma_{\mathrm{w}}}(y_U)\; dy_1 dy_2 \ldots dy_{U_{\mathrm{A}}} \\ &= \int_0^{\infty}\left(\frac{n_{\mathrm{T}}}{p_u}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{y_1^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}}y_1}{p_u}\right) \\ &\quad \cdot \prod_{\substack{i=1\\ i\neq u}}^{U_{\mathrm{A}}}\left(1 - e^{-\frac{n_{\mathrm{T}}y_1}{p_i}}\sum_{v=0}^{n_{\mathrm{T}}n_{\mathrm{R}}-1}\frac{1}{v!}\left(\frac{n_{\mathrm{T}}y_1}{p_i}\right)^{v}\right) dy_1 \end{aligned} \tag{3.13}$$

applying [GR65, Eq. 3.381] and [GR65, Eq. 8.352.1]. Examining (3.13), the first term of the integral represents the probability that the weighted and normalized SNR value of user u has a value equal to y_1 whereas the second term represents the probability that the weighted and normalized SNR values of the remaining $U_{\mathrm{A}}-1$ other users are smaller than the value y_1.

Performing some transformations and applying [GR65, Eq. 3.381.4] and [GR65, Eq. 8.339.1] to (3.13), the channel access probability $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p})$ of user u can be written in closed form as

$$\begin{aligned} P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}) &= \sum_{v=1}^{U_{\mathrm{A}}}\frac{(-1)^{v-1}}{p_u^{n_{\mathrm{T}}n_{\mathrm{R}}}}\sum_{|\eta|=v-1}\sum_{l=0}^{(v-1)\cdot(n_{\mathrm{T}}n_{\mathrm{R}}-1)}\cdot\sum_{|\nu|=l}\left(\frac{1}{\left(\prod_{i=1}^{v-1}\nu_i!\right)}\right) \\ &\quad \cdot\frac{\left(\sum_{i=1}^{v-1}\nu_i + n_{\mathrm{T}}n_{\mathrm{R}}-1\right)!}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!}\cdot\frac{\prod_{i=1}^{v-1}\left(\frac{1}{p_{r(\eta,i)+1}}\right)^{\nu_i}}{\left(\frac{1}{p_u}+\sum_{i=1}^{U_{\mathrm{A}}-1}\frac{\eta_i}{p_{i+1}}\right)^{\sum_{i=1}^{v-1}\nu_i + n_{\mathrm{T}}n_{\mathrm{R}}}} \end{aligned} \tag{3.14}$$

with the multi-indices $\eta = [\eta_1, \eta_2, ..., \eta_{U_\mathrm{A}-1}]$ with $\eta_j \in \{0,1\}\ \forall\ j = 1,..,U_\mathrm{A}-1$ and $\nu = [\nu_1, \nu_2, ..., \nu_{v-1}]$ with $\nu_j \in \{0,1,..,n_\mathrm{T}\cdot n_\mathrm{R}-1\}\ \forall\ j = 1,..,v-1$. The function $r(\eta, i)$ returns the index of the i-th 1 in the multi-index η.

From this, it follows that the average number of resource units allocated to an adaptive user u in an OSTBC-MRC system applying the Non-Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = W_\mathrm{A} \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}). \tag{3.15}$$

Note that $E\{N_{\mathrm{ru},u}\}$ does not have be an integer number, i.e., $E\{N_{\mathrm{ru},u}\}$ can be a fractional number, e.g., if in different time frames different numbers of resource units are allocated to a certain user which can occur when applying WPFS.

3.6.2.1.2.3 Calculation of the channel access probability for adaptive users applying TAS-MRC Using TAS instead of OSTBC at the transmitter, the channel access probability $P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p})$ for user u also changes. In order to derive $P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p})$, the PDF $p_{\gamma_\mathrm{w}}(\gamma_\mathrm{w})$ in (3.13) has to be exchanged by the PDF $p_{\gamma_{\mathrm{w}_{n_\mathrm{T}}}}(\gamma_{\mathrm{w}_{n_\mathrm{T}}})$ of the best out of n_T weighted and normalized SNR values resulting from transmitting with only one transmit antenna and performing MRC with n_R receive antennas given by

$$\begin{aligned} p_{\gamma_{\mathrm{w}_{n_\mathrm{T}}}}(\gamma_{\mathrm{w}_{n_\mathrm{T}}}) \;=\; & \frac{n_\mathrm{T}}{p_u^{n_\mathrm{R}}} \cdot \frac{\gamma_{\mathrm{w}_{n_\mathrm{T}}}^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot \exp\left(-\frac{\gamma_{\mathrm{w}_{n_\mathrm{T}}}}{p_u}\right) \\ & \cdot \left(1 - \exp\left(-\frac{\gamma_{\mathrm{w}_{n_\mathrm{T}}}}{p_u}\right) \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{\gamma_{\mathrm{w}_{n_\mathrm{T}}}}{p_u}\right)^v\right)^{n_\mathrm{T}-1}, \end{aligned} \tag{3.16}$$

leading to

$$\begin{aligned} P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p}) \;=\; & \int_0^\infty \frac{n_\mathrm{T}}{p_u^{n_\mathrm{R}}} \cdot \frac{y_1^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{y_1}{p_u}} \cdot \left(1 - e^{-\frac{y_1}{p_u}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{y_1}{p_u}\right)^v\right)^{n_\mathrm{T}-1} \\ & \cdot \prod_{\substack{i=1\\ i\neq u}}^{U} \left(1 - e^{-\frac{y_1}{p_i}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{y_1}{p_i}\right)^v\right)^{n_\mathrm{T}} \, dy_1 \end{aligned} \tag{3.17}$$

which can be rewritten as

$$\begin{aligned} P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p}) = & \int_0^\infty n_\mathrm{T} \cdot \left(\frac{1}{p'_u}\right)^{n_\mathrm{R}} \cdot \frac{y_1^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{y_1}{p'_u}} \\ & \cdot \prod_{\substack{i=1\\ i\neq u}}^{n_\mathrm{T}\cdot U} \left(1 - e^{-\frac{y_1}{p'_i}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{y_1}{p'_i}\right)^v\right) \, dy_1 \end{aligned} \tag{3.18}$$

with the extended weighting vector $\mathbf{p}'$ of length $n_\mathrm{T}\cdot U$ given by

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{n_\mathrm{T}\ \text{times}}. \tag{3.19}$$

Comparing (3.18) with (3.13), it can be seen that the integrals are similar besides the factor n_T at the beginning. From this, it follows that the channel access probability in a TAS system can be calculated using the channel access probability of an OSTBC system with $\mathbf{p}'$, $U'_\mathrm{A} = n_\mathrm{T} \cdot U_\mathrm{A}$, $n'_\mathrm{T} = 1$ and $n'_\mathrm{R} = n_\mathrm{R}$, given by

$$P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p}) = n_\mathrm{T} \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U'_\mathrm{A}, n'_\mathrm{T}, n'_\mathrm{R}). \tag{3.20}$$

The average number of resource units allocated to an adaptive user u in a TAS-MRC applying the Non-Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = W_\mathrm{A} \cdot P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p}). \tag{3.21}$$

An alternative way to derive (3.20) is to interpret the multi-user TAS system with n_T transmit antennas and U_A users as a system employing only one transmit antenna but with $n_\mathrm{T} \cdot UA$ virtual users. Thus, TAS can be interpreted as a special case of a multi-user OSTBC system with $U'_\mathrm{A} = n_\mathrm{T} \cdot U_\mathrm{A}$, $n'_\mathrm{T} = 1$ and $n'_\mathrm{R} = n_\mathrm{R}$. Hence, one adaptive user must compete against $n_\mathrm{T} \cdot U_\mathrm{A} - 1$ other virtual users to get access to a resource unit. However, each user u is related to n_T virtual users, i.e., the chance that user u is selected is factor n_T larger resulting in (3.20).

3.6.2.1.2.4 Calculation of weighting factors Until now, it was assumed that the weighting factors $\mathbf{p}$ were given. In this section, it it shown how to adjust the weights in order to fulfill the user demands $\mathbf{D}$ of the different users. Without loss of generality, it is assumed that the users are sorted by their user demand in descending order, i.e. $D_{u-1} \geq D_u \geq D_{u+1}$.

In the following, it is shown how to incorporate the fact that different users can have the same user demand when calculation the weighting factors to simplify the calculation. Recalling the definition of the demand groups $\mathcal{G}_i$ and the number of demand groups G introduced in Section 2.6, users having the same channel access demand D_u are arranged into demand groups $\mathcal{G}_i$. From this, it follows that there exists G different weighting factors $\tilde{p}_i$ with $i = 1, .., G$ stacked into the vector $\tilde{\mathbf{p}}$ and G different weighting factors $\tilde{D}_i$ with $i = 1, .., G$ stacked into the vector $\tilde{\mathbf{D}}$. The i-th weighting factor occurs $|\mathcal{G}_i|$ times with $|\cdot|$ denoting the cardinality of a group. Without loss of generality, the weighting factors of the users with the lowest channel access gain corresponding to demand group $\mathcal{G}_G$ are set to 1, resulting in

$$\tilde{\mathbf{p}} = [\tilde{p}_1,\ \tilde{p}_2, .., \ \tilde{p}_{G-1},\ 1] \tag{3.22}$$

The channel access vector $\tilde{\mathbf{D}}$ is given by

$$\tilde{\mathbf{D}} = [\tilde{D}_1,\ \tilde{D}_2, .., \ \tilde{D}_G] \tag{3.23}$$

Thus, the original weighting vector $\mathbf{p}$ can be replaced by

$$\breve{\mathbf{p}} = [\underbrace{\tilde{p}_1,\ \tilde{p}_1, .., \ \tilde{p}_1}_{|\mathcal{G}_1|\text{- times}}, ..., \underbrace{\tilde{p}_{G-1},\ \tilde{p}_{G-1}, .., \ \tilde{p}_{G-1}}_{|\mathcal{G}_{G-1}|\text{- times}}, \underbrace{1,\ 1, .., \ 1}_{|\mathcal{G}_G|\text{- times}}], \tag{3.24}$$

i.e., $\breve{\mathbf{p}}$ can be interpreted as a function of $\tilde{\mathbf{p}}$

$$\breve{\mathbf{p}} = f(\tilde{\mathbf{p}}). \tag{3.25}$$

The following example shall illustrate this. In a system with $U_\mathrm{A} = 5$ users, the channel access demand vector is given by

$$\mathbf{D} = [10,\ 10,\ 7,\ 5,\ 5].$$

Thus, there are $G = 3$ demand groups namely $\mathcal{G}_1 = \{1, 2\}$, $\mathcal{G}_2 = \{3\}$ and $\mathcal{G}_3 = \{4, 5\}$. Hence, the original weighting vector $\mathbf{p} = [p_1,\ p_2,\ p_3,\ p_4,\ p_5]$ can be replaced by

$$\breve{\mathbf{p}} = [\tilde{p}_1,\ \tilde{p}_1,\ \tilde{p}_2,\ 1,\ 1].$$

Due to the replacement of $\mathbf{p}$ by $\breve{\mathbf{p}}$, only $G - 1$ different weighting factors $\tilde{p}_i$ with $i = 1, .., G - 1$ have to be found such that

$$E\{N_{\mathrm{ru},u}\} = W_\mathrm{A} \cdot P_\mathrm{NAF}(i, \breve{\mathbf{p}}) = \tilde{D}_i \quad \forall\ i = 1, .., G - 1, \tag{3.26}$$

i.e., the average number of allocated resource units equals the number of demanded resource units for each user. This can be done by solving the following constrained nonlinear optimization problem

$$\begin{aligned} \tilde{\mathbf{p}}^\star \quad &= \quad \arg\min_{\tilde{\mathbf{p}}} \left\{ \sum_{i=1}^{G-1} \left| P^{(i)}_\mathrm{NAF}(f(\tilde{\mathbf{p}})) - \frac{\tilde{D}_i}{W_\mathrm{A}} \right| \right\} \\ \text{subject to} & \\ &\quad \tilde{p}_u \geq 1. \end{aligned} \tag{3.27}$$

using for example the *fmincon* function in MATLAB™. The corresponding weighting vector $\mathbf{p}$ is then given by

$$\mathbf{p} = [\underbrace{\tilde{p}^\star_1, \tilde{p}^\star_1, .., \tilde{p}^\star_1}_{|\mathcal{G}_1|\text{- times}}, ..., \underbrace{\tilde{p}^\star_{G-1}, \tilde{p}^\star_{G-1}, .., \tilde{p}^\star_{G-1}}_{|\mathcal{G}_{G-1}|\text{- times}}, \underbrace{1, 1, .., 1}_{|\mathcal{G}_G|\text{- times}}], \tag{3.28}$$

The following example shall illustrate the weighting factor calculation. Let us assume a system with $U_{\mathrm{A}} = 5$ adaptive users and $W_{\mathrm{A}} = 100$ available resource units with $n_{\mathrm{T}} = 2$ transmit antennas and $n_{\mathrm{R}} = 1$ receive antenna. In a totally fair system, each user u demands access to $D_u = \frac{W_{\mathrm{A}}}{U_{\mathrm{A}}} = 20$ resource units, i.e., $\mathbf{D} = [20, 20, 20, 20, 20]$. For both OSTBC and TAS, the weighting factors in this case are given by

$$\mathbf{p}_{\mathrm{STC}} = \mathbf{p}_{\mathrm{TAS}} = [1,\ 1,\ 1,\ 1,\ 1]$$

since no weighting has to be done as WPFS converges to PFS. If the channel access demand vector is for example given by $\mathbf{D} = [40,\ 30,\ 20,\ 5,\ 5]$, the weighting factor which minimize (3.27) in case of OSTBC are given by

$$\mathbf{p}_{\mathrm{STC}} = [2.426,\ 2.074,\ 1.703,\ 1,\ 1]$$

and in case of TAS given by

$$\mathbf{p}_{\mathrm{TAS}} = [2.543,\ 2.158,\ 1.755,\ 1,\ 1],$$

i.e., due to the different SNR statistic applying either OSTBC or TAS, the weighting factors have to be adjusted differently. Note that for the calculation of the weighting factors, no knowledge about the channel quality is needed as $P_{\mathrm{NAF}}^{(u)}(\mathbf{p})$ only depends on n_{T}, n_{R}, U_{A} and $\mathbf{p}$, see (3.14). For brevity and to ease the illustration, it is now assumed that there is only one high demand user and 4 low demand users leading to

$$\mathbf{D} = \left[D,\ \frac{100-D}{4},\ \frac{100-D}{4},\ \frac{100-D}{4},\ \frac{100-D}{4}\right],$$

i.e., $\mathbf{p} = [p,\ 1,\ 1,\ 1,\ 1]$. From Figure 3.5, it can be seen that for different antenna constellations the weighting factor p has to be adjusted differently in order to guarantee a certain user demand D. For example, if the high demand user should get access to $D = 60$ resource units, i.e., three times more channel resources compared to the fair case with $D = 20$, the weighting factor has to be set to $p = 3.75$ in a SISO system. In a 2×1 system, the weighting factor has to be set to $p = 2.6$ applying TAS and to $p = 2.48$ applying OSTBC, respectively, and for a 2×2 system, $p = 2.0$ applying TAS-MRC and $p = 1.9$ applying OSTBC-MRC have to be chosen. Moreover, it can be seen that the more antennas are used in the system, the less the increase of the weighting factor p for an increasing user demand D. This can be explained by the spatial diversity which is brought into the system using multiple antennas. The more spatial diversity, the less are the variations of the resulting normalized SNR values of the different users due to the averaging effect. Hence, in order to successfully compete against the other users, only a slight SNR boost is needed corresponding to a small weighting factor increment. In case of SISO, the SNR variations of the different users are rather high, meaning that it requires a larger weighting factor increment to successfully compete against the other users for an increasing user demand D.

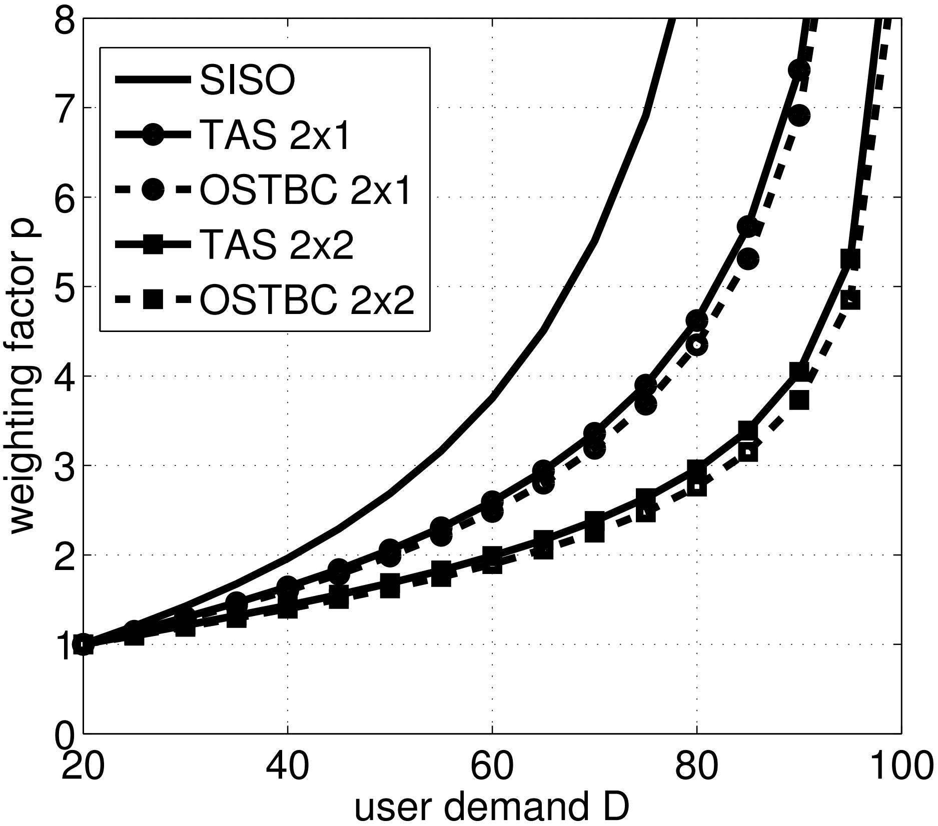


Figure 3.5. Weighting factor p vs. user demand D

3.6.2.1.3 SNR distribution

3.6.2.1.3.1 Introduction In the following, the distribution of the resulting SNR of a resource unit which is allocated to user u will be derived for the non-adaptively served users and adaptively served users applying the Non-Adaptive First scheme assuming any given number $U_{\mathrm{A}} = \vartheta^{\mathrm{T}}\vartheta$ of adaptive users and any given weighting vector $\mathbf{p}$, i.e.. any given user demand vector $\mathbf{D}$. These PDFs will then be used to analytically determine the performance of the system later on. Again, the indices n and k are omitted since the calculations are valid for each resource unit and time frame.

In case of non-adaptively served users, there are no adaptive scheduling decisions or adaptive modulation scheme selections to be made. Hence, only the PDF of the resulting SNR at the receiver of an allocated user is of interest to determine the performance of non-adaptively served users. As stated in Section 2.2, perfect R-CSI is assumed.

In case of adaptively served users, the system performance strongly depends on the quality of the measured SNR values which are required performing adaptive resource allocation and adaptive modulation. In a TDD system, these SNR values are assumed to be outdated and noisy estimates as stated in Section 2.9. Hence, for determining the performance of an adaptively served user, the PDF and Cumulative Density Function (CDF) of the SNR values of an allocated user measured at the BS are of interest since both the adaptive resource allocation and adaptive modulation are performed at the transmitter side.

3.6.2.1.3.2 Non-adaptive users As derived in Section 2.7.3, the resulting SNR $\gamma_{\mathrm{IDFT},u}$ at the receiver of user u applying the non-adaptive transmission mode is given by (2.52). In order to determine the PDF of $\gamma_{\mathrm{IDFT},u}$, several steps have to be performed. First, the PDF of the resulting SNR

$$\gamma_{\mathrm{OM},u} = \frac{1}{n_{\mathrm{T}}} \cdot \sum_{i'=1}^{n_{\mathrm{T}} \cdot n_{\mathrm{R}}} \gamma_u^{(i')}(q) \tag{3.29}$$

obtained from OSTBC and MRC is introduced. With (2.9) and keeping in mind that the real and imaginary parts of the channel transfer function H are modeled as independent Gaussian distributed random variables, $\gamma_u^{(i')}(q)$ from Eq. (2.52) is an exponentially distributed random variable. Thus, $\gamma_{\mathrm{OM},u}$ can be modelled as a weighted sum of $2n_{\mathrm{T}}n_{\mathrm{R}}$ independent exponentially distributed random variables. From [Pro95] it is known that such a sum of random variables is chi-squared distributed with $2n_{\mathrm{T}}n_{\mathrm{R}}$ degrees of freedom with the PDF given by

$$p_{\gamma_{\mathrm{OM}}}^{(u)}(\gamma_{\mathrm{OM}}) = \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_u}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{\gamma_{\mathrm{OM}}^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}} \cdot \gamma_{\mathrm{OM}}}{\bar{\gamma}_u}\right). \tag{3.30}$$

Next, let us introduce z_u denoting the reciprocal value of $\gamma_{\mathrm{OM},u}$ weighted by $\frac{1}{D_u}$ given by

$$z_u = \frac{1}{D_u} \cdot \frac{1}{\gamma_{\mathrm{OM},u}}. \tag{3.31}$$

From this, it follows that z_u is inverse chi-squared distributed with the PDF given by

$$p_z^{(u)}(z) = \left(\frac{n_{\mathrm{T}}}{D_u \cdot \bar{\gamma}_u}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{z^{-n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}}}{D_u \cdot \bar{\gamma}_u \cdot z}\right). \tag{3.32}$$

Actually, (3.32) can be written as a scaled inverse chi-squared distribution with

$$\psi = 2 \cdot n_{\mathrm{T}} \cdot n_{\mathrm{R}} \tag{3.33}$$

degrees of freedom and the scaling parameter

$$\varsigma^2 = \frac{1}{D_u \cdot \bar{\gamma}_u \cdot n_{\mathrm{R}}}. \tag{3.34}$$

The PDF of a scaled inverse chi-squared random variable z with scaling parameter ς^2 and ψ degrees of freedom is given by

$$p(z, \psi, \varsigma^2) = \frac{(\varsigma^2 \cdot \psi/2)^{\psi/2}}{(\psi/2-1)!} \cdot \frac{\exp\left(-\frac{\psi \cdot \varsigma^2}{2z}\right)}{z^{1+\psi/2}}. \tag{3.35}$$

For such scaled inverse chi-squared distributed random variables, the mean value and variance are known to be

$$E\{z_u\} = \frac{\psi \cdot \varsigma^2}{\psi - 2} = \frac{n_\mathrm{T}}{D_u \cdot \bar{\gamma}_u \cdot (n_\mathrm{T} n_\mathrm{R} - 1)} \tag{3.36}$$

and

$$\mathrm{Var}\{z_u\} = \frac{2 \cdot \psi^2 \cdot \varsigma^4}{(\psi-2)^2(\psi-4)} = \frac{n_\mathrm{T}^2}{D_u^2 \cdot \bar{\gamma}_u^2 \cdot (n_\mathrm{T} n_\mathrm{R} - 1)^2 \cdot (n_\mathrm{T} n_\mathrm{R} - 2)}, \tag{3.37}$$

respectively [KK51]. The next step in determining the PDF of $\gamma_{\mathrm{IDFT},u}$ is to compute the PDF of the sum ϱ_u of the D_u independent random variables z_u given by

$$\varrho_u = \sum_{q=1}^{D_u} z_u(q). \tag{3.38}$$

Since to the best knowledge of the author there exist no closed form solution for the PDF of the sum of inverse chi-squared distributed random variables in the literature, the PDF of ϱ_u is approximated applying the Central Limit Theorem (CLT) for continuous random variables [Kay06]. With the mean value and variance of z_u given in (3.36) and (3.37), the PDF $p_\varrho^{(u)}(\varrho)$ can be approximated by a Gaussian PDF with mean

$$\mu_{\mathrm{CLT},u} = D_u \cdot E\{z_u\} = \frac{n_\mathrm{T}}{\bar{\gamma}_u \cdot (n_\mathrm{T} n_\mathrm{R} - 1)} \tag{3.39}$$

and variance

$$\sigma_{\mathrm{CLT},u}^2 = D_u \cdot \mathrm{Var}\{z_u\} = \frac{n_\mathrm{T}^2}{D_u \cdot \bar{\gamma}_u^2 \cdot (n_\mathrm{T} n_\mathrm{R} - 1)^2 \cdot (n_\mathrm{T} n_\mathrm{R} - 2)} \tag{3.40}$$

given by

$$p_\varrho^{(u)}(\varrho) = \frac{1}{\sqrt{2\pi \cdot \sigma_{\mathrm{CLT},u}^2}} \cdot \exp\left(-\frac{(\varrho - \mu_{\mathrm{CLT},u})^2}{2\sigma_{\mathrm{CLT},u}^2}\right). \tag{3.41}$$

Note that this approximation is only feasible for cases with $n_\mathrm{T} \cdot n_\mathrm{R} > 2$ since otherwise the variance in (3.37) is not defined.

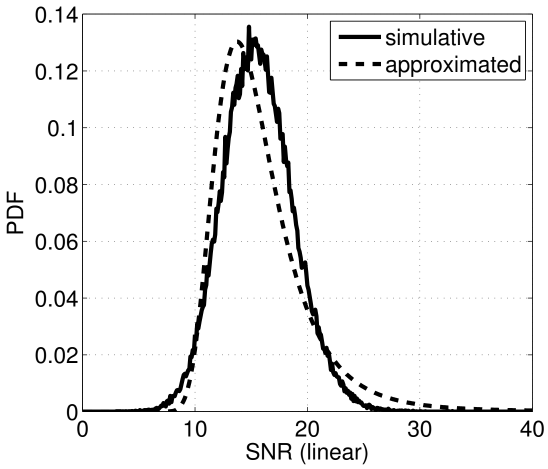
The last step in determining the PDF of $\gamma_{\mathrm{IDFT},u}$ is to compute the PDF of the inverse of ϱ_u,

$$\gamma_{\mathrm{IDFT},u} = \frac{1}{\varrho_u}, \tag{3.42}$$

which can be done by performing a random variable transformation of (3.41) resulting in the PDF of $\gamma_{\mathrm{IDFT},u}$ given by

$$p_{\gamma_{\mathrm{IDFT}}}^{(u)}(\gamma_{\mathrm{IDFT}}) = \frac{1}{\sqrt{2\pi \cdot \sigma_{\mathrm{CLT},u}^2 \cdot \gamma_{\mathrm{IDFT}}^2}} \cdot \exp\left(-\frac{\left(1-\mu_{\mathrm{CLT},u} \cdot \gamma_{\mathrm{IDFT}}\right)^2}{2\sigma_{\mathrm{CLT},u}^2 \cdot \gamma_{\mathrm{IDFT}}}\right). \tag{3.43}$$

In Fig. 3.6(a), the PDF of $\gamma_{\mathrm{IDFT},u}$ is depicted for a system with $n_{\mathrm{T}} = 2$, $n_{\mathrm{R}} = 2$, $\bar{\gamma}_u = 10$ dB and $D_u = 10$ resource units. The solid curve represents the simulative PDF after 10000 independent simulation runs while the dashed curve represents the PDF approximation according to (3.43). In Fig. 3.6(b) the same is shown for $D_u = 25$ resource units. It can be seen that the approximation becomes better as D_u increases.



(a) (b)

Figure 3.6. Simulative PDF and approximated PDF of $\gamma_{\mathrm{IDFT},u}$ for (a) $D_u = 10$ and (b) $D_u = 25$ resource units.

3.6.2.1.3.3 Adaptive users applying OSTBC-MRC In the following, the PDF and CDF of the SNR value of a scheduled resource unit that was measured at the BS in an OSTBC-MRC system are derived in dependency of the weighting vector $\mathbf{p}$ and the number U_{A} of adaptive users. Recalling the WPFS policy, a resource unit is allocated to the user which has the highest weighted and normalized SNR value, i.e., all the $U_{\mathrm{A}} - 1$ other users must have smaller weighted and normalized SNR values such the resource unit is allocated to that given user u. To determine the PDF $p_{\mathrm{STC-NAF},\hat{\gamma}}^{(u)}(\hat{\gamma})$ of the measured SNR $\hat{\gamma}$ of a resource unit allocated to this user u, first the joint PDF of all U_{A} normalized SNR values $X_1, .., X_{U_{\mathrm{A}}}$ has to determined. Since the SNR values of different users are independent from each other and with the knowledge that the measured SNR values are chi-squared distributed, the joint PDF is given by

$$p_{X_1,..,X_{U_{\mathrm{A}}}}(x_1, .., x_{U_{\mathrm{A}}}) = p_{\hat{\gamma}_u}(x_1) \cdots p_{\hat{\gamma}_u}(x_{U_{\mathrm{A}}}) \tag{3.44}$$

with

$$p_{\hat{\gamma}_u}(x) = \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{x^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}} \cdot x}{\bar{\gamma}_{E,u}}\right) \tag{3.45}$$

and $\bar{\gamma}_{E,u} = \bar{\gamma}_u \cdot (1 + \sigma^2_{\mathrm{E},u})$ taking into account that the SNR values measured at the BS are noisy estimates modeled according to (2.70).

The sought after PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ of the SNR $\hat{\gamma}$ of a resource unit allocated to user u measured at the BS is then the marginal PDF calculated by determining the integral over the joint PDF leading to

$$\begin{aligned} p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{STC-NAF}}(u) \\ &\quad \cdot \underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \cdots \int_0^{\frac{p_u}{p_{U_{\mathrm{A}}}}\hat{\gamma}}}_{U_{\mathrm{A}}-1\,\text{times}} p_{X_1,..X_{U_{\mathrm{A}}}}(\hat{\gamma}, y_1, .., y_{U_{\mathrm{A}}-1}) dy_1 \ldots dy_{U_{\mathrm{A}}-1} \\ &= a_{\mathrm{STC-NAF}}(u) \cdot \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \cdot e^{-\frac{n_{\mathrm{T}}\cdot\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\ &\quad \cdot \prod_{\substack{i=1 \\ i\neq u}}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{T}}n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}} \cdot p_u \cdot \hat{\gamma}}{p_i \cdot \bar{\gamma}_{E,u}}\right)^v\right), \end{aligned} \tag{3.46}$$

where the factor $a_{\mathrm{STC-NAF}}(u)$ ensures that

$$\int_0^\infty p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \tag{3.47}$$

Performing the substitution of the variable $y_1 = \frac{\hat{\gamma}\cdot p_u}{\gamma_{\bar{E},u}}$ in the integral of (3.13), it can be seen that the integrals in (3.13) and (3.47) are identical except for the factor $a_{\mathrm{STC-NAF}}(u)$, leading to

$$a_{\mathrm{STC-NAF}}(u) = \frac{1}{P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p})}. \tag{3.48}$$

The following example shall illustrate the calculation of the PDF of the SNR values of allocated resource units. Let us assume a system with $U_{\mathrm{A}} = 3$ adaptively served users where all users have the same average SNR $\bar{\gamma}_u = 10$ dB and perfect CQI. The weighting vector is given by

$$\mathbf{p} = [5,\ 2,\ 1].$$

In Figure 3.7(a), the PDF of the measured SNR values of the resource units allocated to user $u = 1$ is depicted. The dashed curve represents the analytical PDF according to (3.46 and the solid lines represent the PDF evaluated from 10000 simulation runs. Fig. 3.7(b) and Fig. 3.7(c) the PDFs for user $u = 2$ and user $u = 3$ is depicted. One can see that the analytical PDFs are consistent with the simulative ones. Furthermore, it can be seen that the probability of small SNR values is larger for user $u = 1$ than for user $u = 2$ and $u = 3$. The reason for that is the higher weighting factor of user $u = 1$ compared to users $u = 2$ and $u = 3$, i.e., in order to successfully compete against the other users, the actual SNR value of user $u = 1$ does not have to be as high due to the SNR boosting of the WPFS while for user $u = 3$, the SNR values have to be rather high in order to be considered for allocation.

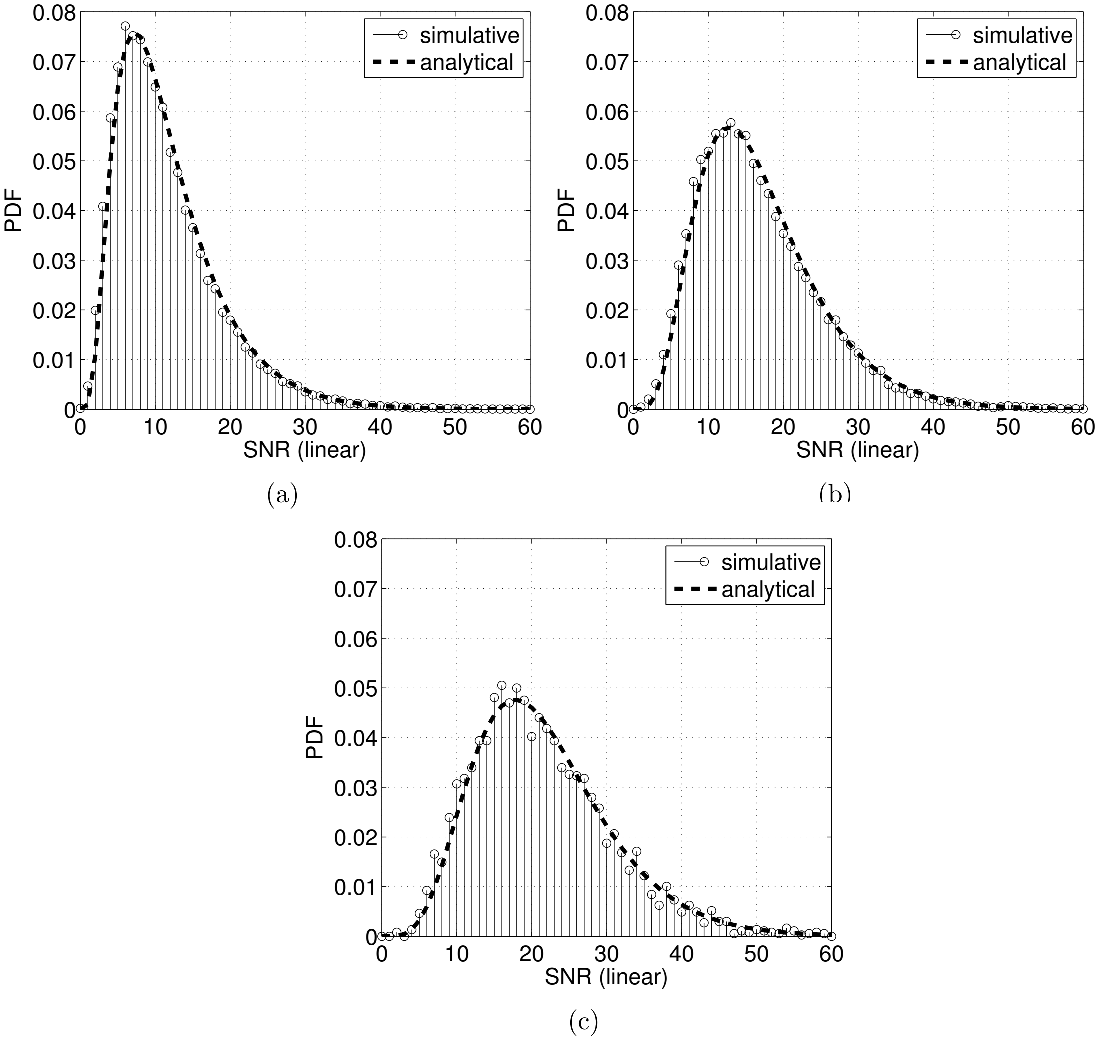


Figure 3.7. Analytical PDF and simulative PDF of the SNR of allocated resource units for user (a) $u = 1$ and (b) $u = 2$ and (c) $u = 3$ applying the Non-Adaptive First scheme.

Finally, the CDF $F^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ of the measured SNR of the resource unit allocated to user u is determined by integrating (3.46) resulting in

$$\begin{aligned} F^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}) &= \frac{a_{\mathrm{STC-NAF}}(u)}{p_u^{n_\mathrm{T}\cdot n_\mathrm{R}}} \cdot \sum_{v=1}^{U} (-1)^{v-1} \sum_{|\eta|=v-1} \sum_{l=0}^{(v-1)\cdot(n_\mathrm{T}n_\mathrm{R}-1)} \sum_{|\nu|=l} \\ &\quad \frac{\left(\sum_{i=1}^{v-1}\nu_i + n_\mathrm{T}n_\mathrm{R} - 1\right)!}{(n_\mathrm{T}n_\mathrm{R}-1)!} \cdot \frac{\left(\frac{1}{(\prod_{i=1}^{v-1}\nu_i!)}\right)\cdot\left(\prod_{i=1}^{v-1}\left(\frac{1}{p_{r(\eta,i)+1}}\right)^{\nu_i}\right)}{\Lambda(\mathbf{p},\eta)^{\sum_{i=1}^{v-1}\nu_i+n_\mathrm{T}n_\mathrm{R}}} \\ &\quad \cdot \left[1 - e^{-\frac{-n_\mathrm{T}p_u\hat{\gamma}\cdot\Lambda(\mathbf{p},\eta)}{\bar{\gamma}_{E,u}}} \sum_{\kappa=0}^{\sum_{i=1}^{v-1}\nu_i+n_\mathrm{T}n_\mathrm{R}-1} (\kappa!)^{-1}\left(\frac{n_\mathrm{T}p_u\hat{\gamma}\Lambda(\mathbf{p},\eta)}{\bar{\gamma}_{E,u}}\right)\right] \end{aligned} \quad (3.49)$$

with $\Lambda(\mathbf{p},\eta) = \frac{1}{p_u} + \sum_{i=1}^{U-1}\frac{\eta_i}{p_{i+1}}$ and η, ν and $r(\eta,i)$ as defined in (3.14).

3.6.2.1.3.4 Adaptive users applying TAS-MRC To determine the PDF and CDF of the SNR of the resource unit allocated to user u which was measured at the BS in a TAS system, the same derivation steps as in (3.44) to (3.49) have to be done. However, PDF $p_{\hat{\gamma}_u}(x)$ which represents a chi-squared distribution has to be exchanged by the PDF $p^{(n_\mathrm{T})}_{\hat{\gamma}_u}(x)$ which represents a best of n_T chi-squared distribution to incorporate the fact that the SNR is a result of a selection process out of n_T transmit antennas. From [Dav81] it is known that $p_{\hat{\gamma}_{u_{n_\mathrm{T}}}}(x)$ is given by

$$p^{(n_\mathrm{T})}_{\hat{\gamma}_u}(x) = \frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \frac{x^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{x}{\bar{\gamma}_{E,u}}} \cdot \left(1 - e^{-\frac{x}{\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{x}{\bar{\gamma}_{E,u}}\right)^v\right)^{n_\mathrm{T}-1}. \quad (3.50)$$

Hence, the PDF $p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma})$ of the SNR of the resource unit allocated to user u which was measured at the BS results in

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{TAS-NAF}}(u) \cdot \frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\ &\quad \cdot \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}\right)^v\right)^{n_\mathrm{T}-1} \\ &\quad \cdot \prod_{\substack{i=1\\ i\neq u}}^{U_\mathrm{A}} \left(1 - e^{-\frac{p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \cdot \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{p_u\cdot\hat{\gamma}}{p_i\cdot\bar{\gamma}_{E,u}}\right)^v\right)^{n_\mathrm{T}}, \end{aligned} \quad (3.51)$$

which can be rewritten as

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{TAS-NAF}}(u) \cdot \frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\ &\quad \cdot \prod_{\substack{i=1\\ i\neq u}}^{nT\cdot U_\mathrm{A}} \left(1 - e^{-\frac{p'_u\hat{\gamma}}{p'_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{p'_u\cdot\hat{\gamma}}{p'_i\cdot\bar{\gamma}_{E,u}}\right)^v\right) \end{aligned} \quad (3.52)$$

with $\mathbf{p}'$ as defined in (3.19). Again, the factor $a_{\mathrm{TAS-NAF}}(u)$, which ensures that

$$\int_0^\infty p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma})d\hat{\gamma} = 1, \tag{3.53}$$

can be determined by performing a substitution of the variable $y_1 = \frac{\hat{\gamma}\cdot p_u}{\gamma_{\bar{E},u}}$ in the integral of (3.18). It can be seen that the integrals in (3.18) and (3.53) are identical except for the factor $a_{TAS}(u)$, leading to

$$\begin{aligned} a_{\mathrm{TAS-NAF}}(u) &= \frac{1}{P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p})} \\ &= \frac{1}{n_{\mathrm{T}} \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U', n'_{\mathrm{T}}, n'_{\mathrm{R}})}. \end{aligned} \tag{3.54}$$

Comparing (3.52) and (3.54) with (3.46) and (3.48), it can be seen that the PDF $p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma})$ can be determined using the PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ given by

$$p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}) = p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U', n'_{\mathrm{T}}, n'_{\mathrm{R}}) \tag{3.55}$$

with $U' = n_{\mathrm{T}} \cdot U$, $n'_{\mathrm{T}} = 1$, $n'_{\mathrm{R}} = n_{\mathrm{R}}$ and $\mathbf{p}'$ as defined in (3.19).

Due to (3.55), the CDF $F^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma})$ can also be determined using the CDF $F^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ given by

$$F^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}) = F^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U', n'_{\mathrm{T}}, n'_{\mathrm{R}}) \tag{3.56}$$

with $U' = n_{\mathrm{T}} \cdot U_L$, $n'_{\mathrm{T}} = 1$, $n'_{\mathrm{R}} = n_{\mathrm{R}}$ and $\mathbf{p}'$ as defined in (3.19).

3.6.2.1.4 Average user data rate and BER taking into account imperfect CQI

3.6.2.1.4.1 Introduction Now being able to determine the distribution of the SNR values of the allocated resource units for any given channel access demand $\mathbf{D}$ and user serving vector ϑ, the system performance can be derived analytically. For the case of adaptively served users, the fact that the measured SNRs are only outdated and noisy CQI values which results in suboptimal resource allocation and modulation scheme selection decisions has to be taken into account when determining the system performance. In the following, the average user data rate and user Bit Error Rate (BER) are derived for the non-adaptively served users and the adaptively served users applying the Non-Adaptive First scheme assuming that the user serving vector ϑ is given.

3.6.2.1.4.2 Non-adaptive users As presented in Section 2.7, D_u resource units are allocated to user u independently from any CQI where one fixed modulation scheme m is used for all resource units with $m = 1, .., M$ and M denoting the number of available modulation schemes. Let b_m denotes the number of bits per symbol corresponding to the applied modulation scheme. From this, it follows that the bit rate $R_b^{(u)}$ of user u expressed in bits per second (b/s) is given by

$$R_b^{(u)} = \frac{D_u \cdot Q_{\text{sub}} \cdot b_m}{T_S} \tag{3.57}$$

with Q_{sub} denoting the number of adjacent subcarriers per resource unit and T_S the symbol duration of an OFDMA symbol neglecting the guard interval. The user data rate $\bar{R}_{\text{N}}^{(u)}$ for the non-adaptively served user u expressed in bits per second per Hertz (b/s/Hz) is then given by

$$\bar{R}_{\text{N}}^{(u)} = \frac{R_b^{(u)}}{D_u \cdot Q_{\text{sub}} \cdot \Delta f} = b_m \tag{3.58}$$

with $\Delta f = \frac{1}{T_S}$ denoting the subcarrier spacing. With the PDF of the SNR γ_{IDFT} at the output of the receiver derived in (3.43), the average BER using the modulation scheme with index m is then determined by

$$\overline{BER}_{\text{N}}^{(u)} = \int_0^\infty BER_m(\gamma_{\text{IDFT}}) \cdot p_{\gamma_{\text{IDFT}}}^{(u)}(\gamma_{\text{IDFT}})\, d\gamma_{\text{IDFT}}, \tag{3.59}$$

where BER_m determines the bit error rate of the applied modulation scheme with index m. In the following, the approximation for the BER of M-QAM and M-PSK modulation introduced in [CG01] is used which is given by

$$BER_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \tag{3.60}$$

with $\beta_m = \frac{1.6}{2^{b_m}-1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9 b_m}+1}$ using M-PSK modulation, respectively. Inserting (3.60) in (3.59) leads to

$$\overline{BER}_{\text{N}}^{(u)} = \frac{1}{\sqrt{2\pi \cdot \sigma_{\text{CLT},u}^2}} \cdot \int_0^\infty \frac{\exp(-\beta_m \gamma_{\text{IDFT}})}{\gamma_{\text{IDFT}}^2} \cdot \exp\left(-\frac{(1-\mu_{\text{CLT},u} \cdot \gamma_{\text{IDFT}})^2}{2\sigma_{\text{CLT},u}^2 \cdot \gamma_{\text{IDFT}}}\right) d\gamma_{\text{IDFT}} \tag{3.61}$$

with

$$\mu_{\text{CLT},u} = \frac{n_{\text{T}}}{\bar{\gamma}_u \cdot (n_{\text{T}} n_{\text{R}} - 1)} \tag{3.62}$$

and

$$\sigma_{\text{CLT},u}^2 = \frac{n_{\text{T}}^2}{D_u \cdot \bar{\gamma}_u^2 \cdot (n_{\text{T}} n_{\text{R}} - 1)^2 \cdot (n_{\text{T}} n_{\text{R}} - 2)}. \tag{3.63}$$

Note that the integral in (3.61) can only be solved numerically. Examining the user date rate and BER of non-adaptively served users, it can be seen that the performance only depends on the number of allocated resource units D_u, the average SNR $\bar{\gamma}_u$ and the applied modulation scheme.

3.6.2.1.4.3 Adaptive users applying OSTBC-MRC In case of adaptively served user u, different modulation schemes m with $m = 1, .., M$ are applied for the allocated resource units according to the instantaneous SNR condition and the SNR threshold vector $\gamma_{\text{th}}^{(u)} = [\gamma_{\text{th},0}^{(u)}, \gamma_{\text{th},1}^{(u)}, ..., \gamma_{\text{th},M}^{(u)}]^{\text{T}}$ which contains the SNR threshold values determining the interval in which a particular modulation scheme is applied, where $\gamma_{\text{th},0}^{(u)} = 0$ and $\gamma_{\text{th},M}^{(u)} = \infty$ for all users. Like in the case of non-adaptively served users, the user data rate $R_{\text{STC-NAF}}^{(u)}(m)$ (in b/s/Hz) of an adaptively served users applying the m-th modulation scheme in an OSTBC-MRC system is given by

$$R_{\text{STC-NAF}}^{(u)}(m) = r_{n_{\text{T}}} \cdot b_m \tag{3.64}$$

with $r_{n_{\text{T}}}$ denoting the data rate of the Space Time Block Code as a function of n_{T} and b_m the number of bits per symbol applying modulation scheme m.

The average user data rate $\bar{R}_{A,\text{STC-NAF}}^{(u)}$ taking into account that different modulation schemes are applied is then defined as sum rate of the different user data rates $R_{\text{STC-NAF}}^{(u)}(m)$ weighted by the probability that modulation scheme m is applied, i.e., that the SNR value lies in the particular SNR interval $[\gamma_{\text{th},m-1}^{(u)}, \gamma_{\text{th},m}^{(u)}]$. Thus, the average data rate of user u can be formulated as

$$\bar{R}_{A,\text{STC-NAF}}^{(u)} = \sum_{m=1}^{M} r_{n_{\text{T}}} \cdot b_m \cdot \int_{\gamma_{\text{th},m-1}^{(u)}}^{\gamma_{\text{th},m}^{(u)}} p_{\text{STC-NAF},\hat{\gamma}}^{(u)}(\hat{\gamma})\, d\hat{\gamma}. \tag{3.65}$$

Using (3.49), the average user data rate for OSTBC systems is given by

$$\bar{R}_{A,\text{STC-NAF}}^{(u)} = r_{n_{\text{T}}} \cdot \sum_{m=1}^{M} b_m \cdot \left(F_{\text{STC-NAF},\hat{\gamma}}^{(u)}(\gamma_{\text{th},m}^{(u)}) - F_{\text{STC-NAF},\hat{\gamma}}^{(u)}(\gamma_{\text{th},m-1}^{(u)}) \right). \tag{3.66}$$

To define the average BER $\overline{BER}_{\text{STC-NAF}}^{(u)}$ of an adaptively served user u in an OSTBC-MRC system, the impact of outdated and noisy CQI has to be taken into account. Therefore, the actual BER $\widehat{BER}_m^{(u)}(\hat{\gamma})$ of user u applying the m-th modulation scheme based on outdated and noisy SNR information $\hat{\gamma}$ has to be derived. Recalling that the interdependency between the BER BER_m when applying modulation scheme m and the actual SNR γ is given by (3.60), $\widehat{BER}_m^{(u)}(\hat{\gamma})$ can be calculated by

$$\widehat{BER}_m^{(u)}(\hat{\gamma}) = \int_0^{\infty} BER_m(\gamma) \cdot p_{\gamma|\hat{\gamma}}^{(u)}(\gamma|\hat{\gamma})\, d\gamma \tag{3.67}$$

with $p_{\gamma|\hat{\gamma}}^{(u)}(\gamma|\hat{\gamma})$ denoting the conditional PDF of the actual SNR γ and the outdated and noisy SNR $\hat{\gamma}$ of user u when applying OSTBC at the transmitter side and MRC at the receiver side. With the conditional PDF of $p_{H|\hat{H}}(H|\hat{H})$ (2.85), the SNR definition (2.9) and [Pro95, p. 43], $p^{(u)}_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ is given by

$$p^{(u)}_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{n_\mathrm{T}}{\bar{\gamma}_u\sigma^2_{r,u}} \cdot \exp\left(-\frac{\mu_u^2 \cdot \hat{\gamma} + \gamma}{\bar{\gamma}_u\sigma^2_{r,u}}\right) \cdot \left(\frac{\gamma}{\mu_u^2\hat{\gamma}}\right)^{(n_\mathrm{T}n_\mathrm{R}-1)/2} \cdot I_{n_\mathrm{T}n_\mathrm{R}-1}\left(\frac{2n_\mathrm{T}\mu_u\sqrt{\gamma\cdot\hat{\gamma}}}{\bar{\gamma}_u\sigma^2_{r,u}}\right), \tag{3.68}$$

with

$$\mu_u = \frac{\rho_u}{1+\sigma^2_{\mathrm{E},u}}, \tag{3.69}$$

$$\sigma^2_{r,u} = \frac{1+\sigma^2_{\mathrm{E},u}-\rho_u^2}{1+\sigma^2_{\mathrm{E},u}} \tag{3.70}$$

and $I_n(x)$ denoting the nth-order modified Bessel function of the first kind. Inserting (3.68) in (3.67) leads to

$$\widehat{BER}^{(u)}_m(\hat{\gamma}) = 0.2 \cdot \left(\frac{n_\mathrm{T}}{n_\mathrm{T}+\beta_m\bar{\gamma}_u\sigma^2_{r,u}}\right)^{n_\mathrm{T}n_\mathrm{R}} \cdot \exp\left(-\frac{\hat{\gamma}n_\mathrm{T}\mu_u^2\beta_m}{n_\mathrm{T}+\beta_m\bar{\gamma}_u\sigma^2_{r,u}}\right) \tag{3.71}$$

applying the identities [GR65, 6.643.4] and [GR65, 8.970.1].

The average rate $\bar{R}^{(u)}_{\mathrm{eb}}$ of incorrectly detected bits at the receiver of user u is then defined as sum of the average rates of incorrectly detected bits applying the different modulation schemes $m = 1, .., M$ as

$$\bar{R}^{(u)}_{\mathrm{eb}} = \sum_{m=1}^{M} \int_{\gamma^{(u)}_{\mathrm{th},m-1}}^{\gamma^{(u)}_{\mathrm{th},m}} r_{n_\mathrm{T}} \cdot b_m \cdot p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}) \cdot \widehat{BER}^{(u)}_m(\hat{\gamma})\, d\hat{\gamma}. \tag{3.72}$$

Finally, the average bit error rate $\overline{BER}^{(u)}_{\mathrm{STC-NAF}}$ of an adaptively served user u in an OSTBC-MRC system is defined as the average rate $\bar{R}^{(u)}_{\mathrm{eb}}$ of erroneous detected bits divided by the average bit rate $\bar{R}^{(u)}_{A,\mathrm{STC-NAF}}$ [MT05] given by

$$\overline{BER}^{(u)}_{A,\mathrm{STC-NAF}} = \frac{1}{\bar{R}^{(u)}_{A,\mathrm{STC-NAF}}} \cdot \sum_{m=1}^{M} \int_{\gamma^{(u)}_{\mathrm{th},m-1}}^{\gamma^{(u)}_{\mathrm{th},m}} r_{n_\mathrm{T}} \cdot b_m \cdot p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}) \cdot \widehat{BER}^{(u)}_m(\hat{\gamma})\, d\hat{\gamma} \tag{3.73}$$

Inserting (3.46) and (3.71) in (3.73) and introducing the functions

$$\Upsilon(m,\eta) = \left(1 + \sum_{i=1}^{U_\mathrm{A}-1} \frac{p_u \cdot \eta_i}{p_{i+1}}\right) \cdot (n_\mathrm{T} + \beta_m\bar{\gamma}_u\sigma^2_{r,u}) + \bar{\gamma}_{E,u}\beta_m\mu_u^2. \tag{3.74}$$

and

$$\Psi(m) = n_{\mathrm{T}} + \beta_m \bar{\gamma}_u \sigma_{r,u}^2 \tag{3.75}$$

and with η, ν and $r(\eta, i)$ as defined in (3.14), (3.73) can be written in closed form as

$$\begin{aligned}\overline{BER}_{A,\mathrm{STC-NAF}}^{(u)} = {} & \frac{a_{\mathrm{STC-NAF}}(u) \cdot r_{n_{\mathrm{T}}}}{5 \cdot \bar{R}_{A,\mathrm{STC-NAF}}^{(u)}} \cdot \sum_{m=1}^{M} b_m \cdot \sum_{v=1}^{U_{\mathrm{A}}} (-1)^{v-1} \sum_{|\eta|=v-1} \sum_{l=0}^{(v-1)\cdot(n_{\mathrm{T}} n_{\mathrm{R}}-1)} \\ & \sum_{|\nu|=l} \left(\frac{1}{\left(\prod_{i=1}^{v-1} \nu_i!\right)} \right) \cdot \frac{\left(\sum_{i=1}^{v-1} \nu_i + n_{\mathrm{T}} n_{\mathrm{R}} - 1\right)!}{(n_{\mathrm{T}} n_{\mathrm{R}} - 1)!} \cdot \left(\prod_{i=1}^{v-1} \left(\frac{1}{p_{r(i)+1}} \right)^{\nu_i} \right) \\ & \cdot \left(\frac{p_u \cdot \Psi(m)}{\Upsilon(m,\eta)} \right)^{\sum_{i=1}^{v-1} \nu_i} \cdot \left(\frac{n_{\mathrm{T}}}{\Upsilon(m,\eta)} \right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \sum_{\kappa=0}^{\sum_{i=1}^{v-1} \nu_i + n_{\mathrm{T}} n_{\mathrm{R}} - 1} (\kappa)^{-1} \\ & \cdot \left[e^{\frac{-\gamma_{\mathrm{th},m-1}^{(u)} n_{\mathrm{T}} \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)}} \left(\frac{\gamma_{\mathrm{th},m-1}^{(u)} n_{\mathrm{T}} \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right)^{\kappa} - e^{\frac{-\gamma_{\mathrm{th},m}^{(u)} n_{\mathrm{T}} \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)}} \left(\frac{\gamma_{\mathrm{th},m}^{(u)} n_{\mathrm{T}} \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right)^{\kappa} \right].\end{aligned} \tag{3.76}$$

With (3.76), the average BER of user u can be determined as a function of the impairment parameters ρ_u and $\sigma_{\mathrm{E},u}^2$, the weighting vector $\mathbf{p}$, the number of transmit and receive antennas n_{T} and n_{R}, the average SNR $\bar{\gamma}_u$ and the number of adaptively served users U_{A}.

3.6.2.1.4.4 Adaptive users applying TAS-MRC In order to determine the average user data rate $\bar{R}_{A,\mathrm{TAS-NAF}}^{(u)}$ of user u in a TAS-MRC system, (3.65) can also be used, however, $r_{n_{\mathrm{T}}}$ is set to 1 since no Space Time Coding is applied, resulting in

$$\bar{R}_{A,\mathrm{TAS-NAF}}^{(u)} = \sum_{m=1}^{M} b_m \cdot \left(F_{\mathrm{TAS-NAF},\hat{\gamma}}^{(u)}(\gamma_{\mathrm{th},m}^{(u)}) - F_{\mathrm{TAS-NAF},\hat{\gamma}}^{(u)}(\gamma_{\mathrm{th},m-1}^{(u)}) \right). \tag{3.77}$$

Exploiting (3.55), the average BER $\overline{BER}_{A,\mathrm{TAS-NAF}}^{(u)}$ of user u in a TAS-MRC system can be written as

$$\overline{BER}_{A,\mathrm{TAS-NAF}}^{(u)} = \overline{BER}_{A,\mathrm{STC-NAF}}^{(u)}(\mathbf{p}', U', n_{\mathrm{T}}', n_{\mathrm{R}}', r_{n_{\mathrm{T}}}') \tag{3.78}$$

with $U_{\mathrm{A}}' = n_{\mathrm{T}} \cdot U_{\mathrm{A}}$, $n_{\mathrm{T}}' = 1$, $n_{\mathrm{R}}' = n_{\mathrm{R}}$, $r_{n_{\mathrm{T}}}' = 1$ and $\mathbf{p}'$ as defined in (3.19).

3.6.2.1.4.5 Special case pure adaptive and pure non-adaptive resource allocation As shown in Section 3.3, the two special cases of a conventional pure adaptive transmission scheme and a conventional pure non-adaptive transmission scheme are incorporated in the Non-Adaptive First allocation scheme. If the user serving vector is set to

$$\vartheta = [0,\ 0,\ ...,\ 0],$$

there are no adaptively served users but only U non-adaptively served users with a user data rate and bit error rate given by (3.58) and (3.61).

If the user serving vector is set to

$$\vartheta = [1,\ 1,\ ...,\ 1],$$

there are no non-adaptively served users. Hence, all U users are served adaptively resulting in a user data rate and BER given by (3.66) and (3.76) in case of an OSTBC-MRC system and in case of a TAS-MRC system given by (3.77) and (3.78) with

$$U_{\mathrm{A}} = U.$$

3.6.2.2 Adaptive First

3.6.2.2.1 Introduction In this section, the Adaptive First resource allocation scheme is analyzed concerning the channel access and resulting SNR distribution of the adaptively and non-adaptively allocated resource units assuming that the user serving vector ϑ is given.

3.6.2.2.2 Channel access As shown in Section 3.3, when applying the Adaptive First scheme, first all available resource units N_{ru} are allocated to the $U_{\mathrm{A}} = \vartheta^T\vartheta$ adaptive users following the WPFS policy. Now, part of the allocated resource units have to be re-assigned to the non-adaptively served users. With the channel access demand vector $\mathbf{D}$, the number of resource units which are demanded by the total number $U_{\mathrm{N}} = U - U_{\mathrm{A}}$ of non-adaptive users is given by

$$W_{\mathrm{N}} = \sum_{\substack{u=1\\ \vartheta_u=0}}^{U} D_u, \tag{3.79}$$

i.e., the channel access demand of the non-adaptively served users is fulfilled. From this it follows that only

$$W_{\mathrm{A}} = N_{\mathrm{ru}} - W_{\mathrm{N}} \tag{3.80}$$

resource units are available for adaptive users. To determine which of the N_{ru} resource units are allocated to adaptive users, simply the W_{A} out of N_{ru} resource units with the best weighted and normalized SNR values are taken into account. Like with the Non-Adaptive First scheme, the probability that a resource unit is allocated to a given adaptively served user u depends on the weighting factors $\mathbf{p}$. Again, the channel access probability for adaptive users has to be determined in order to be able to adjust the weighting factors $\mathbf{p}$ such that D_u resource units are allocated to each adaptive user u on average.

3.6.2.2.2.1 Calculation of the channel access probability for adaptive users applying OSTBC-MRC In order to determine how many resource units are allocated to an adaptive user u applying the Adaptive First scheme in an OSTBC-MRC system depending on the weighting factors $\mathbf{p}$ of all users, it is important to identify the possibilities a resource unit is allocated to a given user u. Assuming there are N_{ru} resource units from which W_{A} are taken into account for scheduling, the probability $P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p})$ that the w-th best resource unit with $w = 1, .., W_{\mathrm{A}}$ is allocated to user u is given by

$$\begin{aligned} P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}) \;=\; & N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \int_0^\infty \left(\frac{n_{\mathrm{T}}}{p_u}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\gamma^{n_{\mathrm{T}} n_{\mathrm{R}}-1}}{(n_{\mathrm{T}} n_{\mathrm{R}}-1)!} \\ & \cdot e^{-\frac{n_{\mathrm{T}}\gamma}{p_u}} \cdot \left(\prod_{\substack{i=1 \\ i\neq u}}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p_i}\right)^v \right) \right) \\ & \cdot \left(1 - \prod_{i=1}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p_i}\right)^v \right) \right)^{w-1} \\ & \cdot \left(\prod_{i=1}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p_i}\right)^v \right) \right)^{N_{\mathrm{ru}}-w} d\gamma. \end{aligned} \tag{3.81}$$

The first terms in the integral of (3.81) outside the bracket represents the probability that the weighted and normalized SNR value of user u has the value γ. Note that the whole range of SNR values from $\gamma = 0$ to $\gamma = \infty$ is considered in the integral, i.e., it does not matter if the probability of the SNR to have the value γ is almost zero. The first bracket term represents the probability that the weighted and normalized SNR value of all other users in this resource unit is smaller than γ, i.e., user u has the highest WPFS ratio for this resource unit. The second bracket term represents the probability that there are $w-1$ resource units whose highest WPFS ratio is higher than the value γ. The third bracket term represents the probability that there are $N_{\mathrm{ru}} - w$ resource units whose highest WPFS ratio is smaller than the value γ, i.e., user u has the highest WPFS ratio in the w-th best resource unit out of N_{ru} resource units. The factor N_{ru} in front of the integral takes into account the N_{ru} possible positions of the w-th best resource unit inside the total number N_{ru} of resource units. The factor

$\binom{N_{\mathrm{ru}}-1}{w-1}$ takes into account the possible positions of the $w-1$ better resource units inside the remaining $N_{\mathrm{ru}}-1$ resource units.

Applying the binomial theorem, (3.81) can be rewritten as

$$\begin{aligned} P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\cdot \int_0^{\infty} \left(\frac{n_{\mathrm{T}}}{p_u}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\gamma^{n_{\mathrm{T}} n_{\mathrm{R}}-1}}{(n_{\mathrm{T}} n_{\mathrm{R}}-1)!} \cdot e^{-\frac{n_{\mathrm{T}}\gamma}{p_u}} \\ &\cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p_i}\right)^{v} \right) \right) \\ &\cdot \left(\prod_{i=1}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p_i}\right)^{v} \right) \right)^{\varepsilon+N-w} d\gamma. \end{aligned} \quad (3.82)$$

With the extended weighting vector $\mathbf{p}'$ of length $(\varepsilon + N_{\mathrm{ru}} - w + 1) \cdot U_{\mathrm{A}}$ given by

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{(\varepsilon+N_{\mathrm{ru}}-w+1)\text{ - times}} , \quad (3.83)$$

(3.82) can be written as

$$\begin{aligned} P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\cdot \int_0^{\infty} \left(\frac{n_{\mathrm{T}}}{p'_u}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\gamma^{n_{\mathrm{T}} n_{\mathrm{R}}-1}}{(n_{\mathrm{T}} n_{\mathrm{R}}-1)!} \cdot e^{-\frac{n_{\mathrm{T}}\gamma}{p'_u}} \\ &\cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_{\mathrm{A}}\cdot(\varepsilon+N_{\mathrm{ru}}-w+1)} \left(1 - e^{-\frac{n_{\mathrm{T}}\gamma}{p'_i}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}}\gamma}{p'_i}\right)^{v} \right) \right) d\gamma. \end{aligned} \quad (3.84)$$

Comparing (3.84) with (3.13), it can be seen that

$$\begin{aligned} P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)), \end{aligned} \quad (3.85)$$

i.e., the channel access probability applying the Adaptive First scheme can be calculated utilizing the channel access probability for the Non-Adaptive First scheme.

From this, it follows that the average number of resource units allocated to user u is given by

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{STC-AF}}(i, N_{\mathrm{ru}}, \mathbf{p}). \quad (3.86)$$

3.6.2.2.2.2 Calculation of the channel access probability for adaptive users applying TAS-MRC To determine the the probability $P^{(u)}_{\text{TAS-AF}}(w, N_{\text{ru}}, \mathbf{p})$ of an adaptive user u to get access to the w-best out of the N_{ru} available resource units, (3.20) can be utilized, i.e., the channel access probability $P_{\text{TAS-NAF}}(u, \mathbf{p})$ of user u in a TAS-MRC system applying the Non-Adaptive First scheme can be written as a special case of the channel access probability $P_{\text{STC-NAF}}(u, \mathbf{p})$ of user u in an OSTBC-MRC system applying the Non-Adaptive First scheme. Thus, applying (3.85) to a TAS-MRC system leads to

$$\begin{aligned} P^{(u)}_{\text{TAS-AF}}(w, N_{\text{ru}}, \mathbf{p}) &= N_{\text{ru}} \cdot \binom{N_{\text{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\quad \cdot n_{\text{T}} \cdot P^{(u)}_{\text{STC-NAF}}(\mathbf{p}', U'_{\text{A}}, n'_{\text{T}}, n'_{\text{R}}) \end{aligned} \tag{3.87}$$

with

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad ... \quad \mathbf{p}]}_{n_{\text{T}}\cdot(N_{\text{ru}}+\varepsilon+w-1)\text{- times}} , \tag{3.88}$$

$U'_{\text{A}} = U_{\text{A}} \cdot (N_{\text{ru}} + \varepsilon + w - 1)$, $n'_{\text{T}} = 1$ and $n'_{\text{R}} = n_{\text{R}}$.

Hence, the average number of allocated resource units to user u in a TAS-MRC system applying the Adaptive First scheme is given by

$$E\{N_{\text{ru},u}\} = \sum_{i=1}^{W_{\text{A}}} P^{(u)}_{\text{TAS-AF}}(i, N_{\text{ru}}, \mathbf{p}). \tag{3.89}$$

3.6.2.2.2.3 Calculation of weighting factors The calculation of the weighting factors applying the Adaptive First scheme can be done similarly as for the Non-Adaptive First scheme. In this case, the following equation must be hold for all users such that the user demands are fulfilled:

$$E\{N_{\text{ru},i}\} = \sum_{\eta=1}^{W_{\text{A}}} P^{(i)}_{\text{AF}}(\eta, N_{\text{ru}}, \mathbf{p}) = D_i \quad \forall\, i = 1, .., U_{\text{A}} - 1. \tag{3.90}$$

This constrained nonlinear optimization problem

$$\begin{aligned} \mathbf{p}^{\star} &= \arg\min_{\mathbf{p}} \left\{ \sum_{i=1}^{U_{\text{A}}-1} \left| \sum_{\eta=1}^{W_{\text{A}}} P^{(i)}_{\text{AF}}(\eta, N_{\text{ru}}, \mathbf{p}) - D_i \right| \right\} \\ &\text{subject to} \\ &\quad p_u \geq 1 \end{aligned} \tag{3.91}$$

can be solved as shown in Section 3.6.2.1.2.4.

3.6.2.2.3 SNR distribution

3.6.2.2.3.1 Introduction As for the Non-Adaptive First scheme, the distribution of the SNR values of the allocated resource units has to be derived for the Adaptive First scheme in order to analytically derive the performance of the system.

3.6.2.2.3.2 Non-adaptive users Since it is assumed that the channels of adjacent resource units are uncorrelated, the SNR distribution, the average data rate $\bar{R}_N^{(u)}$ and BER $\overline{BER}_N^{(u)}$ of a non-adaptive user u applying the Adaptive First strategy do *not* change compared to the case of the Non-Adaptive First since in both cases, the allocation is performed without using any CQI, i.e. randomly.

3.6.2.2.3.3 Adaptive users applying OSTBC-MRC For the case of adaptively served users, the PDF of the measured SNR value of the allocated resource units does change compared to the case of applying the Non-Adaptive First scheme. In a first step, the PDF $p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma})$ of the measured SNR of the allocated resource units from the w-th best out of N_{ru} resource units is derived.

To determine $p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma})$, first the joint PDF of all $N_{\mathrm{ru}} \cdot U_{\mathrm{A}}$ normalized SNR values $X_1, .., X_{U_{N_{\mathrm{ru}} \cdot \mathrm{A}}}$ in the system has to determined. Since the SNR values of different users and resource units are independent of each other and with the knowledge that the measured SNR values are chi-squared distributed, the joint PDF is given by

$$p_{X_1,..,X_{N_{\mathrm{ru}} \cdot U_{\mathrm{A}}}}(x_1, .., x_{N_{\mathrm{ru}} \cdot U_{\mathrm{A}}}) = p_{\hat{\gamma}_u}(x_1) \cdots p_{\hat{\gamma}_u}(x_{N_{\mathrm{ru}} \cdot U_{\mathrm{A}}}) \tag{3.92}$$

with

$$p_{\hat{\gamma}_u}(x) = \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{x^{n_{\mathrm{T}} n_{\mathrm{R}} - 1}}{(n_{\mathrm{T}} n_{\mathrm{R}} - 1)!} \cdot \exp\left(-\frac{n_{\mathrm{T}} \cdot x}{\bar{\gamma}_{E,u}}\right). \tag{3.93}$$

PDF $p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma})$ is then given by the marginal PDF resulting from determining the integral over the joint PDF leading to

$$
\begin{aligned}
p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{STC-AF},w}(u) \cdot \underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \dots \int_0^{\frac{p_u}{p_{U_\mathrm{A}}}\hat{\gamma}}}_{U_\mathrm{A}-1 \text{ - times}} \underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \dots \int_0^{\frac{p_u}{p_{U_\mathrm{A}}}\hat{\gamma}}}_{(N_\mathrm{ru}-w)\cdot U_\mathrm{A} \text{ - times}} \\
&\quad \underbrace{\int_{\frac{p_u}{p_1}\hat{\gamma}}^{\infty} \dots \int_{\frac{p_u}{p_{U_\mathrm{A}}}\hat{\gamma}}^{\infty}}_{(w-1)\cdot U_\mathrm{A} \text{ - times}} p_{X_1,..X_{N_\mathrm{ru}\cdot U_\mathrm{A}}}(\hat{\gamma}, y_1, .., y_{N_\mathrm{ru}\cdot U_\mathrm{A}-1})\, dy_1 \dots dy_{N_\mathrm{ru}\cdot U_\mathrm{A}-1} \\
&= a_{\mathrm{STC-AF},w}(u) \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}}\right)^{n_\mathrm{T}n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T}n_\mathrm{R}-1}}{(n_\mathrm{T}n_\mathrm{R}-1)!} \cdot e^{-\frac{n_\mathrm{T}\cdot\hat{\gamma}}{\bar{\gamma}_{E,u}}} \qquad (3.94) \\
&\quad \cdot \prod_{\substack{i=1 \\ i\neq u}}^{U_\mathrm{A}} \left(1 - e^{-\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{T}n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{n_\mathrm{T}\cdot p_u \cdot \hat{\gamma}}{p_i \cdot \bar{\gamma}_{E,u}}\right)^v\right) \\
&\quad \cdot \left(1 - \prod_{i=1}^{U_\mathrm{A}} \left(1 - e^{-\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{T}n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}\right)^v\right)\right)^{w-1} \\
&\quad \cdot \left(\prod_{i=1}^{U_\mathrm{A}} \left(1 - e^{-\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{T}n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}\right)^v\right)\right)^{N_\mathrm{ru}-w}
\end{aligned}
$$

where the factor $a_{\mathrm{STC-AF},w}(u)$ ensures that

$$
\int_0^{\infty} p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \qquad (3.95)
$$

Applying the binomial theorem, (3.94) can be rewritten as

$$
\begin{aligned}
p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{STC-AF},w}(u) \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \qquad (3.96) \\
&\quad \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}}\right)^{n_\mathrm{T}n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T}n_\mathrm{R}-1}}{(n_\mathrm{T}n_\mathrm{R}-1)!} \cdot e^{-\frac{n_\mathrm{T}\cdot\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\
&\quad \cdot \left(\prod_{\substack{i=1 \\ i\neq u}}^{U_\mathrm{A}} \left(1 - e^{-\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{T}n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}\right)^v\right)\right) \\
&\quad \cdot \left(\prod_{i=1}^{U_\mathrm{A}} \left(1 - e^{-\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{T}n_\mathrm{R}-1} \frac{1}{v!}\left(\frac{n_\mathrm{T}p_u\hat{\gamma}}{p_i\bar{\gamma}_{E,u}}\right)^v\right)\right)^{\varepsilon+N-w}.
\end{aligned}
$$

Introducing the extended weighting vector $\mathbf{p}'$ of length $(\varepsilon + N_\mathrm{ru} - w + 1) \cdot U_\mathrm{A}$ given by

$$
\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{(\varepsilon+N_\mathrm{ru}-w+1) \text{ - times}}, \qquad (3.97)
$$

(3.96) can be written as

$$
\begin{aligned}
p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma}) &= \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\
&\quad \cdot a_{\mathrm{STC-AF},w}(u) \cdot \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{T}} n_{\mathrm{R}}-1}}{(n_{\mathrm{T}} n_{\mathrm{R}}-1)!} \cdot e^{-\frac{n_{\mathrm{T}} \cdot \hat{\gamma}}{\bar{\gamma}_{E,u}}} \\
&\quad \cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)} \left(1 - e^{-\frac{n_{\mathrm{T}} p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}} p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}} \right)^{v} \right) \right)
\end{aligned} \tag{3.98}
$$

Performing the substitution of the variable $\gamma = \frac{\hat{\gamma} \cdot p_u}{\gamma_{\bar{E},u}}$ in the integral of (3.84), it can be seen that the integrals in (3.84) and (3.95) are identical except for the factor $a_{\mathrm{STC-AF},w}(u)$ leading to

$$
a_{\mathrm{STC-AF},w}(u) = \frac{1}{P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p})}. \tag{3.99}
$$

To finally determine the PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma})$ of the measured SNR values of the resource units allocated to user u, the sum over the W_{A} PDFs $p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma})$ with $w = 1, .., W_{\mathrm{A}}$ weighted by the probability $P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p})$ has to be calculated leading to

$$
\begin{aligned}
p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma}) &= \sum_{w=1}^{W_{\mathrm{A}}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_{\mathrm{ru}}, \mathbf{p})} \right) \cdot p^{(u)}_{\mathrm{STC-AF},w,\hat{\gamma}}(\hat{\gamma}) \\
&= \sum_{w=1}^{W_{\mathrm{A}}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_{\mathrm{ru}}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\
&\quad \cdot a_{\mathrm{STC-AF},w}(u) \cdot \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{T}} n_{\mathrm{R}}-1}}{(n_{\mathrm{T}} n_{\mathrm{R}}-1)!} \cdot e^{-\frac{n_{\mathrm{T}} \cdot \hat{\gamma}}{\bar{\gamma}_{E,u}}} \\
&\quad \cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)} \left(1 - e^{-\frac{n_{\mathrm{T}} p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}} p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}} \right)^{v} \right) \right)
\end{aligned} \tag{3.100}
$$

where the factor $\left(\sum_{\xi=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_{\mathrm{ru}}, \mathbf{p})\right)^{-1}$ ensures that

$$
\int_0^{\infty} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \tag{3.101}
$$

In Figure 3.8(a), the PDF of the measured SNR values of the resource units allocated to user $u = 1$ is depicted assuming a system with $U_{\mathrm{A}} = 3$ adaptively served users where all users have the same average SNR $\bar{\gamma}_u = 10$ dB and perfect CQI. In total, there are $N_{\mathrm{ru}} = 10$ resource units available where $W_{\mathrm{A}} = 6$ resource units are only used for the adaptive users. The weighting vector is given by

$$\mathbf{p} = [5,\ 2,\ 1].$$

The dashed line represents the analytical PDF according to (3.100) where the solid lines represents the PDF evaluated from 10000 simulation runs. Fig. 3.7(b) and 3.7(c) show the PDFs for user $u = 2$ and user $u = 3$. Again, one can see that the analytical PDFs are consistent with the simulative ones. Similarly to the Non-Adaptive First scheme, the probability of small SNR values is larger for users with a high weighting factor due to the SNR boosting of the WPFS.

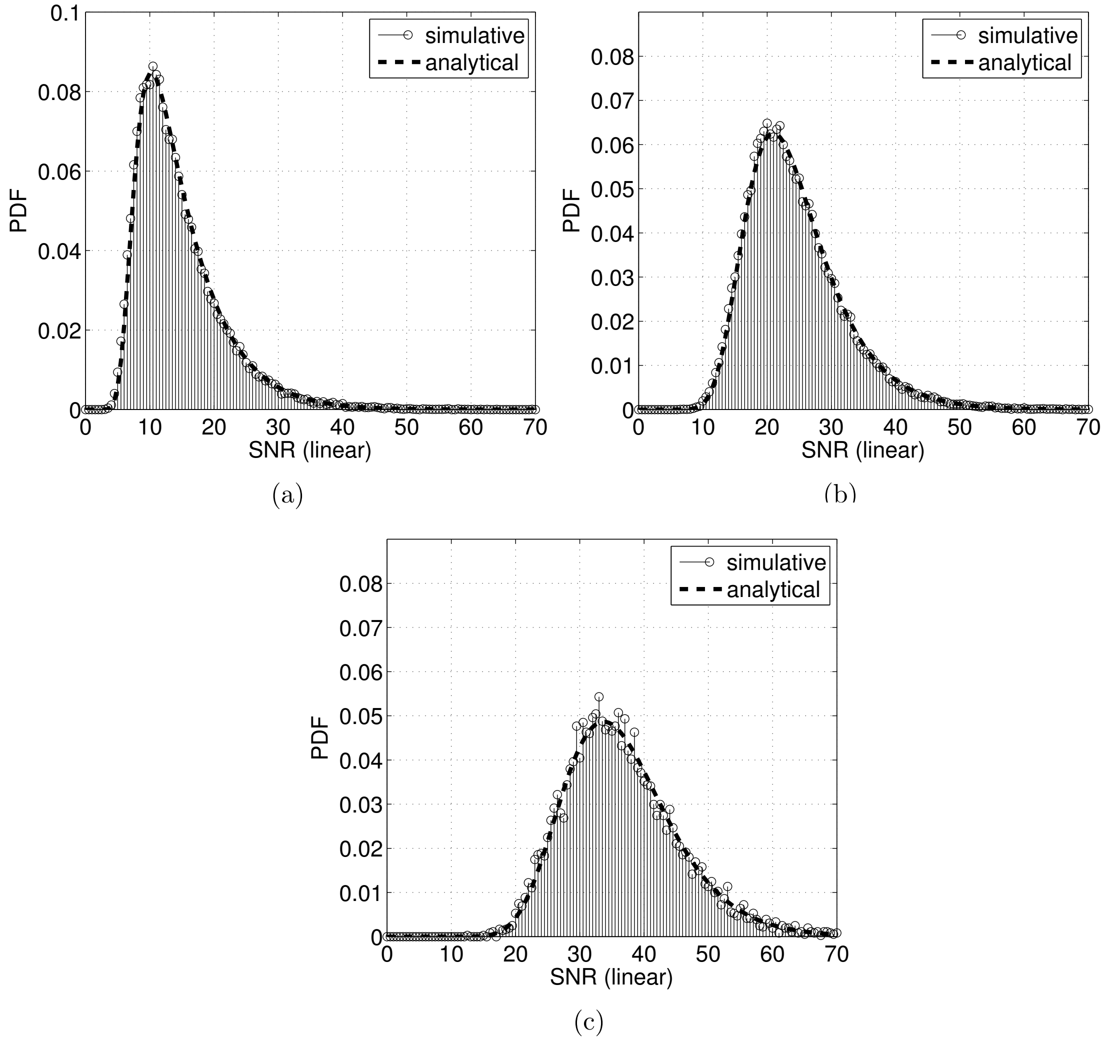


Figure 3.8. Analytical PDF and simulative PDF of the SNR of allocated resource units for user (a) $u = 1$ and (b) $u = 2$ and (c) $u = 3$ applying the Adaptive First scheme.

Examining (3.46), (3.98) and (3.100), it can be seen that the PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma})$ applying the Adaptive First scheme can be written as a weighted double sum of special cases of the PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ applying the Non-Adaptive first scheme given by

$$\begin{aligned} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma}) \;=\; & \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ & \cdot p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{STC-NAF}}(u)) \end{aligned} \tag{3.102}$$

with

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \ldots \quad \mathbf{p}]}_{(\varepsilon+N_\mathrm{ru}-w+1)\text{ - times}}, \tag{3.103}$$

$$U'_\mathrm{A} = U_\mathrm{A} \cdot (\varepsilon + N_\mathrm{ru} - w + 1) \tag{3.104}$$

and

$$a'_{\mathrm{STC-NAF}}(u) = a_{\mathrm{STC-AF},w}(u). \tag{3.105}$$

Thus, the CDF $F^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma})$ of the measured SNR of a resource unit allocated to user u applying the Adaptive first scheme is then given by

$$\begin{aligned} F^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma}) \;=\; & \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ & \cdot F^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{STC-NAF}}(u)) \end{aligned} \tag{3.106}$$

with $\mathbf{p}'$, U'_A and $a'_{\mathrm{STC-NAF}}(u)$ as defined in (3.103) to (3.105).

3.6.2.2.3.4 Adaptive users applying TAS-MRC Determining the PDF $p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma})$ of the SNR values of the resource units allocated to user u applying the Adaptive First scheme in a TAS system, first the PDF of the SNR value of the w-th best out of N_ru resource units has to be derived. To do so, the same derivation steps shown in (3.96) to (3.99) have to be done. However, PDF $p_{\hat{\gamma}_u}(x)$ has to be exchanged by the PDF $p^{(n_\mathrm{T})}_{\hat{\gamma}_{u_{n_\mathrm{T}}}}(x)$ given by (3.50) to incorporate the fact that the SNR is a result of a selection process out of n_T transmit antennas. Hence, the PDF $p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma})$ of the SNR of the w-th best resource unit allocated to user u applying the Adaptive First scheme results in

$$
\begin{aligned}
p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma}) &= a_{\mathrm{TAS-AF},w}(u) \cdot \frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{(n_{\mathrm{R}}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\
&\cdot \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}\right)^{v}\right)^{n_{\mathrm{T}}-1} \\
&\cdot \prod_{\substack{i=1\\ i\neq u}}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}} \cdot \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{p_u \cdot \hat{\gamma}}{p_i \cdot \bar{\gamma}_{E,u}}\right)^{v}\right)^{n_{\mathrm{T}}} \\
&\cdot \left(1 - \prod_{i=1}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}} p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}} p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}\right)^{v}\right)^{n_{\mathrm{T}}}\right)^{w-1} \\
&\cdot \left(\prod_{i=1}^{U_{\mathrm{A}}} \left(1 - e^{-\frac{n_{\mathrm{T}} p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{T}} n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{n_{\mathrm{T}} p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}\right)^{v}\right)^{n_{\mathrm{T}}}\right)^{N_{\mathrm{ru}}-w}
\end{aligned}
\tag{3.107}
$$

which can be rewritten as

$$
\begin{aligned}
p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma}) &= \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\
&\cdot a_{\mathrm{TAS-AF},w}(u) \cdot \frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{(n_{\mathrm{R}}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\
&\cdot \left(\prod_{\substack{i=1\\ i\neq u}}^{n_{\mathrm{T}} \cdot U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)} \left(1 - e^{-\frac{p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}}\right)^{v}\right)\right)
\end{aligned}
\tag{3.108}
$$

with $\mathbf{p}'$ given by

$$
\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{n_{\mathrm{T}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)\text{ - times}} \quad , \tag{3.109}
$$

Again, the factor $a_{\mathrm{TAS-AF},w}(u)$, which ensures that

$$
\int_0^{\infty} p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \tag{3.110}
$$

with

$$
\begin{aligned}
a_{\mathrm{TAS-AF},w}(u) &= \frac{1}{P^{(u)}_{\mathrm{TAS-AF}}(\mathbf{p})} \\
&= \frac{1}{n_{\mathrm{T}} \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U'_{\mathrm{A}}, n'_{\mathrm{T}}, n'_{\mathrm{R}})}.
\end{aligned}
\tag{3.111}
$$

with $U'_{\mathrm{a}} = n_{\mathrm{T}} \cdot U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)$, $n'_{\mathrm{T}} = 1$, $n'_{\mathrm{R}} = n_{\mathrm{R}}$ and $\mathbf{p}'$ as defined in (3.109).

Like in the case of OSTBC-MRC, the PDF $p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma})$ of the measured SNR values of the resource units allocated to user u is calculated by the sum over the W_A PDFs $p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma})$ with $w = 1, .., W_\mathrm{A}$ weighted by the probability $P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})$ leading to

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma}) &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \cdot p^{(u)}_{\mathrm{TAS-AF},w,\hat{\gamma}}(\hat{\gamma}) \qquad (3.112)\\ &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^\varepsilon \\ &\cdot a_{\mathrm{TAS-AF},w}(u) \cdot \frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \\ &\cdot \left(\prod_{\substack{i=1\\ i\neq u}}^{n_\mathrm{T}\cdot U_\mathrm{A}\cdot(\varepsilon+N_\mathrm{ru}-w+1)} \left(1 - e^{-\frac{p'_u\hat{\gamma}}{p'_i\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{p'_u\hat{\gamma}}{p'_i\bar{\gamma}_{E,u}} \right)^v \right) \right) \end{aligned}$$

where the factor $\left(\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})\right)^{-1}$ ensures that

$$\int_0^\infty p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \qquad (3.113)$$

As done in the case of an OSTBC-MRC system applying the Adaptive First scheme, the PDF $p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma})$ for a TAS-MRC system applying the Adaptive First scheme can be written as a weighted double sum of the PDF $p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma})$ applying the Non-Adaptive First scheme given by

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma}) &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^\varepsilon \\ &\cdot p^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{STC-NAF}}(u)) \qquad (3.114) \end{aligned}$$

with

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad ... \quad \mathbf{p}]}_{(\varepsilon+N_\mathrm{ru}-w+1)\text{ - times}}, \qquad (3.115)$$

$$U'_\mathrm{A} = U_\mathrm{A} \cdot (\varepsilon + N_\mathrm{ru} - w + 1) \qquad (3.116)$$

and

$$a'_{\mathrm{TAS-NAF}}(u) = a_{\mathrm{TAS-AF},w}(u). \qquad (3.117)$$

From this, follows that the CDF $F^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma})$ of the measured SNR of a resource unit allocated to user u applying the Adaptive first scheme in a TAS-MRC system is then given by

$$F^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\hat{\gamma}) = \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^\varepsilon \cdot F^{(u)}_{\mathrm{TAS-NAF},\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{TAS-NAF}}(u)) \qquad (3.118)$$

with $\mathbf{p}'$, U'_A and $a'_{\mathrm{TAS-NAF}}(u)$ as defined in (3.115) to (3.117).

3.6.2.2.4 Average user data rate and BER taking into account imperfect CQI

3.6.2.2.4.1 Non-adaptive users As mentioned in Section 3.6.2.2.3, the average data rate $\bar{R}^{(u)}_N$ and BER $\overline{BER}^{(u)}_N$ of a non-adaptive user u applying the Adaptive First scheme is equivalent to the average data rate and BER applying the Non-Adaptive First scheme derived in Section 3.6.2.1.4.

3.6.2.2.4.2 Adaptive users applying OSTBC-MRC Similar to the Non-Adaptive First scheme, the average user data $\bar{R}^{(u)}_{A,\mathrm{STC-AF}}$ of user u can be determined using the definition of (3.65) while exchanging the PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ by the PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma})$. With (3.106), the average user data rate $\bar{R}^{(u)}_{A,\mathrm{STC-AF}}$ applying the Adaptive First scheme in an OSTBC-MRC system is given by

$$\bar{R}^{(u)}_{A,\mathrm{STC-AF}} = r_{n_\mathrm{T}} \cdot \sum_{m=1}^{M} b_m \cdot \left(F^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\gamma^{(u)}_{\mathrm{th},m}) - F^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\gamma^{(u)}_{\mathrm{th},m-1}) \right). \qquad (3.119)$$

For the calculation of the average BER, the definition (3.73) can be used. Again, PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ has to be exchanged by PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma}}(\hat{\gamma})$. Keeping in mind that $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ can be written as a sum of special cases of PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ as shown in (3.102), the average user BER $\overline{BER}^{(u)}_{A,\mathrm{STC-AF}}$ of user u applying the Adaptive First scheme in an OSTBC-MRC system can be written as a sum of the average user BER $\overline{BER}^{(u)}_{A,\mathrm{STC-NAF}}$ applying the Non-Adaptive First scheme in an OSTBC-MRC system leading to

$$\overline{BER}^{(u)}_{A,\mathrm{STC-AF}} = \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^\varepsilon \cdot \overline{BER}^{(u)}_{A,\mathrm{STC-NAF}}(\mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{STC-NAF}}(u)) \qquad (3.120)$$

with $\overline{BER}^{(u)}_{A,\mathrm{STC-NAF}}$ as defined in (3.76) and $\mathbf{p}'$, U'_A and $a'_{\mathrm{STC-NAF}}(u)$ as defined in (3.103) to (3.105).

3.6.2.2.4.3 Adaptive users applying TAS-MRC The average user data rate $\bar{R}^{(u)}_{A,\mathrm{TAS-AF}}$ applying the Adaptive First scheme in a TAS-MRC system is given by

$$\bar{R}^{(u)}_{A,\mathrm{TAS-AF}} = \sum_{m=1}^{M} b_m \cdot \left(F^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\gamma^{(u)}_{\mathrm{th},m}) - F^{(u)}_{\mathrm{TAS-AF},\hat{\gamma}}(\gamma^{(u)}_{\mathrm{th},m-1}) \right). \tag{3.121}$$

With the same considerations done in (3.120), the average user BER $\overline{BER}^{(u)}_{A,\mathrm{TAS-NAF}}$ applying the Adaptive First scheme can be written as a sum of the average user BER $\overline{BER}^{(u)}_{A,\mathrm{TAS-NAF}}$ applying the Non-Adaptive First scheme in an OSTBC-MRC system leading to

$$\begin{aligned} \overline{BER}^{(u)}_{A,\mathrm{TAS-AF}} &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-AF}}(w, N_\mathrm{ru}, \mathbf{p})}{\sum_{\xi=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-AF}}(\xi, N_\mathrm{ru}, \mathbf{p})} \right) \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\quad \cdot \overline{BER}^{(u)}_{A,\mathrm{TAS-NAF}}(\mathbf{p}', U'_\mathrm{A}, a'_{\mathrm{TAS-NAF}}(u)) \end{aligned} \tag{3.122}$$

with $\overline{BER}^{(u)}_{A,\mathrm{TAS-NAF}}$ as defined in (3.78) and $\mathbf{p}'$, U'_A and $a'_{\mathrm{TAS-NAF}}(u)$ as defined in (3.115) to (3.117).

3.6.2.2.4.4 Special case pure adaptive and pure non-adaptive resource allocation Applying the Adaptive First scheme, also the two special cases of pure adaptive and pure non-adaptive resource allocation are incorporated. For the special case

$$\vartheta = [0,\ 0,\ ...,\ 0],$$

with U non-adaptively served users, the user data rate and bit error rate are given by (3.58) and (3.61) since there is no difference in the performance compared to the Non-Adaptive First scheme as denoted in Section 3.6.2.2.3.

For the second special case

$$\vartheta = [1,\ 1,\ ...,\ 1],$$

with U non-adaptively served users, there is no longer a different between Non-Adaptive First and Adaptive First since there are no non-adaptive users. Thus, the user data rate and BER are given by (3.66) and (3.76) in case of an OSTBC-MRC system and in case of a TAS-MRC system given by (3.77) and (3.78) with

$$U_\mathrm{A} = U.$$

Note that the equations for the average user data rate and BER derived for the Adaptive First scheme also lead to results given by (3.66), (3.76), (3.77) and (3.78) for the case that $U_\mathrm{A} = U$, i.e., $W_\mathrm{A} = N_\mathrm{ru}$.

3.6.2.3 Optimizing SNR thresholds

3.6.2.3.1 Non-adaptive users In the following, the optimal SNR threshold vector $\gamma_{\text{th}}^{(u)}$ which solves the SNR threshold problem (3.8) has to be found.

Maximizing the data rate of non-adaptive users, subproblem (3.8) can be simplified to

$$\begin{aligned} \bar{R}_{\text{N,max}}^{(u)} &= \max_{m} \left(\bar{R}_{\text{N}}^{(u)} \right) \\ & \quad \text{subject to} \\ & \quad \overline{BER}_{\text{N}}^{(u)}(m) \leq BER_{\text{T}}. \end{aligned} \tag{3.123}$$

since only one modulation scheme is used for each user. As $\overline{BER}_{\text{N}}^{(u)}(m)$ cannot be written in closed form, the modulation scheme m which maximizes the user date rate $\bar{R}_{\text{N}}^{(u)}$ for a given average SNR $\bar{\gamma}_u$ and number D_u of allocated resource units subject to the target BER cannot be determined analytically but has to betermined by testing all M possible modulation schemes, where in a realistic scenario the number M of available modulation schemes can be assumed to be smaller than $M < 10$. Note that this optimization problem can be done off-line for a finite number of values for $\bar{\gamma}_u$ and D_u and the results can be stored in a look-up table.

3.6.2.3.2 Adaptive users To solve (3.8) for adaptive users, a Lagrange multiplier approach similar to [MT05] is performed where the objective function is given by

$$\Phi^{(u)}(\gamma_{\text{th}}^{(u)}) = \bar{R}_{A}^{(u)}(\gamma^{(u)}) + \lambda \cdot \left(\bar{R}_{A}^{(u)}(\gamma_{\text{th}}^{(u)}) \overline{BER}_{A}^{(u)}(\gamma_{\text{th}}^{(u)}) - \bar{R}_{A}^{(u)}(\gamma_{\text{th}}^{(u)}) BER_{T} \right) \tag{3.124}$$

with λ denoting the Lagrange multiplier. Note that $\overline{BER}_{A}^{(u)}(\gamma_{\text{th}}^{(u)})$ and $\bar{R}_{A}^{(u)}(\gamma_{\text{th}}^{(u)})$ represent the BER and data rate applying both resource allocation strategies NAF and AF in both OSTBC-MRC and TAS-MRC systems, respectively. Using (3.73) and (3.71), the objective function can be rewritten as

$$\begin{aligned} \Phi^{(u)}(\gamma_{\text{th}}^{(u)}) &= (1 - \lambda BER_{T}) \sum_{m=1}^{M} b_m \int_{\gamma_{\text{th},m-1}^{(u)}}^{\gamma_{\text{th},m}^{(u)}} p_{\hat{\gamma}}^{(u)}(\hat{\gamma})\, d\hat{\gamma} \\ & \quad + \lambda \sum_{m=1}^{M} b_m \int_{\gamma_{\text{th},m-1}^{(u)}}^{\gamma_{\text{th},m}^{(u)}} p_{\hat{\gamma}}^{(u)}(\hat{\gamma}) \cdot \widehat{BER}_{m}^{(u)}(\hat{\gamma})\, d\hat{\gamma}. \end{aligned} \tag{3.125}$$

In order to determine the optimal threshold vector $\gamma_{\text{th,opt}}^{(u)}$, $\Phi^{(u)}(\gamma_{\text{th}}^{(u)})$ has to be differentiated with respect to the elements of $\gamma_{\text{th}}^{(u)}$, where

$$\frac{\partial \Phi^{(u)}(\gamma_{\text{th,opt}}^{(u)})}{\partial \gamma_{\text{th},m}^{(u)}} = 0 \tag{3.126}$$

must hold for all $m = 1, .., M-1$. Using the fact that

$$\frac{\partial}{\partial x}\left(\int_0^x f(z) \cdot G(z)\, dz\right) = f(x) \cdot G(x), \tag{3.127}$$

the derivation results in $M-1$ equations given by

$$\frac{(1-\lambda BER_T)}{\lambda} = \frac{\widehat{BER}_m^{(u)}(\gamma_{\mathrm{th},m}^{(u)}) \cdot b_m - \widehat{BER}_{m+1}^{(u)}(\gamma_{\mathrm{th},m}^{(u)}) \cdot b_{m+1}}{b_{m+1} - b_m}. \tag{3.128}$$

From (3.128), it can be seen that each element $\gamma_{\mathrm{th},m}^{(u)}$ of the optimal threshold vector $\gamma_{\mathrm{th,opt}}^{(u)}$ can be calculated using an initial value $\gamma_{\mathrm{th},1}^{(u)}$. Thus, each threshold vector $\gamma_{\mathrm{th}}^{(u)}$ is a function of the initial value $\gamma_{\mathrm{th},1}^{(u)}$, i.e.,

$$\gamma_{\mathrm{th}}^{(u)} = f(\gamma_{\mathrm{th},1}^{(u)}). \tag{3.129}$$

Determining the maximum average data rate subject to the target BER, the optimal initial value $\gamma_{\mathrm{th,opt},1}^{(u)}$ has to be found which fulfills

$$\overline{BER}_A^{(u)}(f(\gamma_{\mathrm{th,opt},1}^{(u)})) \leq BER_T \tag{3.130}$$

resulting in

$$\bar{R}_{A,max}^{(u)} = \bar{R}_A^{(u)}\left(\gamma_{\mathrm{th,opt}}^{(u)}\right), \tag{3.131}$$

which can be done numerically using for example the *fzero* function in MATLAB™.

3.6.3 FDD systems

3.6.3.1 Non-Adaptive First

3.6.3.1.1 Introduction In this section, the Non-Adaptive First resource allocation scheme in an FDD system is analyzed concerning the channel access and resulting SNR distribution of the adaptively and non-adaptively allocated resource units assuming that the user serving vector ϑ is given.

3.6.3.1.2 Channel access For a non-adaptively served user u with $\vartheta_u = 0$ nothing changes compared to a TDD system regarding channel access since in both TDD and FDD systems, the resource allocation is performed without considering any instantaneous CQI. Each non-adaptive gets access to D_u resource units following a round robin policy, i.e. the channel access demand is fulfilled for the non-adaptive users.

The remaining W_{A} resource units calculated according to (3.10) are then allocated to the $U_{\mathrm{A}} = \vartheta^T \vartheta$ adaptive users following the QWPFS policy as shown in Section 2.8.4. Similar to WPFS, QWPFS employs a user-dependent weighting factor p_u to adjust the probability of getting access to the channel. However, the SNR values are quantized and no longer continuous as in case of a TDD system. Hence, the calculation of the channel access probability $P^{(u)}(\mathbf{p})$ of an adaptively served user u with $\vartheta_u = 1$ is different compared to the TDD case. In the following, it is shown how to compute the channel access probability for adaptive users and how to adjust the weighting factors $\mathbf{p}$ such that each adaptive user u is allocated to D_u resource units on average.

3.6.3.1.2.1 Calculation of the channel access probability for adaptive users applying OSTBC-MRC The probability $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p})$ of the adaptive user u to get access to a resource unit in a system applying OSTBC at the transmitter and MRC at the receiver is now derived as a function of the weighting vector $\mathbf{p}$. For sake of a better understanding, it is firstly assumed that the feedback link is error-free ($p_b = 0$), i.e., the quantization levels of the SNR values of the different users are perfectly known at the BS. Later on, also the case with imperfect feedback link is discussed.

Recalling the QWPFS policy given by (2.63), it is can be seen that only the user $u^{\star}(n,k)$ with the highest normalized quantized and weighted SNR value gets access to resource unit n in time frame k. In case that several users have the same weighted SNR value, one user is randomly selected. Note that it is assumed that the SNR thresholds are fixed and the same for each user.

In the following, the events which have to occur in order that a given resource unit is allocated to user u with weighting factor p_u and a normalized SNR value $\frac{\gamma_u(n,k)}{\bar{\gamma}_u}$ which lies in the q-th quantization level $[\gamma_{\mathrm{th},q-1}, \gamma_{\mathrm{th},q}]$ are specified:

1. The normalized SNR value of user u must lie in the q-th quantization level.
2. User u must successfully compete against all users which have a weighting factor equivalent to p_u and whose normalized SNR also lies in the q-th quantization level.
3. All other users which have the same weighting factor as user u must have an SNR value which lies beneath the q-th quantization level.
4. User u must successfully compete against all users which have a higher weighting factor p_v with $p_v > p_u$ but whose SNR value lies in a lower quantization level l with $l < q$ such that the resulting weighted SNR value $p_v \cdot l = p_u \cdot q$.

5. All other users which have a higher weighting factor p_v must have an SNR value lying in the l-th quantization level such that the resulting weighted SNR value $p_v \cdot l < p_u \cdot q$.

6. User u must successfully compete against all users which have a lower weighting factor p_v with $p_v < p_u$ but whose SNR value lies in a higher quantization level l with $l > q$ such that the resulting weighted SNR value $p_v \cdot l = p_u \cdot q$.

7. All other users which have a lower weighting factor p_v must have an SNR value lying in the l-th quantization level such that the resulting weighted SNR value $p_v \cdot l < p_u \cdot q$.

To determine the access probability, three different sets of users are introduced: First, the set $\mathcal{S}_{\mathrm{sw}}^{(u)}$ of users which have the same weighting factor as user u. Second, the set $\mathcal{S}_{\mathrm{hw}}^{(u)}$ of users which have a higher weighting factor than user u. Third, the set $\mathcal{S}_{\mathrm{lw}}^{(u)}$ of users which have a lower weighting factor than user u.

Furthermore, the sets $\mathcal{S}_{\mathrm{sw}}^{(u)}$ and $\mathcal{S}_{\mathrm{hw}}^{(u)}$ have to be further subdivided to determine the access probability.

First, for each quantization level $q = 1, .., L$ with $L = 2^{N_Q}$ it has to be checked whether there are users with a weighting factor p_v higher than p_u but with a quantization level l_v lower than q such that $p_v \cdot l_v = p_u \cdot q$. Hence, for each user v of set $\mathcal{S}_{\mathrm{hw}}^{(u)}$ it has to be determined whether

$$l_v = \frac{p_u \cdot q}{p_v} \tag{3.132}$$

is an integer number. If this the case for user v of set $\mathcal{S}_{\mathrm{hw}}^{(u)}$, user v is put in the set $\mathcal{S}_{\mathrm{hw},q}^{(u)}$. The corresponding quantization level l_v is stored in the vector $\mathbf{l}_{\mathrm{hw},q}^{(u)}$.

Next, for each quantization level $q = 1, .., L$ it is checked whether there are users with a weighting factor p_v lower than p_u but with a quantization level l higher than q such that $p_v \cdot l = p_u \cdot q$. Hence, for each user v of set $\mathcal{S}_{\mathrm{sw}}^{(u)}$ it has to be determined whether

$$l_v = \frac{p_u \cdot q}{p_v}$$

is an integer number with $q < l_v < L$. If this the case for user v of set $\mathcal{S}_{\mathrm{sw}}^{(u)}$, user v is put in the set $\mathcal{S}_{\mathrm{sw},q}^{(u)}$ where the corresponding quantization level l_v is stored in the vector $\mathbf{l}_{\mathrm{sw},q}^{(u)}$.

The following example shall illustrate this procedure. Let us assume a system with $U_\mathrm{A} = 6$ users, with $N_Q = 2$ quantization bits (i.e., $q = 1, .., 4$) and with the weighting vector $\mathbf{p}$ given by

$$\mathbf{p} = \left[6,\ 4,\ 2,\ 2,\ \frac{4}{3}\ 1\right].$$

The user under consideration is user $u = 3$ and the considered quantization level is $q = 2$. The three sets of users are then given by

$$\begin{aligned} \mathcal{S}_\mathrm{sw}^{(3)} &= \{4\}, \\ \mathcal{S}_\mathrm{hw}^{(3)} &= \{1,\ 2\}, \\ \mathcal{S}_\mathrm{lw}^{(3)} &= \{5,\ 6\}, \end{aligned}$$

respectively, i.e., user $u = 4$ has the same weighting factor as user $u = 3$ while users $u = 1$ and $u = 2$ have higher weighting factors and users $u = 5$ and $u = 6$ have lower weighting factors. For the quantization level $q = 2$, the set $\mathcal{S}_{\mathrm{hw},q}^{(u)}$ and the vector $\mathbf{l}_{\mathrm{hw},q}^{(u)}$ are given by

$$\begin{aligned} \mathcal{S}_{\mathrm{hw},2}^{(3)} &= \{2\}; \\ \mathbf{l}_{\mathrm{hw},2}^{(u)} &= [1], \end{aligned}$$

respectively, i.e., if user $u = 2$ has a normalized SNR value which lies in the 1-st quantization level, the weighted SNR values of users $u = 3$ and $u = 6$ are equal, since $p_3 \cdot 2 = p_2 \cdot 1 = 4$.

The set $\mathcal{S}_{\mathrm{sw},q}^{(u)}$ and the vector $\mathbf{l}_{\mathrm{sw},q}^{(u)}$ for $q = 2$ are given by

$$\begin{aligned} \mathcal{S}_{\mathrm{sw},2}^{(3)} &= \{5,\ 6\}, \\ \mathbf{l}_{\mathrm{sw},1}^{(u)} &= [3,\ 4], \end{aligned}$$

respectively, i.e., if user $u = 5$ has a normalized SNR value which lies in the 3-rd quantization level, the weighted SNR values of users $u = 3$ and $u = 6$ are equal, since $p_3 \cdot 2 = p_5 \cdot 3 = 4$. Further on, if user $u = 6$ has a normalized SNR value which lies in the 4-th quantization level, the weighted SNR values of users $u = 3$ and $u = 6$ are equal, since $p_3 \cdot 2 = p_6 \cdot 1 = 4$.

Having defined all sets of users, the probability $P_{\mathrm{STC-NAF}}^{(u)}(q, \mathbf{p})$ of user u to get access to a resource unit with an SNR lying in the q-th quantization level can be calculated by determing the probability of the seven events mentioned above.

The probability P_q for the first event in an OSTBC-MRC system is given by

$$P_q = \sum_{v=0}^{n_\mathrm{T} n_\mathrm{R} - 1} \frac{1}{v!} \left(e^{-n_\mathrm{T} \cdot \gamma_{\mathrm{th},q-1}} (n_\mathrm{T} \cdot \gamma_{\mathrm{th},q-1})^v - e^{-n_\mathrm{T} \cdot \gamma_{\mathrm{th},q}} (n_\mathrm{T} \cdot \gamma_{\mathrm{th},q})^v \right) \tag{3.133}$$

using the fact that the normalized SNR values are chi-squared distributed with $2n_\mathrm{T}n_\mathrm{R}$ degrees of freedom.

The probability of the second and third event, i.e., the probability $P_{\mathrm{Ev2,3}}$ that there are only users which have the same weighting factor not exceeding the q-th quantization interval is given by

$$P_{\mathrm{Ev2,3}} = \sum_{\iota=0}^{|\mathcal{S}_{\mathrm{sw}}^{(u)}|} \binom{|\mathcal{S}_{\mathrm{sw}}^{(u)}|}{\iota} \cdot [P_q]^{\iota} \cdot [P_{<q}]^{|\mathcal{S}_{\mathrm{sw}}^{(u)}|-\iota} \tag{3.134}$$

with $|\mathcal{S}_{\mathrm{sw}}^{(u)}|$ the cardinality of the set of users with equal weighting factor compared to user u and $P_{<q}$ denoting the probability that a normalized SNR value lies below the q-th quantization level given by

$$P_{<q} = \sum_{\kappa=1}^{q-1} P_q. \tag{3.135}$$

The probability P_{Ev4} of the fourth event, i.e., the probability that the weighted SNR values of users of set $\mathcal{S}_{\mathrm{hw},q}^{(u)}$ are at most equal to the weighted SNR of user u in quantization level q is given by

$$P_{\mathrm{Ev4}} = \sum_{\zeta=0}^{|\mathcal{S}_{\mathrm{hw},q}^{(u)}|} \sum_{|\mathbf{a}|=\zeta} \left(\prod_{\psi=1}^{|\mathcal{S}_{\mathrm{hw},q}^{(u)}|} a_\psi \cdot P_{\mathbf{l}_{\mathrm{hw},q}(\psi)} + (1-a_\psi) \cdot P_{<\mathbf{l}_{\mathrm{hw},q}(\psi)} \right) \tag{3.136}$$

with vector $\mathbf{a} = [a_1, .., a_{|\mathcal{S}_{\mathrm{hw},q}^{(u)}|}]$ and $a_\psi \in \{0,1\}$.

The probability P_{Ev5} of the fifth event is given by

$$P_{\mathrm{Ev5}} \quad = \prod_{\psi \,\in\, \mathcal{S}_{\mathrm{hw}}^{(u)} \setminus \mathcal{S}_{\mathrm{hw},q}^{(u)}} P_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \tag{3.137}$$

with $\mathcal{S}_{\mathrm{hw}}^{(u)} \setminus \mathcal{S}_{\mathrm{hw},q}^{(u)}$ the set of users with a higher weighting factor than user u which have not been considered in the fourth event.

Considering the sixth event, the probability P_{Ev6} that the weighted SNR values of users of set $\mathcal{S}_{\mathrm{sw},q}^{(u)}$ are at most equal to the weighted SNR of user u in quantization level q is given by

$$P_{\mathrm{Ev6}} = \sum_{\eta=0}^{|\mathcal{S}_{\mathrm{sw},q}^{(u)}|} \sum_{|\mathbf{b}|=\eta} \left(\prod_{\psi=1}^{|\mathcal{S}_{\mathrm{sw},q}^{(u)}|} b_\psi \cdot P_{\mathbf{l}_{\mathrm{sw},q}(\psi)} + (1-b_\psi) \cdot P_{<\mathbf{l}_{\mathrm{sw},q}(\psi)} \right) \tag{3.138}$$

with vector $\mathbf{b} = [b_1, .., b_{|\mathcal{S}_{\mathrm{sw},q}^{(u)}|}]$ and $b_\psi \in \{0,1\}$.

Finally, the probability P_{Ev7} of the seventh event is given by

$$P_{\mathrm{Ev7}} = \prod_{\psi\,\in\,\mathcal{S}_{\mathrm{sw}}^{(u)} \setminus \mathcal{S}_{\mathrm{sw},q}^{(u)}} P_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \tag{3.139}$$

with $\mathcal{S}_{\mathrm{sw}}^{(u)} \setminus \mathcal{S}_{\mathrm{sw},q}^{(u)}$ the set of users with a lower weighting factor than user u which not have been considered in the sixth event.

Multiplying the probabilities of these seven events, the probability $P_{\mathrm{STC-NAF}}^{(u)}(q, \mathbf{p})$ of user u to get access to a resource unit with an SNR lying in the q-th quantization level is given by

$$\begin{aligned} P_{\mathrm{STC-NAF}}^{(u)}(q, \mathbf{p}) = \; & P_q \cdot \left(\prod_{\psi\,\in\,\mathcal{S}_{\mathrm{hw}}^{(u)} \setminus \mathcal{S}_{\mathrm{hw},q}^{(u)}} P_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right) \cdot \left(\prod_{\psi\,\in\,\mathcal{S}_{\mathrm{sw}}^{(u)} \setminus \mathcal{S}_{\mathrm{sw},q}^{(u)}} P_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right) \\ & \sum_{\zeta=0}^{|\mathcal{S}_{\mathrm{hw},q}^{(u)}|} \sum_{|\mathbf{a}|=\zeta} \left(\prod_{\psi=1}^{|\mathcal{S}_{\mathrm{hw},q}^{(u)}|} a_\psi \cdot P_{\mathbf{l}_{\mathrm{hw},q}(\psi)} + (1 - a_\psi) \cdot P_{<\mathbf{l}_{\mathrm{hw},q}(\psi)} \right) \\ & \sum_{\eta=0}^{|\mathcal{S}_{\mathrm{sw},q}^{(u)}|} \sum_{|\mathbf{b}|=\eta} \left(\prod_{\psi=1}^{|\mathcal{S}_{\mathrm{sw},q}^{(u)}|} b_\psi \cdot P_{\mathbf{l}_{\mathrm{sw},q}(\psi)} + (1 - b_\psi) \cdot P_{<\mathbf{l}_{\mathrm{sw},q}(\psi)} \right) \\ & \sum_{\iota=0}^{|\mathcal{S}_{\mathrm{sw}}^{(u)}|} \binom{|\mathcal{S}_{\mathrm{sw}}^{(u)}|}{\iota} \cdot [P_q]^\iota \cdot [P_{<q}]^{|\mathcal{S}_{\mathrm{sw}}^{(u)}| - \iota} \cdot \frac{1}{1 + \iota + \zeta + \eta} \end{aligned} \tag{3.140}$$

with $\mathbf{a}$ and $\mathbf{b}$ as defined in (3.136) and (3.138). The factor $\frac{1}{1+\iota+\zeta+\eta}$ takes into account the number of users user u must compete against in the random selection process performed when several users have an equivalent weighted SNR.

Thus, the probability $P_{\mathrm{STC-NAF}}^{(u)}(\mathbf{p})$ of user u to get access to a resource unit in total is given by

$$P_{\mathrm{STC-NAF}}^{(u)}(\mathbf{p}) = \sum_{q=1}^{L} P_{\mathrm{STC-NAF}}^{(u)}(q, \mathbf{p}). \tag{3.141}$$

Until now, it was assumed that the feedback link for the CQI is error-free. In the following, it is assumed that the CQI feedback bits are detected with a BER rate p_b as defined in Section 2.9.6. Thus, it is possible that a normalized SNR value which was measured to be in the y-th quantization level at the MS is now assumed to be in the x-th quantization level at the BS due to detection errors. As shown in Section 2.9.6, the probability $e_{x,y}$ of this event is given by

$$e_{x,y} = (1 - p_b)^{N_Q - b_{x,y}} \cdot p_b^{b_{x,y}}, \tag{3.142}$$

with $b_{x,y}$ the x, y-th element of the Hamming distance matrix $\mathbf{B}$ introduced in (2.90) and (2.91), respectively, with $x, y, = 1, .., L$. Note that

$$\sum_{x=1}^{L} e_{x,y} = 1 \tag{3.143}$$

since the sum of the probabilities has to be one as the BS always assumes a certain quantization level for each resource unit of each user.

Hence, the probability $\tilde{P}_q$ that the normalized SNR value is assumed to be in the q-th quantization level is given by

$$\tilde{P}_q = \sum_{\upsilon=1}^{L} e_{q,\upsilon} P_\upsilon \tag{3.144}$$

with P_υ as defined in (3.133).
The probability $\tilde{P}_{<q}$ that a normalized SNR value is assumed to lie in a quantization level below the q-th quantization level is given by

$$\tilde{P}_{<q} = \sum_{\kappa=1}^{q-1} \tilde{P}_q. \tag{3.145}$$

For $p_b = 0$, $\tilde{P}_q = P_q$ and $\tilde{P}_{<q} = P_{<q}$ since matrix $\mathbf{E}$ with the elements $e_{x,y}$ becomes an identity matrix.

To determine the channel access probability $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}, p_b)$ of user u in a system with a CQI feedback BER of p_b, the probabilities P_q and $P_{<q}$ in (3.140) and (3.141) have to be exchanged by probabilities $\tilde{P}_q$ and $\tilde{P}_{<q}$ leading to

$$P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}, p_b) = \sum_{q=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(q, \mathbf{p}, p_b). \tag{3.146}$$

with

$$
\begin{aligned}
P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b) &= \tilde{P}_q \cdot \left(\prod_{\psi\,\in\,\mathcal{S}^{(u)}_{\mathrm{hw}} \backslash \mathcal{S}^{(u)}_{\mathrm{hw},q}} \tilde{P}_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right) \\
&\cdot \left(\prod_{\psi\,\in\,\mathcal{S}^{(u)}_{\mathrm{sw}} \backslash \mathcal{S}^{(u)}_{\mathrm{sw},q}} \tilde{P}_{<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right) \\
&\cdot \sum_{\zeta=0}^{|\mathcal{S}^{(u)}_{\mathrm{hw},q}|} \sum_{|\mathbf{a}|=\zeta} \left(\prod_{\psi=1}^{|\mathcal{S}^{(u)}_{\mathrm{hw},q}|} a_\psi \cdot \tilde{P}_{\mathbf{l}_{\mathrm{hw},q}(\psi)} + (1-a_\psi) \cdot \tilde{P}_{<\mathbf{l}_{\mathrm{hw},q}(\psi)} \right) \\
&\sum_{\eta=0}^{|\mathcal{S}^{(u)}_{\mathrm{sw},q}|} \sum_{|\mathbf{b}|=\eta} \left(\prod_{\psi=1}^{|\mathcal{S}^{(u)}_{\mathrm{sw},q}|} b_\psi \cdot \tilde{P}_{\mathbf{l}_{\mathrm{sw},q}(\psi)} + (1-b_\psi) \cdot \tilde{P}_{<\mathbf{l}_{\mathrm{sw},q}(\psi)} \right) \\
&\sum_{\iota=0}^{|\mathcal{S}^{(u)}_{\mathrm{sw}}|} \binom{|\mathcal{S}^{(u)}_{\mathrm{sw}}|}{\iota} \cdot [\tilde{P}_q]^\iota \cdot [\tilde{P}_{<q}]^{|\mathcal{S}^{(u)}_{\mathrm{sw}}|-\iota} \frac{1}{1+\iota+\zeta+\eta}
\end{aligned}
\tag{3.147}
$$

and $\mathbf{a}$ and $\mathbf{b}$ as defined in (3.136) and (3.138).

The average number of allocated resource units to user u in an OSTBC-MRC system applying the Non-Adaptive First scheme is given by

$$
E\{N_{\mathrm{ru},u}\} = W_{\mathrm{A}} \cdot P_{\mathrm{STC-NAF}}(\mathbf{p},p_b). \tag{3.148}
$$

Note that (3.146) is true for all possible SNR thresholds γ_{th} as long as each user u applies the same SNR thresholds γ_{th}. However, if the SNR thresholds are defined such that the probability of a normalized SNR value to lie in the q-th quantization level $[\gamma_{\mathrm{th},q-1}, \gamma_{\mathrm{th},q}]$ is the same for all L intervals, i.e.,

$$
P_q = \frac{1}{L} \;\forall\; q = 1,..,L, \tag{3.149}
$$

then the probability $\tilde{P}_q(p_b)$ that the normalized SNR value is assumed to be in the q-th quantization level is given by

$$
\tilde{P}_q = \sum_{v=1}^{L} d_{q,v} \cdot P_v = \sum_{v=1}^{L} d_{q,v} \cdot \frac{1}{L} = \frac{1}{L} \cdot \sum_{v=1}^{L} d_{q,v} = \frac{1}{L} = P_q, \tag{3.150}
$$

i.e., $\tilde{P}_q$ becomes independent of the CQI feedback BER p_b. Further on, also the channel access probability $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p},p_b)$ of user u in a system with a CQI feedback BER of p_b becomes independent of p_b. This eases the calculation of the weighting factors since a given channel access demand vector $\mathbf{D}$ will always lead to the same weighting vector $\mathbf{p}$ independent of the user-dependent channel knowledge impairment parameters such as the CQI feedback BER p_b.

In the following, it is shown how to compute the SNR thresholds $\gamma_{\mathrm{th},q}$ with $q = 0, .., L$ and $\gamma_{\mathrm{th},0} = 0$ and $\gamma_{\mathrm{th},L} = \infty$ such that (3.149) is fulfilled. The probability $P(\gamma)$ that a normalized SNR has at most the value γ is given by

$$P(\gamma) = 1 - e^{-n_{\mathrm{T}}\cdot\gamma} \cdot \sum_{v=0}^{n_{\mathrm{T}}n_{\mathrm{R}}-1} \frac{1}{v!}(n_{\mathrm{T}} \cdot \gamma)^v. \tag{3.151}$$

From this it follows that the following equation must hold:

$$P(\gamma_{\mathrm{th},q}) = \frac{q}{L} \;\forall\; q = 1, .., L-1. \tag{3.152}$$

To determine the SNR threshold $\gamma_{\mathrm{th},q}$, the root of the function

$$g_{\mathrm{STC}}(\gamma_{\mathrm{th},q}) = 1 - \frac{q}{L} - e^{-n_{\mathrm{T}}\cdot\gamma_{\mathrm{th},q}} \cdot \sum_{v=0}^{n_{\mathrm{T}}n_{\mathrm{R}}-1} \frac{1}{v!}(n_{\mathrm{T}} \cdot \gamma_{\mathrm{th},q})^v \tag{3.153}$$

has to be determined which can be done using the *fzero* function in MATLAB™.

3.6.3.1.2.2 Calculation of the channel access probability for adaptive users applying TAS-MRC As introduced in Section 2.5.3, there are two types of TAS schemes in an FDD system differing in the CQI feedback. With TAS-FA, each MS of user u feeds back all n_{T} CQI values of a resource unit and the transmit antenna selection is performed at the BS. With TAS-FB, only the best out of n_{T} CQI values is fed back to the BS along with the antenna label of the antenna providing the best SNR. Thus, the antenna selection is performed at the MSs. These facts have to be taken into account when calculating the channel access probability $P^{(u)}_{\mathrm{TAS-NAF}}(\mathbf{p})$ for a TAS-MRC system applying the Non-Adaptive First scheme.

First, the case of TAS-FA is considered. Like in the case of TAS in a TDD system, where the antenna selection is also done at the BS, a TAS-FA-MRC system with n_{T} transmit antennas, n_{R} receiver antennas and U_{A} adaptive users can be interpreted as an OSTBC-MRC system with $n'_{\mathrm{T}} = 1$ transmit antennas, $n'_{\mathrm{R}} = n_{\mathrm{R}}$ receiver antennas and $U'_{\mathrm{A}} = n_{\mathrm{T}} \cdot U_{\mathrm{A}}$ virtual adaptive users resulting in

$$P^{(u)}_{\mathrm{TAS-FA-NAF}}(\mathbf{p}, p_b) = n_{\mathrm{T}} \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', p_b, U'_{\mathrm{A}}, n'_{\mathrm{T}}, n'_{\mathrm{R}}) \tag{3.154}$$

with

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{n_{\mathrm{T}}\ \text{times}}. \tag{3.155}$$

To achieve that $P^{(u)}_{\mathrm{TAS-FA-NAF}}(\mathbf{p}, p_b)$ becomes independent of the CQI feedback BER p_b to ease the calculations of the weighting factor as mentioned before, the SNR thresholds $\gamma_{\mathrm{th},q}$ have to be the roots of the function

$$g_{\mathrm{TAS-AF}}(\gamma_{\mathrm{th},q}) = 1 - \frac{q}{L} - e^{-\gamma_{\mathrm{th},q}} \cdot \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!}(\gamma_{\mathrm{th},q})^v. \tag{3.156}$$

The average number of allocated resource units to user u in a TAS-FA-MRC system applying the Non-Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = W_{\mathrm{A}} \cdot P_{\mathrm{TAS-FA-NAF}}(\mathbf{p}, p_b). \tag{3.157}$$

For the case of TAS-FB, it has to be taken into account that the antenna selection is already done at the MSs leading to the probability $P_{\mathrm{TAS-FB},q}$ that the best normalized SNR value out of n_{T} values lies in the q-th quantization level given by

$$P_{\mathrm{TAS-FB},q} = \left(1 - e^{-\gamma_{\mathrm{th},q}} \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!}(\gamma_{\mathrm{th},q})^v\right)^{n_{\mathrm{T}}} - \left(1 - e^{-\gamma_{\mathrm{th},q-1}} \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!}(\gamma_{\mathrm{th},q-1})^v\right)^{n_{\mathrm{T}}}. \tag{3.158}$$

From this, it follows that the probability $\tilde{P}_{\mathrm{TAS-FB},q}$ that the best normalized SNR value out of n_{T} values is assumed to be in the q-th quantization level is given by

$$\tilde{P}_{\mathrm{TAS-FB},q} = \sum_{v=1}^{L} e_{q,v} P_{\mathrm{TAS-FB},v}. \tag{3.159}$$

The probability $\tilde{P}_{\mathrm{TAS-FB},<q}$ that the best normalized SNR value out of n_{T} SNR values is assumed to lie in a quantization level below the q-th quantization level is given by

$$\tilde{P}_{\mathrm{TAS-FB},<q} = \sum_{\kappa=1}^{q-1} \tilde{P}_{\mathrm{TAS-FB},q}. \tag{3.160}$$

Thus, the channel access probability $P^{(u)}_{\mathrm{TAS-FB-NAF}}(\mathbf{p}, p_b)$ for an adaptive user u in a TAS-FB-MRC system applying the Non-Adaptive First scheme can be calculated by exchanging $\tilde{P}_q$ and $\tilde{P}_{<q}$ with $\tilde{P}_{\mathrm{TAS-FB},q}$ and $\tilde{P}_{\mathrm{TAS-FB},<q}$ in (3.146) and (3.147) leading to

$$P^{(u)}_{\mathrm{TAS-FB-NAF}}(\mathbf{p}, p_b) = \sum_{q=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(q, \mathbf{p}, p_b). \tag{3.161}$$

with

$$P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b) = \tilde{P}_{\mathrm{TAS-FB},q} \cdot \left(\prod_{\psi \in \mathcal{S}^{(u)}_{\mathrm{hw}} \backslash \mathcal{S}^{(u)}_{\mathrm{hw},q}} \tilde{P}_{\mathrm{TAS-FB},<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right) \tag{3.162}$$
$$\cdot \left(\prod_{\psi \in \mathcal{S}^{(u)}_{\mathrm{sw}} \backslash \mathcal{S}^{(u)}_{\mathrm{sw},q}} \tilde{P}_{\mathrm{TAS-FB},<\lceil \frac{p_u \cdot q}{p_\psi} \rceil} \right)$$
$$\cdot \sum_{\zeta=0}^{|\mathcal{S}^{(u)}_{\mathrm{hw},q}|} \sum_{|\mathbf{a}|=\zeta} \left(\prod_{\psi=1}^{|\mathcal{S}^{(u)}_{\mathrm{hw},q}|} a_\psi \cdot \tilde{P}_{\mathrm{TAS-FB},\mathbf{l}_{\mathrm{hw},q}(\psi)} + (1-a_\psi) \cdot \tilde{P}_{\mathrm{TAS-FB},<\mathbf{l}_{\mathrm{hw},q}(\psi)} \right)$$
$$\sum_{\eta=0}^{|\mathcal{S}^{(u)}_{\mathrm{sw},q}|} \sum_{|\mathbf{b}|=\eta} \left(\prod_{\psi=1}^{|\mathcal{S}^{(u)}_{\mathrm{sw},q}|} b_\psi \cdot \tilde{P}_{\mathrm{TAS-FB},\mathbf{l}_{\mathrm{sw},q}(\psi)} + (1-b_\psi) \cdot \tilde{P}_{\mathrm{TAS-FB},<\mathbf{l}_{\mathrm{sw},q}(\psi)} \right)$$
$$\sum_{\iota=0}^{|\mathcal{S}^{(u)}_{\mathrm{sw}}|} \binom{|\mathcal{S}^{(u)}_{\mathrm{sw}}|}{\iota} \cdot [\tilde{P}_{\mathrm{TAS-FB},q}]^{\iota} \cdot [\tilde{P}_{\mathrm{TAS-FB},<q}]^{|\mathcal{S}^{(u)}_{\mathrm{sw}}|-\iota} \frac{1}{1+\iota+\zeta+\eta}$$

and **a** and **b** as defined in (3.136) and (3.138).

The average number of allocated resource units to user u in a TAS-FB-MRC system applying the Non-Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = W_{\mathrm{A}} \cdot P_{\mathrm{TAS-FB-NAF}}(\mathbf{p},p_b). \tag{3.163}$$

In order to accomplish that $P^{(u)}_{\mathrm{TAS-FB-NAF}}(\mathbf{p},p_b)$ becomes independent of the CQI feedback BER p_b to ease the calculations of the weighting factors, the SNR thresholds $\gamma_{\mathrm{th},q}$ have to be the roots of the function

$$g_{\mathrm{TAS-BF}}(\gamma_{\mathrm{th},q}) = \left(1 - e^{-\gamma_{\mathrm{th},q}} \cdot \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} (\gamma_{\mathrm{th},q})^v \right)^{n_{\mathrm{T}}} - \frac{q}{L}. \tag{3.164}$$

3.6.3.1.2.3 Calculation of weighting factors In order to determine the weighting factors **p** to fulfill the user demand **D**, the same problem of (3.27) shown in Section 3.6.2.1.2.4 for a TDD system has to be solved.

However, the channel access probability $P_{\mathrm{NAF}}(i, f(\tilde{\mathbf{p}}))$ is no longer a continuous function with respect to **p**. In contrast to a TDD system, where the channel access probability of a user can have any value between 0 and 1 by adjusting **p**, there is only a finite number of values which $P_{\mathrm{NAF}}(i, f(\tilde{\mathbf{p}}))$ can have due to the quantization of the SNR feedback in an FDD system.

Thus, it is possible that for a given user demand vector $\mathbf{D}$, there is no weighting vector $\tilde{\mathbf{p}}^{\star}$ which perfectly accomplishes the required user demands. Note that the more quantization bits are used, the better the granularity of possible values of $P_{\mathrm{NAF}}(i, f(\tilde{\mathbf{p}}))$.

3.6.3.1.3 SNR distribution

3.6.3.1.3.1 Introduction Like in the analysis of the TDD system, the SNR distribution of the SNR values of the allocated resource units has to be derived for the Non-Adaptive First Scheme in order to analytically derive the performance of the system.

3.6.3.1.3.2 Non-adaptive users For the non-adaptively served users, there is no difference in the SNR distribution compared to the case of a TDD system, since no CQI is used for the resource allocation. Thus, it does not matter whether the CQI is quantized or not, i.e., the SNR distribution is the same as derived in Section 3.6.2.1.3.2 given by (3.43).

3.6.3.1.3.3 Adaptive users applying OSTBC-MRC To ease the derivation of the distribution of the SNR values of allocated resource units, it is initially assumed that the feedback link is error-free, i.e., $p_b = 0$. Later on, also the case with $p_b > 0$ is considered.

If a resource unit is allocated to adaptive user u whose normalized SNR value lies in the q-th quantization level, the exactly measured SNR value $\hat{\gamma}$ is not known. However, it is known that $\hat{\gamma}$ lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$ with the average SNR $\bar{\gamma}_{E,u}$ measured at the MS and used for the normalization given by $\bar{\gamma}_{E,u} = \bar{\gamma}_u \cdot (1 + \sigma_{E,u}^2)$. Further on, it is known that $\hat{\gamma}$ is chi-squared distributed. Hence, the PDF $p^{(u)}_{\mathrm{STC-NAF},\gamma,q}(\gamma)$ of $\hat{\gamma}$ of a resource allocated to user u in the q-th quantization level in an OSTBC-MRC system applying the Non-Adaptive First scheme is given by

$$
\begin{aligned}
p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},q}(\hat{\gamma}) \quad &= \quad a_{\mathrm{STC-NAF},q} \cdot \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}} n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{T}} n_{\mathrm{R}} - 1}}{(n_{\mathrm{T}} n_{\mathrm{R}} - 1)!} \\
&\quad \cdot e^{-\frac{n_{\mathrm{T}} \hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma} - \gamma_{\mathrm{th},q-1}) - \delta(\hat{\gamma} - \gamma_{\mathrm{th},q})\right]
\end{aligned}
\tag{3.165}
$$

with $\delta(\hat{\gamma})$ denoting the step function given by

$$\delta(\hat{\gamma}) = \begin{cases} 1 & \hat{\gamma} \geq 0, \\ 0 & \text{else.} \end{cases} \tag{3.166}$$

The factor $a_{\text{STC-NAF},q}$ ensures that

$$\int_0^\infty p^{(u)}_{\text{STC-NAF},\hat{\gamma},q}(\hat{\gamma})\ d\hat{\gamma} = 1. \tag{3.167}$$

With the probability P_q given by (3.133) denoting the probability that a measured SNR value lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\text{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\text{th},q}$, the factor $a_{\text{STC-NAF},q}$ is given by

$$a_{\text{STC-NAF},q} = \frac{1}{P_q}. \tag{3.168}$$

Finally, the PDF $p^{(u)}_{\text{STC-NAF},\hat{\gamma}}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u taking into account all L quantization levels is determined by summing up the PDFs $p^{(u)}_{\text{STC-NAF},\gamma,q}(\gamma)$ weighted by the probability that the allocated resource unit has a normalized SNR value that lies in the q-th quantization level which is given by the probability $P^{(u)}_{\text{STC-NAF}}(q,\mathbf{p})$ given in (3.140). Thus,

$$\begin{aligned} p^{(u)}_{\text{STC-NAF},\hat{\gamma}}(\hat{\gamma}) &= \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\text{STC-NAF}}(q,\mathbf{p})}{\sum_{v=1}^{L} P^{(u)}_{\text{STC-NAF}}(v,\mathbf{p})} \right) \cdot p^{(u)}_{\text{STC-NAF},\hat{\gamma},q}(\hat{\gamma}) \\ &= \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\text{STC-NAF}}(q,\mathbf{p})}{\sum_{v=1}^{L} P^{(u)}_{\text{STC-NAF}}(v,\mathbf{p})} \right) \cdot a_{\text{STC-NAF},q} \\ &\quad \cdot \left(\frac{n_\text{T}}{\bar{\gamma}_{E,u}} \right)^{n_\text{T} n_\text{R}} \cdot \frac{\hat{\gamma}^{n_\text{T} n_\text{R}-1}}{(n_\text{T} n_\text{R}-1)!} \cdot e^{-\frac{n_\text{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot [\delta(\hat{\gamma}-\gamma_{\text{th},q-1}) - \delta(\hat{\gamma}-\gamma_{\text{th},q})] \end{aligned} \tag{3.169}$$

where the factor $\left[\sum_{v=1}^{L} P_{\text{STC-NAF}}(v,\mathbf{p})\right]^{-1}$ ensures that

$$\int_0^\infty p^{(u)}_{\text{STC-NAF},\hat{\gamma}}(\hat{\gamma})\ d\hat{\gamma} = 1. \tag{3.170}$$

Now, the case of $p_b > 0$ is considered. If the CQI feedback is possibly erroneous, it is not known whether the measured SNR value $\hat{\gamma}$ which is assumed to be in the q-th quantization level actually lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\text{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\text{th},q}$ due to feedback bit errors. Thus, it is possible that an SNR value assumed to be in the x-th quantization level actually lies in the y-th quantization level. The probability of this event is $e_{x,y}$ given by (2.93) as introduced in Section 2.9.6. From this, it follows that the actually measured SNR value assumed to be in the q-th quantization level actually lies in the ω-th quantization level with a probability of $e_{q,\omega}$. Knowing that the SNR values from the ω-th quantization level are chi-squared distributed, the PDF $p^{(u)}_{\mathrm{STC-NAF},\gamma,p_b,q}(\gamma)$ of $\hat{\gamma}$ of a resource unit allocated to user u in the q-th quantization level in an OSTBC-MRC system applying the Non-Adaptive First scheme with $p_b > 0$ is given by

$$\begin{aligned} p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma}) &= a_{\mathrm{STC-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}}\right)^{n_\mathrm{T}n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T}n_\mathrm{R}-1}}{(n_\mathrm{T}n_\mathrm{R}-1)!} \\ &\quad \cdot e^{-\frac{n_\mathrm{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega})\right] \end{aligned} \tag{3.171}$$

Again, the factor $a_{\mathrm{STC-NAF},p_b,q}$ ensures that

$$\int_0^\infty p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})\, d\hat{\gamma} = 1. \tag{3.172}$$

With the probability $\tilde{P}_q$ given by (3.144 which denotes the probability that a measured SNR value is assumed to lie between $\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}$ and $\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}$, the factor $a_{\mathrm{STC-NAF},p_b,q}$ is given by

$$a_{\mathrm{STC-NAF},p_b,q} = \frac{1}{\tilde{P}_q}. \tag{3.173}$$

The PDF $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u taking into account all L quantization levels with $p_b > 0$ is given by

$$\begin{aligned} p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b}(\hat{\gamma}) &= \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(\upsilon,\mathbf{p},p_b)}\right) \cdot p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma}) \\ &= \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(\upsilon,\mathbf{p},p_b)}\right) \cdot a_{\mathrm{STC-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\ &\quad \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}}\right)^{n_\mathrm{T}n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T}n_\mathrm{R}-1}}{(n_\mathrm{T}n_\mathrm{R}-1)!} \\ &\quad \cdot e^{-\frac{n_\mathrm{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega})\right]. \end{aligned} \tag{3.174}$$

Note that for $p_b = 0$, (3.174) is equivalent to (3.169) as matrix $\mathbf{E}$ with elements $e_{x,y}$ becomes an identity matrix.

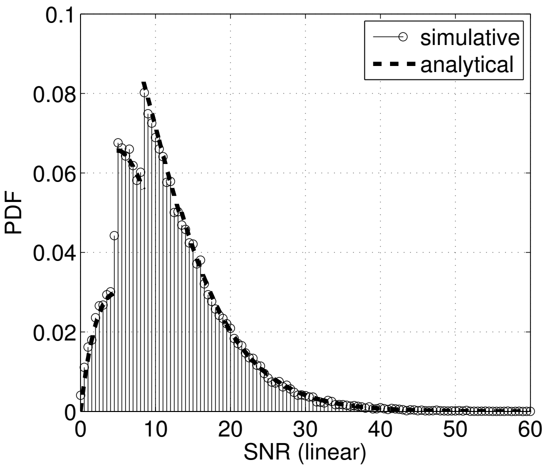
In the following example, the calculation of the PDF of the SNR values of allocated resource units shall be illustrated. A system with $U_\mathrm{A} = 3$ adaptively served users, $n_\mathrm{T} = 2$ transmit antennas and $n_\mathrm{R} = 1$ receive antenna each is assumed where all users have the same average SNR $\bar{\gamma}_u = 10$ dB and perfectly measured CQI ($\sigma^2_{\mathrm{E},u} = 0$). For the CQI feedback, $N_Q = 2$ quantization bits are applied, i.e., there are 4 quantization levels, where the binary bit coding scheme is used. The SNR thresholds are given by

$$\gamma_\mathrm{th} = [0,\ 4.8,\ 8.39,\ 13.46,\ \infty],$$

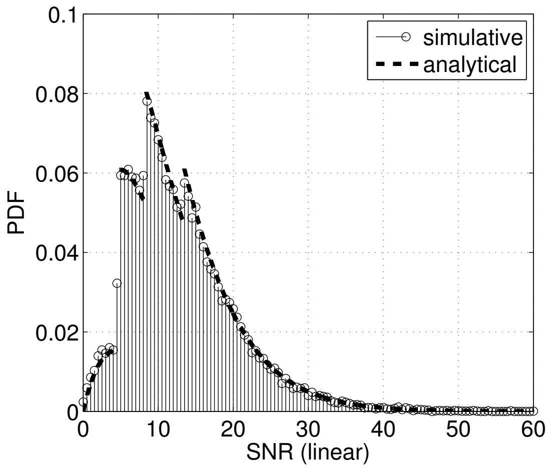
such that the probability of a measured SNR value to lie in any of the 4 quantization levels is $\frac{1}{4}$. Further on, a feedback BER of $p_b = 0.1$ is assumed. The weighting vector is given by

$$\mathbf{p} = [3,\ 2,\ 1].$$

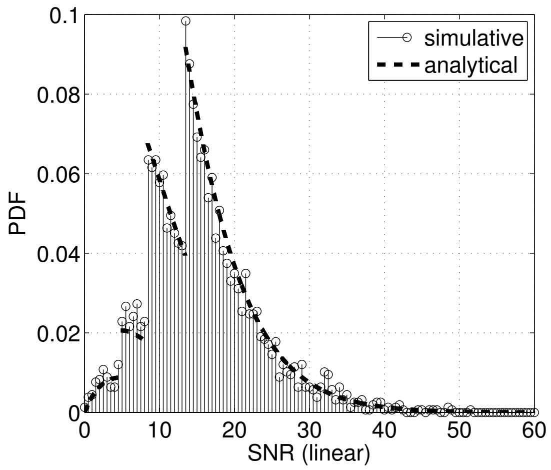
In Figure 3.7(a) to 3.7(c), the PDFs of the measured SNR values of allocated resource units are depicted for user $u = 1$, $u = 2$, and $u = 3$. It can be seen that the simulative PDFs match the analytical ones. The steps in the PDF at the SNR thresholds due to the step functions in (3.174) are clearly visible. Like in the case of a TDD system, it can be seen that the probability of small SNR values is larger for user $u = 1$ than for user $u = 2$ and $u = 3$ due to the SNR boosting of the QWPFS.



(a)



(b)



(c)

Figure 3.9. Analytical PDF and simulative PDF of the SNR of allocated resource units for user (a) $u = 1$ and (b) $u = 2$ and (c) $u = 3$ applying the Non-Adaptive First scheme.

3.6.3.1.3.4 Adaptive users applying TAS-MRC For the case of a system applying TAS at the transmitter, again the two feedback schemes TAS-FA and TAS-FB have to be considered when deriving the SNR distribution of allocated resource units.

As shown in Section 3.6.3.1.2, applying TAS-FA-MRC in a system with n_{T} transmit antennas, n_{R} receive antennas and U_{A} adaptive users can be interpreted as an OSTBC-MRC system with $n'_{\mathrm{T}} = 1$ transmit antennas, $n'_{\mathrm{R}} = n_{\mathrm{R}}$ receiver antennas and $U'_{\mathrm{A}} = n_{\mathrm{T}} \cdot U_{\mathrm{A}}$ virtual adaptive users. Thus, the PDF $p^{(u)}_{\mathrm{TAS-FA-NAF},\hat{\gamma}}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u when applying TAS-FA-MRC with $p_b > 0$ is given by

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-FA-NAF},\hat{\gamma},p_b}(\hat{\gamma}) \quad = \quad & \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{TAS_FA-NAF}}(q, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FA-NAF}}(\upsilon, \mathbf{p}, p_b)} \right) \\ & \cdot a_{\mathrm{TAS-FA-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \cdot \left(\frac{1}{\bar{\gamma}_{E,u}} \right)^{n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{(n_{\mathrm{R}}-1)!} \\ & \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega}) \right] \end{aligned} \tag{3.175}$$

with

$$a_{\mathrm{STC-FA-NAF},p_b,q} = \frac{1}{\tilde{P}_q(n'_{\mathrm{T}} = 1, n'_{\mathrm{R}} = n_{\mathrm{R}})}. \tag{3.176}$$

Applying TAS-FB-MRC, the MSs feed back the quantized CQI value of the best transmit antenna plus the digitized antenna label of the best antenna. Thus, besides possible errors detecting the feedback bits of the CQI, also possible errors detecting the antenna label have to be taken into account when deriving the SNR of allocated resource units. At the BS, three possible scenarios considering the antenna label are conceivable:

a) The antenna label is correctly received.

b) The antenna label is not correctly received. However, the SNR value of the wrongly selected antenna lies in the same quantization level as that of the correct antenna.

c) The antenna label is not correctly received and the SNR value of the wrongly selected antenna lies in a quantization level below the quantization level of the correct antenna.

Note that the case that the SNR value of the wrongly selected antenna lies in a quantization level *above* the quantization level of the correct antenna does not exist since the SNR value of the correct antenna always lies in a quantization level equal to or higher than the quantization levels of the other antennas due to the selection of the best antenna, i.e., the correct antenna always provides the best SNR.

In the following, the PDFs of the SNR values for these three events are derived. To do so, the function $F_{n_\mathrm{R}}^{(u)}(\hat{\gamma})$ is introduced which denotes the probability that a chi-squared distributed SNR value is smaller than $\hat{\gamma}$ given by

$$F_{n_\mathrm{R}}^{(u)}(\hat{\gamma}) = 1 - \exp\left(-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}\right) \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}\right)^{v}. \tag{3.177}$$

In case that the antenna label is correctly received, the SNR value is the best out of n_T chi-squared distributed SNR values. Thus, PDF $p_{\hat{\gamma},\mathrm{a})}^{(u)}$ is given by

$$p_{\hat{\gamma},\mathrm{a}}^{(u)} = \frac{n_\mathrm{T}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \left[F_{n_\mathrm{R}}^{(u)}(\hat{\gamma})\right]^{n_\mathrm{T}-1}. \tag{3.178}$$

For the second case, it is assumed that the SNR value of the best antenna lies in the q-th quantization level. Now, the CDF $P_{\hat{\gamma},\mathrm{b}}^{(u)}$ of the SNR value of the wrongly selected antenna which also lies in the q-th quantization level $[\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}, \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}]$ has to be determined. Assuming that there are n_T different transmit antennas, the wrongly selected antenna can be the second best, the third best down to the n_T-th best antenna with equal probability. Hence,

$$P_{\hat{\gamma},\mathrm{b}}^{(u)} = \frac{1}{n_\mathrm{T}-1} \sum_{\omega=2}^{n_\mathrm{T}} P_{\hat{\gamma},\mathrm{b},\omega-\mathrm{th\ best}}^{(u)}, \tag{3.179}$$

with $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1} \leq \hat{\gamma} \leq \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$ which can also be written as

$$P_{\hat{\gamma},\mathrm{b}}^{(u)} = \frac{1}{n_\mathrm{T}-1} \sum_{\omega=1}^{n_\mathrm{T}} P_{\hat{\gamma},\mathrm{b},\omega-\mathrm{th\ best}}^{(u)} - P_{\hat{\gamma},\mathrm{b},1-\mathrm{th\ best}}^{(u)}. \tag{3.180}$$

To determine $P_{\hat{\gamma},\mathrm{b},\omega-\mathrm{th\ best}}^{(u)}$ one has to consider all cases where $(\omega-1)$ SNR values lie between $\hat{\gamma}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$ and at least one SNR value lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\hat{\gamma}$. For the special case $\omega = 1$, the CDF $P_{\hat{\gamma},\mathrm{b},1-\mathrm{th\ best}}^{(u)}$ of the best out of n_T chi-squared distributed SNR values which lies in the q-th quantization interval is given by

$$P_{\hat{\gamma},\mathrm{b},1-\mathrm{th\ best}}^{(u)} = \left[F_{n_\mathrm{R}}^{(u)}(\hat{\gamma})\right]^{n_\mathrm{T}} - \left[F_{n_\mathrm{R}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right]^{n_\mathrm{T}}. \tag{3.181}$$

The expression of the CDF $\sum_{\omega=1}^{n_\mathrm{T}} P_{\hat{\gamma},\mathrm{b},\omega-\mathrm{th\ best}}^{(u)}$ of the sum over the best out of n_T SNR values down to the n_T-th best out of n_T SNR values can be simplified considering the following aspects. For all cases from best out of n_T SNR values down to worst out of

n_T SNR values, at least one SNR value must lie between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\hat{\gamma}$. For this one value, there are n_T possible candidates. Further on, the remaining $n_\mathrm{T}-1$ other SNR values must be at least smaller than $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$. Thus,

$$\sum_{\omega=1}^{n_\mathrm{T}} P^{(u)}_{\hat{\gamma},\omega-\mathrm{th\ best}} = n_\mathrm{T} \cdot \left(F^{(u)}_{n_\mathrm{R}}(\hat{\gamma}) - F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right) \cdot \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_\mathrm{T}-1} \quad (3.182)$$

Inserting (3.182) and (3.181) in (3.180) results in

$$\begin{aligned} P^{(u)}_{\hat{\gamma},\mathrm{b}} &= \frac{1}{n_\mathrm{T}-1} \Bigg(n_\mathrm{T} \cdot \left(F^{(u)}_{n_\mathrm{R}}(\hat{\gamma}) - F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right) \cdot \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_\mathrm{T}-1} \\ &\quad - \left(\left[F^{(u)}_{n_\mathrm{R}}(\hat{\gamma})\right]^{n_\mathrm{T}} - \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right]^{n_\mathrm{T}} \right) \Bigg). \end{aligned} \quad (3.183)$$

Differentiating (3.183) with respect to $\hat{\gamma}$ leads to the PDF $p^{(u)}_{\hat{\gamma},\mathrm{b}}$ given by

$$p^{(u)}_{\hat{\gamma},\mathrm{b}} = \frac{n_\mathrm{T}}{(n_\mathrm{T}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_\mathrm{R}-1)!} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{\bar{\gamma}^{n_\mathrm{R}}_{E,u}} \cdot \left(\left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_\mathrm{T}-1} - \left[F^{(u)}_{n_\mathrm{R}}(\hat{\gamma})\right]^{n_\mathrm{T}-1} \right) \quad (3.184)$$

with $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1} \le \hat{\gamma} \le \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$.

In the third case, the SNR value of the wrongly selected antenna lies in the l-th quantization with $l < q$ while the SNR of the best antenna lies in the q-th quantization level. Again, the wrongly selected antenna can have the second best down to the worst SNR value out of n_T with equal probability leading to

$$P^{(u)}_{\hat{\gamma},\mathrm{c}} = \frac{1}{n_\mathrm{T}-1} \sum_{\omega=2}^{n_\mathrm{T}} P^{(u)}_{\hat{\gamma},\mathrm{c},\omega-\mathrm{th\ best}}, \quad (3.185)$$

with $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1} \le \hat{\gamma} \le \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l}$. To determine $P^{(u)}_{\hat{\gamma},\mathrm{c},\omega-\mathrm{th\ best}}$ one has to consider all cases where $(\omega-2)$ SNR values lie between $\hat{\gamma}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$, at least one SNR value lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$ and at least one SNR value lies between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1}$ and $\hat{\gamma}$. Considering the sum $\sum_{\omega=2}^{n_\mathrm{T}} P^{(u)}_{\hat{\gamma},\mathrm{c},\omega-\mathrm{th\ best}}$, it can be seen that for all these cases at least one SNR value must lie between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1}$ and $\hat{\gamma}$. For this one value, there are n_T possible candidates. Further on, the remaining $n_\mathrm{T}-1$ other SNR values must be at least smaller than $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$. However, since at least one value must lie between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$, one has to substract all cases where the remaining $n_\mathrm{T}-1$ SNR values at least are smaller than $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$. Thus,

$$\begin{aligned} \sum_{\omega=2}^{n_\mathrm{T}} P^{(u)}_{\hat{\gamma},c,\omega-\mathrm{th\ best}} &= n_\mathrm{T} \cdot \left(F^{(u)}_{n_\mathrm{R}}(\hat{\gamma}) - F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1})\right) \\ &\quad \cdot \left(\left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_\mathrm{T}-1} - \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right]^{n_\mathrm{T}-1} \right). \end{aligned} \quad (3.186)$$

Inserting (3.186) in (3.185) results in

$$\begin{aligned} P_{\hat{\gamma},\mathrm{c}}^{(u)} &= \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} \cdot \left(F_{n_{\mathrm{R}}}^{(u)}(\hat{\gamma}) - F_{n_{\mathrm{R}}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1})\right) \\ &\cdot \left(\left[F_{n_{\mathrm{R}}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_{\mathrm{T}}-1} - \left[F_{n_{\mathrm{R}}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right]^{n_{\mathrm{T}}-1}\right). \end{aligned} \tag{3.187}$$

Differentiating (3.187) with respect to $\hat{\gamma}$ leads to the PDF $p_{\hat{\gamma},\mathrm{c})}^{(u)}$ given by

$$p_{\hat{\gamma},\mathrm{c})}^{(u)} = \frac{n_{\mathrm{T}}}{(n_{\mathrm{T}}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \cdot \left(\left[F_{n_{\mathrm{R}}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q})\right]^{n_{\mathrm{T}}-1} - \left[F_{n_{\mathrm{R}}}^{(u)}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1})\right]^{n_{\mathrm{T}}-1}\right) \tag{3.188}$$

with $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l-1} \leq \hat{\gamma} \leq \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},l}$.

Next, the PDF $p_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}^{(u)}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u whose SNR value is assumed to be in the q-th quantization level is derived when $p_b > 0$. To do so, the probability P_{AL} that the antenna label is received incorrectly is introduced given by

$$P_{\mathrm{AL}} = 1 - (1-p_b)^{\log_2(n_{\mathrm{T}})}. \tag{3.189}$$

assuming that $\log_2(n_{\mathrm{T}})$ feedback bits are used for signaling the antenna label. With the three cases derived above, $p_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}^{(u)}(\hat{\gamma})$ is given by

$$\begin{aligned} p_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}^{(u)}(\hat{\gamma}) &= a_{\mathrm{TAS-FB-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\ &\cdot P_{\mathrm{AL}} \cdot p_{\hat{\gamma},\mathrm{c}}^{(u)} \cdot \left[\delta(\hat{\gamma}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1})\right] \\ &+ \left(P_{\mathrm{AL}} \cdot p_{\hat{\gamma},\mathrm{b}}^{(u)} + (1-P_{\mathrm{AL}}) \cdot p_{\hat{\gamma},\mathrm{a}}^{(u)}\right) \\ &\cdot \left[\delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega})\right], \end{aligned} \tag{3.190}$$

which can be rewritten as

$$
\begin{aligned}
p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}(\hat{\gamma}) \;=\;& a_{\mathrm{TAS-FB-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& \cdot P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{T}}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \\
& \cdot \left(\left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega})\right]^{n_{\mathrm{T}}-1} - \left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1})\right]^{n_{\mathrm{T}}-1} \right) \\
& \cdot \left[\delta(\hat{\gamma}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1})\right] \\
& + \left(P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{T}}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \right. \\
& \cdot \left(\left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega})\right]^{n_{\mathrm{T}}-1} - \left[F^{(u)}_{n_{\mathrm{R}}}(\hat{\gamma})\right]^{n_{\mathrm{T}}-1} \right) \\
& + (1 - P_{\mathrm{AL}}) \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{R}}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \cdot \left[F^{(u)}_{n_{\mathrm{R}}}(\hat{\gamma})\right]^{n_{\mathrm{T}}-1} \\
& \left. \cdot \left[\delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega})\right] \right).
\end{aligned} \tag{3.191}
$$

Again, the factor $a_{\mathrm{TAS-FB-NAF},p_b,q}$ ensures that

$$
\int_0^\infty p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})\, d\hat{\gamma} = 1. \tag{3.192}
$$

With the probability $\tilde{P}_{\mathrm{TAS-FB},q}$ given in (3.159) which denotes the probability that the best out of n_{T} SNR values is assumed to lie between $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q-1}$ and $\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},q}$, the factor $a_{\mathrm{TAS-FB-NAF},p_b,q}$ is given by

$$
a_{\mathrm{TAS-FB-NAF},p_b,q} = \frac{1}{\tilde{P}_{\mathrm{TAS-FB},q}}. \tag{3.193}
$$

The PDF $p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u taking into account all L quantization levels with $p_b > 0$ is then given by

$$\begin{aligned}
p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b}(\hat{\gamma}) = & \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \\
& a_{\mathrm{TAS-FB-NAF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& \cdot P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{T}}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \\
& \cdot \left(\left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega})\right]^{n_{\mathrm{T}}-1} - \left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega-1})\right]^{n_{\mathrm{T}}-1} \right) \\
& \cdot \left[\delta(\hat{\gamma}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega-1})\right] \\
& + \Bigg(P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{T}}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \\
& \cdot \left(\left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega})\right]^{n_{\mathrm{T}}-1} - \left[F^{(u)}_{n_{\mathrm{R}}}(\hat{\gamma})\right]^{n_{\mathrm{T}}-1} \right) \\
& + (1-P_{\mathrm{AL}}) \cdot \frac{n_{\mathrm{T}}}{(n_{\mathrm{R}}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}_{E,u}^{n_{\mathrm{R}}}} \cdot \left[F^{(u)}_{n_{\mathrm{R}}}(\hat{\gamma})\right]^{n_{\mathrm{T}}-1} \\
& \cdot \left[\delta(\hat{\gamma} - \bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega})\right]\Big) .
\end{aligned} \tag{3.194}$$

3.6.3.1.4 Average user data rate and BER

3.6.3.1.4.1 Non-adaptive users Since the distribution of the SNR values of allocated resource units of non-adaptive users applying the non-Adaptive First scheme in an FDD system is the same as in a TDD system, the average user data rate and BER are equivalent to the user data rate and BER derived in Section 3.6.2.1.4.

3.6.3.1.4.2 Adaptive users applying OSTBC-MRC Determining the average data rate and BER of user u for an OSTBC-MRC system applying the non-Adaptive First scheme taking into account imperfect CQI, the definitions (3.65) and (3.73) for the average data rate and BER can be used again. However, the number M of applied modulation schemes is limited by the number L of quantization levels leading to

$$\begin{aligned}
\bar{R}^{(u)}_{A,\mathrm{STC-NAF},p_b} &= \sum_{q=1}^{L} r_{n_{\mathrm{T}}} \cdot b_q \cdot \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b}(\hat{\gamma})\, d\hat{\gamma}. \qquad (3.195) \\
&= \sum_{q=1}^{L} r_{n_{\mathrm{T}}} \cdot b_q \cdot \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})\, d\hat{\gamma}.
\end{aligned}$$

Inserting (3.171) in (3.195) results in

$$\begin{aligned}
\bar{R}^{(u)}_{A,\mathrm{STC-NAF},p_b} &= \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \\
&\cdot \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} a_{\mathrm{STC-NAF},p_b,q} \\
&\cdot \sum_{\omega=1}^{L} e_{q,\omega} \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}}\right)^{n_\mathrm{T} n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T} n_\mathrm{R}-1}}{(n_\mathrm{T} n_\mathrm{R}-1)!} \cdot e^{-\frac{n_\mathrm{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}}\, d\hat{\gamma} \\
&= \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(\upsilon,\mathbf{p},p_b)} \right)
\end{aligned} \tag{3.196}$$

recalling the calculation of factor $a_{\mathrm{STC-NAF},p_b,q}$ of (3.173).

The average BER of user u is calculated following the definition of (3.73) is given by

$$\begin{aligned}
\overline{BER}^{(u)}_{A,\mathrm{STC-NAF},p_b} &= \frac{1}{\bar{R}^{(u)}_{A,\mathrm{STC-NAF},p_b}} \cdot \sum_{q=1}^{L} \\
&\int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} r_{n_\mathrm{T}} \cdot b_q \cdot p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma}) \cdot \widehat{BER}^{(u)}_q(\hat{\gamma})\, d\hat{\gamma}
\end{aligned} \tag{3.197}$$

with

$$\widehat{BER}^{(u)}_q(\hat{\gamma}) = 0.2 \cdot \left(\frac{n_\mathrm{T}}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right)^{n_\mathrm{T} n_\mathrm{R}} \cdot \exp\left(-\frac{\hat{\gamma} n_\mathrm{T} \mu_u^2 \beta_q}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right). \tag{3.198}$$

Inserting (3.171) and (3.198) in (3.197) results in

$$\begin{aligned}
\overline{BER}^{(u)}_{A,\mathrm{STC-NAF},p_b} &= \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{STC-NAF},p_b}} \cdot \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot a_{\mathrm{STC-NAF},p_b,q} \\
&\cdot \left(\frac{P^{(u)}_{\mathrm{STC-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \left(\frac{n_\mathrm{T}}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right)^{n_\mathrm{T} n_\mathrm{R}} \\
&\Bigg[\exp\left(-\frac{\gamma_{\mathrm{th},\omega-1} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right) \\
&\cdot \sum_{v=0}^{n_\mathrm{T} n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right)^{v} \\
&- \exp\left(-\frac{\gamma_{\mathrm{th},\omega} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right) \\
&\cdot \sum_{v=0}^{n_\mathrm{T} n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}}\right)^{v} \Bigg]
\end{aligned} \tag{3.199}$$

3.6.3.1.4.3 Adaptive users applying TAS-MRC Computing the average user data rate and BER for a TAS-MRC system following the Feedback All policy, one can utilize the fact that TAS-FA-MRC can be interpreted as a special case of an OSTBC-MRC system as derived in Section 3.6.2.1.2.3. Hence, the same derivation steps as shown above can be used to determine the average user data rate given by

$$\bar{R}^{(u)}_{A,\mathrm{TAS-FA-NAF},p_b} = \sum_{q=1}^{L} b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FA-NAF}}(q,\mathbf{p},p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FA-NAF}}(v,\mathbf{p},p_b)} \right) \tag{3.200}$$

and the average BER given by

$$\begin{aligned}
\overline{BER}^{(u)}_{A,\mathrm{TAS-FA-NAF},p_b} = & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FA-NAF},p_b}} \cdot \sum_{q=1}^{L} b_q \cdot a_{\mathrm{TAS-FA-NAF},p_b,q} \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FA-NAF}}(q,\mathbf{p},p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FA-NAF}}(v,\mathbf{p},p_b)} \right) \\
& \cdot \sum_{\omega=1}^{L} e_{q,\omega} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_{\mathrm{R}}} \\
& \cdot \left[\exp\left(- \frac{\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{\bar{\gamma}_{E,u}(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})} \right) \right. \\
& \cdot \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{\bar{\gamma}_{E,u}(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})} \right)^{v} \\
& - \exp\left(- \frac{\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{\bar{\gamma}_{E,u}(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})} \right) \\
& \left. \cdot \sum_{v=0}^{n_{\mathrm{R}}-1} \frac{1}{v!} \left(\frac{\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{\bar{\gamma}_{E,u}(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})} \right)^{v} \right].
\end{aligned} \tag{3.201}$$

For the case of following the Feedback Best policy, the average user data rate in a TAS-FB-MRC system is given by

$$\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} = \sum_{q=1}^{L} b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(v,\mathbf{p},p_b)} \right) \tag{3.202}$$

.

Computing the average BER $\overline{BER}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ in a TAS-FB-MRC system, $p^{(u)}_{\mathrm{STC-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})$ in (3.197) has to be exchanged by $p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})$ given by (3.194). Since $p^{(u)}_{\mathrm{TAS-FB-NAF},\hat{\gamma},p_b,q}(\hat{\gamma})$ consists of three parts, the average BER $\overline{BER}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ is also expressed in three parts $\overline{BER1}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$,

$\overline{BER2}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ and $\overline{BER3}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ to ease the readability. Thus,

$$\begin{aligned}\overline{BER1}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} = & \frac{1}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \sum_{q=2}^{L} \int_{0}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}} a_{\mathrm{TAS-FB-NAF},p_b,q} \\ & \cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \sum_{\omega=1}^{L} e_{q,\omega} \\ & \cdot \frac{P_{\mathrm{AL}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}^{n_{\mathrm{R}}}_{E,u}} \cdot \widehat{BER}^{(u)}_{q}(\hat{\gamma})\, d\hat{\gamma},\end{aligned} \tag{3.203}$$

which can be rewritten as

$$\begin{aligned}\overline{BER1}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} = & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \cdot \sum_{q=2}^{L} b_q \cdot a_{\mathrm{TAS-FB-NAF},p_b,q} \\ & \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\ & P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_{\mathrm{R}}} \\ & \cdot \left(\left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega}) \right]^{n_{\mathrm{T}}-1} - \left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega-1}) \right]^{n_{\mathrm{T}}-1} \right) \\ & \cdot F^{(u)}_{n_{\mathrm{R}}} \left(\frac{\gamma_{\mathrm{th},\omega-1}\cdot(1+\beta_q(\bar{\gamma}_u\sigma^2_{r,u}+\bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q\bar{\gamma}_u\sigma^2_{r,u}} \right).\end{aligned} \tag{3.204}$$

$\overline{BER2}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ is computed as follows:

$$\begin{aligned}\overline{BER2}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} = & \frac{1}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \sum_{q=1}^{L} \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} a_{\mathrm{TAS-FB-NAF},p_b,q} \\ & \cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \sum_{\omega=1}^{L} e_{q,\omega} \\ & \cdot \frac{P_{\mathrm{AL}}}{(n_{\mathrm{R}}-1)!} \cdot \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} \cdot \left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},\omega}) \right]^{n_{\mathrm{T}}-1} \\ & \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}^{n_{\mathrm{R}}}_{E,u}} \cdot \widehat{BER}^{(u)}_{q}(\hat{\gamma})\, d\hat{\gamma},\end{aligned} \tag{3.205}$$

which can be rewritten in closed form as

$$
\begin{aligned}
\overline{BER}_{2\,A,\mathrm{TAS-FB-NAF},p_b}^{(u)} = {} & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \cdot \sum_{q=1}^{L} b_q \cdot a_{\mathrm{TAS-FB-NAF},p_b,q} \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& P_{\mathrm{AL}} \cdot \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_{\mathrm{R}}} \cdot \left[F^{(u)}_{n_{\mathrm{R}}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right]^{n_{\mathrm{T}}-1} \\
& \cdot \left[F^{(u)}_{n_{\mathrm{R}}} \left(\frac{\gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right. \\
& \left. F^{(u)}_{n_{\mathrm{R}}} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right] .
\end{aligned} \tag{3.206}
$$

Finally, the third BER term is calculated given by

$$
\begin{aligned}
\overline{BER}_{3\,A,\mathrm{TAS-FB-NAF},p_b}^{(u)} = {} & \frac{1}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \sum_{q=1}^{L} \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} a_{\mathrm{TAS-FB-NAF},p_b,q} \\
& \cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(\upsilon,\mathbf{p},p_b)} \right) \sum_{\omega=1}^{L} e_{q,\omega} \\
& \cdot \frac{\left(n_{\mathrm{T}}(1-P_{\mathrm{AL}}) - \frac{n_{\mathrm{T}}}{n_{\mathrm{T}}-1} P_{\mathrm{AL}} \right)}{(n_{\mathrm{R}}-1)!} \cdot \left[F^{(u)}_{n_{\mathrm{R}}}(\hat{\gamma}) \right]^{n_{\mathrm{T}}-1} \\
& \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{\bar{\gamma}^{n_{\mathrm{R}}}_{E,u}} \cdot \widehat{BER}^{(u)}_q(\hat{\gamma}) \, d\hat{\gamma},
\end{aligned} \tag{3.207}
$$

which can be written in closed form as

$$\begin{aligned}\overline{BER3}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} &= \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}} \sum_{q=1}^{L} a_{\mathrm{TAS-FB-NAF},p_b,q} \\ &\cdot b_q \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-NAF}}(q,\mathbf{p},p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FB-NAF}}(v,\mathbf{p},p_b)} \right) \sum_{\omega=1}^{L} e_{q,\omega} \\ &\cdot \frac{n_\mathrm{T}}{n_\mathrm{T}-1} \left(n_\mathrm{T}(1-P_\mathrm{AL})-1\right) \cdot \sum_{l=0}^{n_\mathrm{T}-1} \binom{n_\mathrm{T}-1}{l} (-1)^l \\ &\sum_{|\eta|=l} \binom{l}{\eta} \left(\frac{1}{\prod_{v=0}^{n_\mathrm{R}-1} (v!)^{\eta_v}} \right) \frac{(n_\mathrm{R}-1+G)!}{(n_\mathrm{R}-1)!} \\ &\cdot \frac{(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})^G}{\left((l+1)+\beta_q((l+1)\bar{\gamma}_u\sigma^2_{r,u}+\bar{\gamma}_{E,u}\mu_u^2)\right)^{n_\mathrm{R}+G}} \\ &\cdot \left[F^{(u)}_{n_\mathrm{R}+G}\left(\frac{\gamma_{\mathrm{th},\omega}\left((l+1)+\beta_q((l+1)\bar{\gamma}_u\sigma^2_{r,u}+\bar{\gamma}_{E,u}\mu_u^2)\right)}{1+\beta_q\bar{\gamma}_u\sigma^2_{r,u}} \right) \right. \\ &\left. - F^{(u)}_{n_\mathrm{R}+G}\left(\frac{\gamma_{\mathrm{th},\omega-1}\left((l+1)+\beta_q((l+1)\bar{\gamma}_u\sigma^2_{r,u}+\bar{\gamma}_{E,u}\mu_u^2)\right)}{1+\beta_q\bar{\gamma}_u\sigma^2_{r,u}} \right) \right]\end{aligned} \tag{3.208}$$

with $\eta = [\eta_0, .., \eta_{n_\mathrm{T}-1}]$ where $\eta_v \in \{0,1\}$ and $G = \sum_{v=0}^{n_\mathrm{R}-1} v \cdot \eta_v$.

Finally, the average BER of user u is given by

$$\begin{aligned}\overline{BER}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} &= \overline{BER1}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} \\ &+ \overline{BER2}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} + \overline{BER3}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}.\end{aligned} \tag{3.209}$$

3.6.3.1.4.4 Special case pure adaptive and pure non-adaptive resource allocation Like in the case of a TDD system, the two special cases of a pure adaptive and a pure non-adaptive system are incorporated in the expressions of the average user data rate and BER for adaptively and non-adaptively served users derived in the sections above. For $\vartheta = [0, .., 0]$, there are no adaptively served users and the user data rate and BER for all users are calculated as shown in Section 3.6.2.1.4.2. For $\vartheta = [1, .., 1]$, all users are served adaptively, i.e., all $U_\mathrm{A} = U$ users have to be considered when calculating $P^{(u)}_{\mathrm{NAF},p_b}$ as shown in Section 3.6.2.1.4.3 and 3.6.2.1.4.4.

3.6.3.2 Adaptive First

3.6.3.2.1 Introduction In this section, the Adaptive First resource allocation scheme for an FDD system is analyzed concerning the channel access and resulting

SNR distribution of the adaptively and non-adaptively allocated resource units assuming that the user serving vector ϑ is given.

3.6.3.2.2 Channel access Like in the case of a TDD system, all available resource units N_{ru} are allocated to the $U_{\mathrm{A}} = \vartheta^T \vartheta$ adaptive users now following the QWPFS policy. Further on, W_{N} resource units are re-assigned to the total number $U_{\mathrm{N}} = U - U_{\mathrm{A}}$ of non-adaptive users, i.e., the channel access demand of the non-adaptively served users is fulfilled. Like in the TDD system case, it have to be determined which $W_{\mathrm{A}} = N_{\mathrm{ru}} - W_{\mathrm{N}}$ of the N_{ru} resource units are allocated to adaptive users. Again, only the best W_{A} resource units which have the best weighted normalized and quantized SNR value are taken into account for serving the adaptive users. Thus, the channel access probability for adaptive users has to be determined in order to be able to adjust the weighting factors $\mathbf{p}$ such that each adaptive user u is allocated to D_u resource units on average.

3.6.3.2.2.1 Calculation of the channel access probability for adaptive users applying OSTBC-MRC In Section 3.6.2.2.2, it could be shown that in a TDD OSTBC-MRC system applying the Adaptive First scheme, the probability $P_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, u, \mathbf{p})$ of user u to get access to the w-th best resource unit given by (3.85) can be calculated using the channel access probability $P_{\mathrm{STC-NAF}}$ for the Non-Adaptive First scheme. Since the general principle of the Adaptive First scheme does *not* change if applied in an FDD system with quantized CQI values, (3.85) can be also applied in an FDD system simply by exchanging $P_{\mathrm{STC-NAF}}(u, \mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1))$ with $P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b)$ as calculated in (3.146). Hence,

$$
\begin{aligned}
P^{(u)}_{\mathrm{STC-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}} - 1}{w - 1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\
&\quad \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b) \\
&= \sum_{q=1}^{L} N_{\mathrm{ru}} \binom{N_{\mathrm{ru}} - 1}{w - 1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\
&\quad \cdot P^{(u)}_{\mathrm{STC-NAF}}(q, \mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b) \\
&= \sum_{q=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(q, w, N_{\mathrm{ru}}, \mathbf{p}, p_b)
\end{aligned}
\tag{3.210}
$$

with

$$
\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \ldots \quad \mathbf{p}]}_{(\varepsilon + N_{\mathrm{ru}} - w + 1)\ \text{times}} . \tag{3.211}
$$

Hence, the average number of resource units allocated to user u in an OSTBC-MRC system applying the Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{STC-AF}}(i, N_{\mathrm{ru}}, \mathbf{p}, p_b). \tag{3.212}$$

3.6.3.2.2.2 Calculation of the channel access probability for adaptive users applying TAS-MRC For a TAS-FA-MRC system applying the Non-Adaptive first scheme where each MS feeds back all CQI values and the transmit antenna selection is done at the BS, the channel access probability is calculated according to (3.154) , i.e., as a special of an OSTBC system. Hence, in a TAS-FA-MRC FDD system applying the Adaptive First scheme the probability $P^{(u)}_{\mathrm{TAS-FA-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b)$ of user u to get access to the w-th best resource unit is given by

$$\begin{aligned} P^{(u)}_{\mathrm{TAS-FA-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \cdot n_{\mathrm{T}} \\ &\quad \cdot P^{(u)}_{\mathrm{STC-NAF}}(\mathbf{p}^{\perp}, U_{\mathrm{A}}^{\perp} = n_{\mathrm{T}} \cdot U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b) \\ &= n_{\mathrm{T}} \cdot \sum_{q=1}^{L} P^{(u)}_{\mathrm{TAS-FA-AF}}(q, w, N_{\mathrm{ru}}, \mathbf{p}, p_b) \end{aligned} \tag{3.213}$$

with

$$\mathbf{p}^{\perp} = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \dots \quad \mathbf{p}]}_{n_{\mathrm{T}}\cdot(\varepsilon+N_{\mathrm{ru}}-w+1)\text{ - times}} \quad . \tag{3.214}$$

Thus, the average number of resource units allocated to user u in a TAS-FA-MRC system applying the Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{TAS-FA-AF}}(i, N_{\mathrm{ru}}, \mathbf{p}, p_b). \tag{3.215}$$

For a TAS-FB-MRC system applying the Non-Adaptive First scheme where each MS feeds back only the CQI value of the best transmit antenna, the probability $P^{(u)}_{\mathrm{TAS-FB-NAF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b)$ of user u to get access to the w-th best resource unit is given by (3.161). From this, it follows that the probability $P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b)$ of user u to get access to the w-th best resource unit applying the Adaptive First scheme is calculated according to

$$\begin{aligned} P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b) &= N_{\mathrm{ru}} \cdot \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\quad \cdot P^{(u)}_{\mathrm{TAS-FB-NAF}}(\mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b) \\ &= \sum_{q=1}^{L} N_{\mathrm{ru}} \binom{N_{\mathrm{ru}}-1}{w-1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \\ &\quad \cdot P^{(u)}_{\mathrm{TAS-FB-NAF}}(q, \mathbf{p}', U'_{\mathrm{A}} = U_{\mathrm{A}} \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1), p_b) \\ &= \sum_{q=1}^{L} P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_{\mathrm{ru}}, \mathbf{p}, p_b) \end{aligned} \tag{3.216}$$

with

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \ldots \quad \mathbf{p}]}_{(\varepsilon + N_{\mathrm{ru}} - w + 1)\ \mathrm{times}} . \tag{3.217}$$

The average number of resource units allocated to user u in a TAS-FB-MRC system applying the Adaptive First scheme is given by

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{TAS-FB-AF}}(i, N_{\mathrm{ru}}, \mathbf{p}, p_b). \tag{3.218}$$

3.6.3.2.2.3 Calculation of weighting factors To determine the proper weighting factors $\mathbf{p}$ to fulfill the user demand $\mathbf{D}$, the following problem has to be solved

$$\begin{aligned} \tilde{\mathbf{p}}^{\star} \quad &= \quad \arg\min_{\tilde{\mathbf{p}}} \left\{ \sum_{i=1}^{G-1} \left| \sum_{\eta=1}^{W_{\mathrm{A}}} P^{(i)}_{\mathrm{AF}}(\eta, N_{\mathrm{ru}}, f(\tilde{\mathbf{p}}, p_b)) - \frac{\tilde{D}_i}{W_{\mathrm{A}}} \right| \right\} \\ &\text{subject to} \\ &\quad \tilde{p}_u \geq 1 \end{aligned} \tag{3.219}$$

with $\tilde{p}$, $\tilde{D}$ and $f(\tilde{\mathbf{p}})$ as defined in Section 3.6.2.1.2.4. This problem can be solved as shown in Section 3.6.2.1.2.4.

3.6.3.2.3 SNR distribution

3.6.3.2.3.1 Non-adaptive users As stated before, the distribution of the SNR of resource units allocated to non-adaptive users applying the Adaptive First scheme is equivalent to the SNR distribution when applying the Non-Adaptive First scheme.

3.6.3.2.3.2 Adaptive users applying OSTBC-MRC In order to calculate the PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of resource units allocated to adaptive users in an OSTBC-MRC system applying the Adaptive First scheme, PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,q,w}(\hat{\gamma})$ of the SNR of w-th best out of N_{ru} resource units which is assumed to lie in the q-th quantization level has to be determined.

From (3.171), it is known that the PDF of the SNR assumed to lie in the q-th quantization level is a sum of chi-squared distributed PDFs. Thus,

$$\begin{aligned} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,q,w}(\hat{\gamma}) &= a_{\mathrm{STC-AF},p_b,q} \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\ &\cdot \left(\frac{n_{\mathrm{T}}}{\bar{\gamma}_{E,u}}\right)^{n_{\mathrm{T}}n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{T}}n_{\mathrm{R}}-1}}{(n_{\mathrm{T}}n_{\mathrm{R}}-1)!} \\ &\cdot e^{-\frac{n_{\mathrm{T}}\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma}-\gamma_{\mathrm{th},\omega})\right] \end{aligned} \tag{3.220}$$

with

$$a_{\mathrm{STC-AF},p_b,q} = \frac{1}{\tilde{P}_q} \tag{3.221}$$

to ensure that

$$\int_0^{\infty} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,q,w}(\hat{\gamma})\, d\hat{\gamma} = 1. \tag{3.222}$$

The PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w}(\hat{\gamma})$ of the SNR of the w-th best resource unit taking into account all L quantization levels is the weighted sum of the PDFs $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w,q}(\hat{\gamma})$ given by

$$\begin{aligned} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w}(\hat{\gamma}) &= \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(q,w,N_{\mathrm{ru}},\mathbf{p},p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(v,w,N_{\mathrm{ru}},\mathbf{p},p_b)}\right) \\ &\cdot p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,q,w}(\hat{\gamma}), \end{aligned} \tag{3.223}$$

where the factor $\sum_{v=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(v,w,N_{\mathrm{ru}},\mathbf{p},p_b)$ ensures that

$$\int_0^{\infty} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w}(\hat{\gamma})\, d\hat{\gamma} = 1. \tag{3.224}$$

Finally, the PDF $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u is the weighted sum of the PDFs $p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w}(\hat{\gamma})$ given by

$$\begin{aligned} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b}(\hat{\gamma}) &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,w}(\hat{\gamma}) \qquad (3.225) \\ &= \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\ &\quad \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(\upsilon, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot a_{\mathrm{STC-AF},p_b,q} \sum_{\omega=1}^{L} e_{q,\omega} \\ &\quad \cdot \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}} \right)^{n_\mathrm{T} n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T} n_\mathrm{R} - 1}}{(n_\mathrm{T} n_\mathrm{R} - 1)!} \\ &\quad \cdot e^{-\frac{n_\mathrm{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega}) \right] \end{aligned}$$

where the factor $\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)$ ensures that

$$\int_0^\infty p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b}(\hat{\gamma})\, d\hat{\gamma} = 1. \qquad (3.226)$$

In the following example, the calculation of the PDF of the SNR values of allocated resource units shall be illustrated. A system with $U_\mathrm{A} = 3$ adaptively served users, $n_\mathrm{T} = 2$ transmit antennas and $n_\mathrm{R} = 1$ receive antenna each is assumed where all users have the same average SNR $\bar{\gamma}_u = 10$ dB and perfectly measured CQI ($\sigma^2_{\mathrm{E},u} = 0$). For the CQI feedback $N_Q = 2$ quantization bits are applied, i.e., there are 4 quantization levels, where the binary bit coding scheme is used. The SNR thresholds are given by

$$\gamma_\mathrm{th} = [0,\ 4.8,\ 8.39,\ 13.46,\ \infty],$$

such that the probability of a measured SNR value to lie in any of the 4 quantization level is $\frac{1}{4}$. Further on, a feed back BER of $p_b = 0.1$ is assumed. The weighting vector is given by

$$\mathbf{p} = [2,\ 1.5,\ 1].$$

In Figure 3.8(a) to 3.8(c), the PDFs of the measured SNR values of allocated resource units are depicted for user $u = 1$, $u = 2$, and $u = 3$. As shown in the TDD case, the simulative PDFs match the analytical ones. Also, the steps in the PDF at the SNR thresholds due to the step functions in (3.225) are clearly visible. Further on, it can be seen that the probability of small SNR values is larger for user $u = 1$ than for users $u = 2$ and $u = 3$ due to the SNR boosting of the QWPFS.

3.6.3.2.3.3 Adaptive users applying TAS-MRC As TAS-FA-MRC can be interpreted as a special case of OSTBC-MRC with $U'_\mathrm{A} = n_\mathrm{T} \cdot U_\mathrm{A}$, $n'_\mathrm{T} = 1$ and $n'_\mathrm{R} = n_\mathrm{R}$,

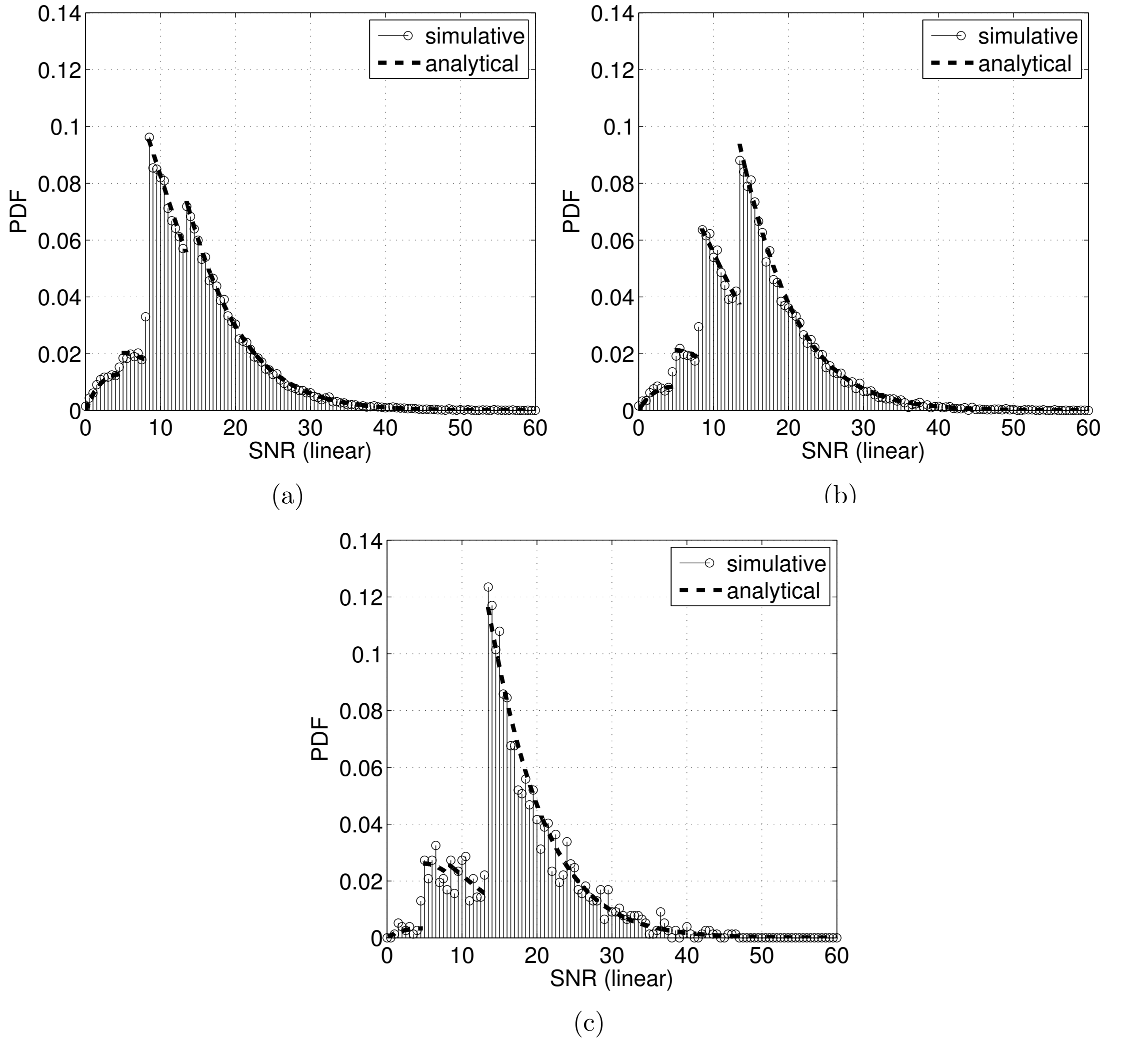


Figure 3.10. Analytical PDF and simulative PDF of the SNR of allocated resource units for user (a) $u = 1$ and (b) $u = 2$ and (c) $u = 3$ applying the Adaptive First scheme.

the PDF $p^{(u)}_{\mathrm{TAS-FA-AF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u in a TAS-FA-MRC system is given by

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-FA-AF},\hat{\gamma},p_b}(\hat{\gamma}) \;=\; & \sum_{w=1}^{W_{\mathrm{A}}} \left(\frac{P^{(u)}_{\mathrm{TAS-FA-AF}}(w, N_{\mathrm{ru}}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_{\mathrm{A}}} P^{(u)}_{\mathrm{TAS-FA-AF}}(\upsilon, N_{\mathrm{ru}}, \mathbf{p}, p_b)} \right) \\ & \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{TAS-FA-AF}}(q, w, N_{\mathrm{ru}}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FA-AF}}(\upsilon, w, N_{\mathrm{ru}}, \mathbf{p}, p_b)} \right) \\ & \cdot a_{\mathrm{TAS-FA-AF},p_b,q} \sum_{\omega=1}^{L} e_{q,\omega} \cdot \left(\frac{1}{\bar{\gamma}_{E,u}} \right)^{n_{\mathrm{R}}} \cdot \frac{\hat{\gamma}^{n_{\mathrm{R}}-1}}{(n_{\mathrm{R}}-1)!} \\ & \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left[\delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega}) \right] \end{aligned} \tag{3.227}$$

with

$$a_{\mathrm{TAS-FA-AF},p_b,q} = \frac{1}{\tilde{P}_q(n'_\mathrm{T}=1, n'_\mathrm{R}=n_\mathrm{R})}. \tag{3.228}$$

For the case of TAS-FB-MRC, it is shown in (3.178) to (3.187) that the PDF of the SNR assumed to lie in the q-th quantization level is a sum of chi-squared distributed PDFs $p^{(u)}_{\hat{\gamma},\mathrm{a}}$, $p^{(u)}_{\hat{\gamma},\mathrm{b}}$ and $p^{(u)}_{\hat{\gamma},\mathrm{c}}$. Performing the same derivation steps as done in the case of OSTBC-MRC applying the Adaptive First scheme, PDF $p^{(u)}_{\mathrm{TAS-FB-AF},\hat{\gamma},p_b}(\hat{\gamma})$ of the SNR of a resource unit allocated to user u in a TAS-FB-MRC system is given by

$$\begin{aligned} p^{(u)}_{\mathrm{TAS-FB-AF},\hat{\gamma},p_b}(\hat{\gamma}) \; = \; & \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\ & \sum_{q=1}^{L} \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FB-AF}}(v, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot a_{\mathrm{TAS-FB-AF},p_b,q} \\ & \sum_{\omega=1}^{L} e_{q,\omega} \left[P_\mathrm{AL} \cdot \frac{n_\mathrm{T}}{(n_\mathrm{T}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_\mathrm{R}-1)!} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \right. \\ & \cdot \left(\left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right]^{n_\mathrm{T}-1} - \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) \right]^{n_\mathrm{T}-1} \right) \\ & \cdot \left[\delta(\hat{\gamma}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) \right] \\ & + \left(P_\mathrm{AL} \cdot \frac{n_\mathrm{T}}{(n_\mathrm{T}-1)} \cdot \frac{e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}}}{(n_\mathrm{R}-1)!} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \right. \\ & \cdot \left(\left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right]^{n_\mathrm{T}-1} - \left[F^{(u)}_{n_\mathrm{R}}(\hat{\gamma}) \right]^{n_\mathrm{T}-1} \right) \\ & + (1-P_\mathrm{AL}) \cdot \frac{n_\mathrm{T}}{(n_\mathrm{R}-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \frac{\hat{\gamma}^{n_\mathrm{R}-1}}{\bar{\gamma}_{E,u}^{n_\mathrm{R}}} \cdot \left[F^{(u)}_{n_\mathrm{R}}(\hat{\gamma}) \right]^{n_\mathrm{T}-1} \\ & \left. \left. \cdot \left[\delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right] \right) \right] \end{aligned} \tag{3.229}$$

with

$$a_{\mathrm{TAS-FB-AF},p_b,q} = \frac{1}{\tilde{P}_{\mathrm{TAS-FB},q}}. \tag{3.230}$$

3.6.3.2.4 Average user data rate and BER

3.6.3.2.4.1 Non-adaptive users The average data rate $\bar{R}^{(u)}_N$ and BER $\overline{BER}^{(u)}_N$ of a non-adaptive user u applying the Adaptive First scheme is equivalent to the average data rate and BER applying the Non-Adaptive First scheme derived in Section 3.6.2.1.4.

3.6.3.2.4.2 Adaptive users applying OSTBC-MRC The average data rate $\bar{R}^{(u)}_{A,\mathrm{STC-AF},p_b}$ of user u for an OSTBC-MRC system applying the Adaptive First scheme taking into account imperfect CQI is computed as follows:

$$\bar{R}^{(u)}_{A,\mathrm{STC-AF},p_b} = \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b}(\hat{\gamma})\, d\hat{\gamma}. \tag{3.231}$$

Inserting (3.225) in (3.231) results in

$$\begin{aligned}
\bar{R}^{(u)}_{A,\mathrm{STC-NAF},p_b} &= \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \qquad (3.232)\\
&\cdot \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(v, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot a_{\mathrm{STC-AF},p_b,q}\\
&\sum_{\omega=1}^{L} e_{q,\omega} \left(\frac{n_\mathrm{T}}{\bar{\gamma}_{E,u}} \right)^{n_\mathrm{T} n_\mathrm{R}} \cdot \frac{\hat{\gamma}^{n_\mathrm{T} n_\mathrm{R}-1}}{(n_\mathrm{T} n_\mathrm{R}-1)!} \cdot e^{-\frac{n_\mathrm{T}\hat{\gamma}}{\bar{\gamma}_{E,u}}}\\
&\left[\delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega-1}) - \delta(\hat{\gamma} - \gamma_{\mathrm{th},\omega}) \right]\, d\hat{\gamma}\\
&= \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot \left(\frac{\sum_{w=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right)
\end{aligned}$$

with $P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b) = \sum_{v=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(v, w, N_\mathrm{ru}, \mathbf{p}, p_b)$ and recalling the calculation of factor $a_{\mathrm{STC-AF},p_b,q}$ of (3.221).

The average BER of user u is calculated given by

$$\overline{BER}^{(u)}_{A,\mathrm{STC-AF},p_b} = \frac{1}{\bar{R}^{(u)}_{A,\mathrm{STC-AF},p_b}} \cdot \sum_{q=1}^{L} \int_{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q-1}}^{\bar{\gamma}_{E,u}\cdot\gamma_{\mathrm{th},q}} r_{n_\mathrm{T}} \cdot b_q \cdot p^{(u)}_{\mathrm{STC-AF},\hat{\gamma},p_b,q}(\hat{\gamma}) \cdot \widehat{BER}^{(u)}_q(\hat{\gamma})\, d\hat{\gamma}. \tag{3.233}$$

Inserting (3.225) and (3.198) in (3.233) results in

$$\begin{aligned}
\overline{BER}^{(u)}_{A,\mathrm{STC-AF},p_b} &= \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{STC-AF},p_b}} \cdot \sum_{w=1}^{W_\mathrm{A}} \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{STC-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\
& \sum_{q=1}^{L} r_{n_\mathrm{T}} \cdot b_q \cdot a_{\mathrm{STC-AF},p_b,q} \cdot \left(\frac{P^{(u)}_{\mathrm{STC-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{STC-AF}}(v, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\
& \sum_{\omega=1}^{L} e_{q,\omega} \left(\frac{n_\mathrm{T}}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_\mathrm{T} n_\mathrm{R}} \\
& \left[\exp\left(-\frac{\gamma_{\mathrm{th},\omega-1} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u} \mu^2_u))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right. \\
& \cdot \sum_{v=0}^{n_\mathrm{T} n_\mathrm{R} - 1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u} \mu^2_u))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^v \\
& - \exp\left(-\frac{\gamma_{\mathrm{th},\omega} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u} \mu^2_u))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \\
& \left. \cdot \sum_{v=0}^{n_\mathrm{T} n_\mathrm{R} - 1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega} \cdot n_\mathrm{T} \cdot (n_\mathrm{T} + \beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u} \mu^2_u))}{n_\mathrm{T} + \beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^v \right].
\end{aligned} \tag{3.234}$$

3.6.3.2.4.3 Adaptive users applying TAS-MRC Like in the case of the Non-Adaptive First scheme, the average user data rate and BER for a TAS-MRC system following the Feedback All policy can be computed utilizing the fact that TAS-FA-MRC can be interpreted as a special case of an OSTBC-MRC system as derived in Section 3.6.2.1.2.3. Hence, the same derivation steps as shown above can be used to determine the average user data rate given by

$$\bar{R}^{(u)}_{A,\mathrm{TAS-FA-AF},p_b} = \sum_{q=1}^{L} b_q \cdot \left(\frac{\sum_{w=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FA-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FA-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \tag{3.235}$$

and the average BER given by

$$\begin{aligned}\overline{BER}^{(u)}_{A,\mathrm{TAS-FA-AF},p_b} &= \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FA-AF},p_b}} \cdot \sum_{w=1}^{W_\mathrm{A}} \sum_{q=1}^{L} b_q \cdot a_{\mathrm{TS-FA-AF},p_b,q} \\ &\cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FA-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FA-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\ &\cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FA-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FA-AF}}(\upsilon, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\ &\sum_{\omega=1}^{L} e_{q,\omega} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_\mathrm{R}} \\ &\left[\exp\left(- \frac{\gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right. \\ &\cdot \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^v \\ &- \exp\left(- \frac{\gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \\ &\left. \cdot \sum_{v=0}^{n_\mathrm{R}-1} \frac{1}{v!} \left(\frac{\gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^v \right]. \end{aligned} \tag{3.236}$$

For the case of following the Feedback Best policy, the average user data rate in a TAS-FB-MRC system applying the Adaptive First scheme is given by

$$\bar{R}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} = \sum_{q=1}^{L} b_q \cdot \left(\frac{\sum_{w=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \tag{3.237}$$

.

Computing the average BER $\overline{BER}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b}$ in a TAS-FB-MRC system applying the Adaptive First scheme, the same derivation steps as for the case of applying the

Non-Adaptive First scheme can be performed resulting in the three BER terms

$$
\begin{aligned}
\overline{BER1}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} = {} & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b}} \sum_{w=1}^{W_\mathrm{A}} \sum_{q=2}^{L} b_q \cdot a_{\mathrm{TAS-FB-AF},p_b,q} \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-AF}}(\upsilon, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& P_\mathrm{AL} \cdot \frac{n_\mathrm{T}}{n_\mathrm{T}-1} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_\mathrm{R}} \\
& \cdot \left(\left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right]^{n_\mathrm{T}-1} - \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega-1}) \right]^{n_\mathrm{T}-1} \right) \\
& \cdot F^{(u)}_{n_\mathrm{R}} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right),
\end{aligned}
\tag{3.238}
$$

$$
\begin{aligned}
\overline{BER2}^{(u)}_{A,\mathrm{TAS-FB-NAF},p_b} = {} & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b}} \sum_{w=1}^{W_\mathrm{A}} \sum_{q=1}^{L} b_q \cdot a_{\mathrm{TAS-FB-AF},p_b,q} \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(\upsilon, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{\upsilon=1}^{L} P^{(u)}_{\mathrm{TAS-FB-AF}}(\upsilon, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& P_\mathrm{AL} \cdot \frac{n_\mathrm{T}}{n_\mathrm{T}-1} \left(\frac{1}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right)^{n_\mathrm{R}} \cdot \left[F^{(u)}_{n_\mathrm{R}}(\bar{\gamma}_{E,u} \cdot \gamma_{\mathrm{th},\omega}) \right]^{n_\mathrm{T}-1} \\
& \cdot \left[F^{(u)}_{n_\mathrm{R}} \left(\frac{\gamma_{\mathrm{th},\omega} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right. \\
& \left. F^{(u)}_{n_\mathrm{R}} \left(\frac{\gamma_{\mathrm{th},\omega-1} \cdot (1+\beta_q(\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu^2_u))}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right],
\end{aligned}
\tag{3.239}
$$

and

$$
\begin{aligned}
\overline{BER_3}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} = & \frac{0.2}{\bar{R}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b}} \sum_{w=1}^{W_\mathrm{A}} \sum_{q=1}^{L} b_q \cdot a_{\mathrm{TAS-FB-AF},p_b,q} \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{W_\mathrm{A}} P^{(u)}_{\mathrm{TAS-FB-AF}}(v, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \\
& \cdot \left(\frac{P^{(u)}_{\mathrm{TAS-FB-AF}}(q, w, N_\mathrm{ru}, \mathbf{p}, p_b)}{\sum_{v=1}^{L} P^{(u)}_{\mathrm{TAS-FB-AF}}(v, w, N_\mathrm{ru}, \mathbf{p}, p_b)} \right) \cdot \sum_{\omega=1}^{L} e_{q,\omega} \\
& \cdot \frac{n_\mathrm{T}}{n_\mathrm{T}-1} \left(n_\mathrm{T}(1-P_\mathrm{AL}) - 1 \right) \cdot \sum_{l=0}^{n_\mathrm{T}-1} \binom{n_\mathrm{T}-1}{l} (-1)^l \\
& \sum_{|\eta|=l} \binom{l}{\eta} \left(\frac{1}{\prod_{v=0}^{n_\mathrm{R}-1} (v!)^{\eta_v}} \right) \frac{(n_\mathrm{R}-1+G)!}{(n_\mathrm{R}-1)!} \\
& \cdot \frac{(1+\beta_q \bar{\gamma}_u \sigma^2_{r,u})^G}{\left((l+1) + \beta_q((l+1)\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2) \right)^{n_\mathrm{R}+G}} \\
& \cdot \left[F^{(u)}_{n_\mathrm{R}+G} \left(\frac{\gamma_{\mathrm{th},\omega} \left((l+1) + \beta_q((l+1)\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2) \right)}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right. \\
& \left. - F^{(u)}_{n_\mathrm{R}+G} \left(\frac{\gamma_{\mathrm{th},\omega-1} \left((l+1) + \beta_q((l+1)\bar{\gamma}_u \sigma^2_{r,u} + \bar{\gamma}_{E,u}\mu_u^2) \right)}{1+\beta_q \bar{\gamma}_u \sigma^2_{r,u}} \right) \right]
\end{aligned} \tag{3.240}
$$

with $\eta = [\eta_0, .., \eta_{n_\mathrm{T}-1}]$ where $\eta_v \in \{0, 1\}$ and $G = \sum_{v=0}^{n_\mathrm{R}-1} v \cdot \eta_v$.

Finally, the average BER of user u is given by

$$
\begin{aligned}
\overline{BER}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} = & \overline{BER_1}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} \\
& + \overline{BER_2}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b} + \overline{BER_3}^{(u)}_{A,\mathrm{TAS-FB-AF},p_b}.
\end{aligned} \tag{3.241}
$$

3.6.3.2.4.4 Special case pure adaptive and pure non-adaptive resource allocation Like in the case of a TDD system, the two special cases of a pure adaptive and a pure non-adaptive system are incorporated in the expressions of the average user data rate and BER for adaptively and non-adaptively served users applying the Adaptive First scheme derived in the sections above. For $\vartheta = [0, .., 0]$, there are no adaptively served users and the user data rate and BER for all users are calculated given by (3.58) and (3.61). For $\vartheta = [1, .., 1]$, all users are served adaptively, i.e., all $U_\mathrm{A} = U$ users have to be considered when calculating the average user data rate and BER according to (3.196) and (3.199) applying OSTBC-MRC, according to (3.200) and (3.201) applying TAS-FA-MRC and according to (3.202) and (3.209) applying TAS-FB-MRC, respectively.

3.6.3.3 Optimizing SNR thresholds

3.6.3.3.1 Non-adaptive users For non-adaptive users in an FDD system, only the modulation scheme m which maximizes the user data rate while fulfilling the target BER has to be found. This can be done as described in Section 3.6.2.3 for a TDD system.

3.6.3.3.2 Adaptive users For adaptive users in the considered FDD system, the SNR thresholds are already pre-determined by the quantization level bounds, i.e., the number of possibly applied modulation scheme is limited to the number L of quantization levels in contrast to a TDD system where M modulation schemes can be applied.

As stated before, it is assumed that the SNR thresholds of the normalized SNR values are equal and fixed for all users, where it is assumed that the spacing of the thresholds is done in such a way that the probability of a normalized SNR value to lie in the q-th quantization level is $\frac{1}{L}$ due to the reasons described in Section 3.6.3.1.2.

Of course, one could consider a system where each user has different SNR thresholds leading to $U \cdot L$ different SNR thresholds. Hence, for each quantization level q of user u, the SNR thresholds $\gamma_{\text{th},q-1}^{(u)}$ and $\gamma_{\text{th},q}^{(u)}$ and the applied modulation scheme represented by the number $b_q^{(u)}$ of bits per symbol would have to be chosen such that the user data rate is maximized subject to the target BER leading to $U \cdot L^2$ degrees of freedom. However, since in an FDD system with quantized CQI values, the probability that user u gets access to the channel and also its user data rate and BER depend on the SNR thresholds of all other users and not only on the its own SNR thresholds like in a TDD system, it is not possible to optimize the SNR thresholds of user u independently of the SNR thresholds of the other users. Hence, the optimization would have to be done in a global manner which becomes infeasible for large numbers U of users, numbers L of quantization levels and numbers M of available modulation schemes.

Another drawback of assuming different SNR thresholds of each user is the fact that the weighting factors $\mathbf{p}$ would also depend on all $U \cdot L$ SNR thresholds. This would imply that when changing the SNR thresholds, the weighting factors also would have to be re-calculated for a given user demand vector $\mathbf{D}$. By assuming fixed SNR thresholds for all users, the weighting factors always remain the same for a given $\mathbf{D}$.

Hence, in order to keep the solution of the problem feasible, the SNR thresholds are assumed to be equal and fixed for each user u accepting some losses compared to the optimal solution as the degrees of freedom are reduced to L, i.e., the remaining degrees of freedom which can be utilized to maximize the user data rate are the modulation schemes $\mathbf{b}^{(u)} = [b_1^{(u)}, .., b_L^{(u)}]$ with $b_q \in \mathbb{N} \; \forall \; q = 1, .., L$ representing the number of bits per data symbol corresponding to the applied modulation scheme when the SNR value of the scheduled user u lies in the q-th quantization level. Thus, the original SNR threshold problem of (3.8) is transformed into a nonlinear integer programming problem:

$$\begin{aligned} \bar{R}_{\mathrm{A,opt}}^{(u)}(\vartheta) &= \max_{\mathbf{b}^{(u)}} \left(\bar{R}_{\mathrm{A}}^{(u)}(\vartheta, \mathbf{b}^{(u)}) \right) && (3.242) \\ & \quad \text{subject to} \\ & \quad \overline{BER}_{\mathrm{A}}^{(u)}(\vartheta, \mathbf{b}^{(u)}) \leq BER_{\mathrm{T}} \\ & \quad b_q^{(u)} \in \mathbb{N} && (3.243) \end{aligned}$$

Due to the complex structure of the expressions $\bar{R}_{\mathrm{A}}^{(u)}(\vartheta, \mathbf{b}^{(u)})$ and $\overline{BER}_{\mathrm{A}}^{(u)}(\vartheta, \mathbf{b}^{(u)})$, an analytical optimization is not feasible to the best knowledge of the author. Assuming there are M different modulation schemes available for each quantization level q,

$$N_{\mathrm{pS}} = M^L \tag{3.244}$$

possible solutions exist. However, in the q-th quantization level it is not reasonable to apply a modulation scheme with less bits per symbol than in the $(q-1)$-th quantization level due to a higher SNR, i.e.,

$$b_{q-1}^{(u)} \leq b_q^{(u)} \tag{3.245}$$

which means that the number N_{rS} of reasonable solutions is smaller than N_{pS}. As shown in Appendix A.5, N_{rS} is given by

$$N_{\mathrm{rS}} = f(L, M), \tag{3.246}$$

where $f(L, M)$ is a recursive function given by

$$\begin{aligned} f(L, M) &= f(L-1, M) + f(L, M-1) && (3.247) \\ \text{with} \quad & f(1, M) = M \\ \text{and} \quad & f(L, 1) = 1. \end{aligned}$$

Fig. 3.11 illustrates the number of solutions which have to be tested in order to find the optimal modulation scheme vector $\mathbf{b}^{(u)}$ such that the user data rate is maximized while fulfilling the target BER as a function of the number L of quantization levels assuming $M = 4$ available modulation schemes. The dashed curve represents the number of reasonable solutions taking into account (3.245), i.e., a Modulation-aware search is performed. The solid curve represents the number of all possible solutions which are to be tested when performing an exhaustive search. It can be seen that huge savings in terms of complexity are achievable considering (3.245). For a realistic example with $L = 4$ quantization levels corresponding to $N_Q = 2$ CQI feedback bits and $M = 4$ modulation schemes, $N_{\mathrm{rS}} = 35$ variations of $\mathbf{b}^{(u)}$ have to be tested while for $L = 8$ ($N_Q = 3$) there are $N_{\mathrm{rS}} = 165$ variations compared to $N_{\mathrm{pS}} = 64$ and $N_{\mathrm{pS}} = 65536$, respectively. These are feasible numbers of operations especially when taking into account the fact that the modulation scheme optimization can be done off-line for a finite number of system parameters and stored in a look-up table.

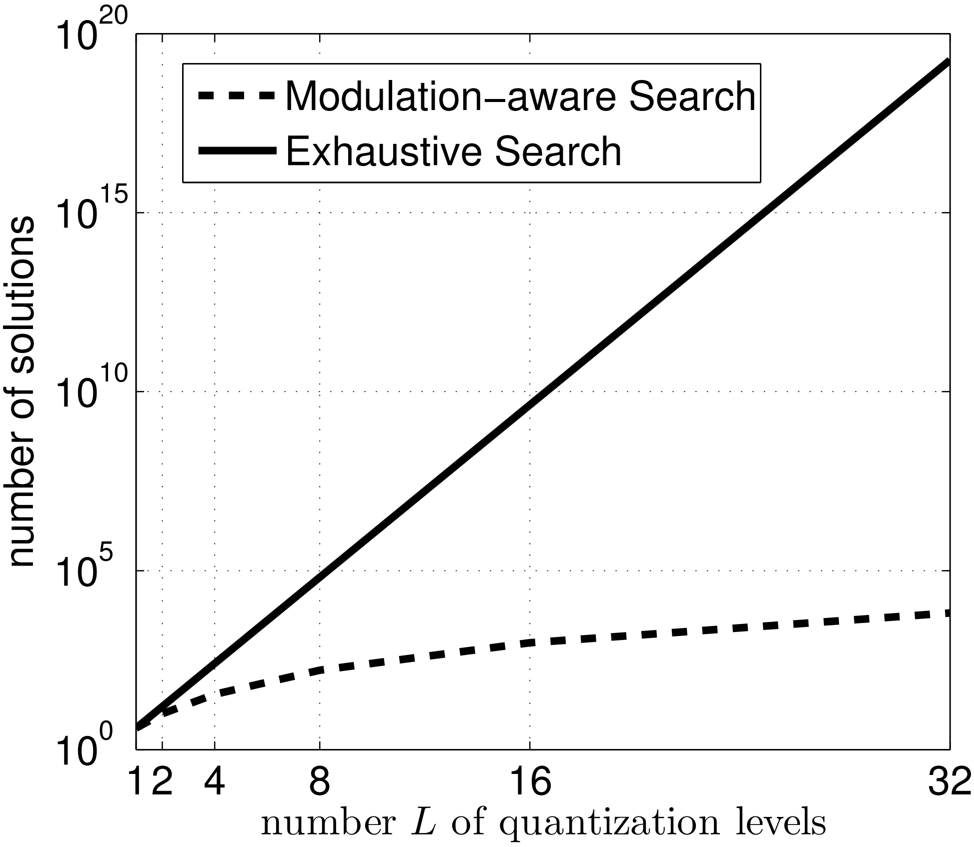


Figure 3.11. Number of solutions vs. number L of quantization levels for $M = 4$ available modulation schemes

3.7 The user serving problem

3.7.1 Introduction

For the analytical calculation of the user performance and the optimization of the SNR thresholds of the applied modulation schemes shown in Section 3.6, it was assumed that the user serving vector ϑ was already given. In the following, it is shown how to determine ϑ such that the average data rate of the system is maximized while all users fulfill the target BER and the minimum data rate requirements as stated in the problem formulation. In Section 3.7.2, solutions for the general case of different user demands are presented. In Section 3.7.3, it is shown that for the special case of equal user demands, the complexity of the algorithm solving the user serving problem can be reduced which will be shown in the concluding complexity analysis in Section 3.7.4.

3.7.2 Solutions for different user demands

3.7.2.1 Introduction

The problem to be solved is given by (3.9) where $\bar{R}^{(u)}_{\mathrm{N,opt}}(\vartheta)$ and $\bar{R}^{(u)}_{\mathrm{A,opt}}(\vartheta)$ denote the average user data rate achievable with optimized SNR thresholds applying the non-adaptive and adaptive transmission mode, respectively, for a given user serving vector ϑ. Since the user data rate $\bar{R}^{(u)}_{\mathrm{N,opt}}(\vartheta)$ of (3.58) of non-adaptively served users does not depend on ϑ as shown in Section 3.6, the expression ϑ can be omitted leading to

$$\bar{R}^{(u)}_{\mathrm{N,opt}}(\vartheta) = \bar{R}^{(u)}_{\mathrm{N,opt}}. \tag{3.248}$$

In the following, it is assumed that the minimum user data rate $\bar{R}^{(u)}_{\mathrm{min}}$ each user u shall achieve is given by the the average user data $\bar{R}^{(u)}_{\mathrm{N,opt}}$ achievable when applying the non-adaptive transmission mode, i.e.,

$$\bar{R}^{(u)}_{\mathrm{min}} = \bar{R}^{(u)}_{\mathrm{N,opt}}. \tag{3.249}$$

That means that no matter how bad the channel conditions are, each user shall achieve at least the data rate achievable when applying the robust non-adaptive transmission scheme, otherwise, any sophisticated adaptive transmission scheme would be pointless.

In the following, an exhaustive search algorithm and a reduced complexity algorithm are presented in Section 3.7.2.2 and 3.7.2.3, respectively.

3.7.2.2 Exhaustive Search

The most time-consuming way to solve (3.9) is an Exhaustive Search (ES), i.e., all possible user serving vectors ϑ are tested to find the best vector according to (3.9), i.e., for each possible number U_{A} of adaptive users there exist $\binom{U}{U_{\mathrm{A}}}$ possible realizations of ϑ. Hence, $\sum_{u=1}^{U}\binom{U}{u} = 2^U$ possible realizations of ϑ have to be tested, which can become prohibitively complex for large numbers U of users.

3.7.2.3 Reduced Complexity Algorithm

From the analytical expressions of the average user data derived in Section 3.6.2 and Section 3.6.3 it could be seen that besides the SNR thresholds, the data rate of user u depends on the weighting vector $\mathbf{p}$. To be more precise, it depends on the number $|\mathcal{G}_i|$ of adaptive users in each demand group $\mathcal{G}_i$ with $i = 1, .., G$, i.e., the number of users with a certain weighting factor p_i against which user u has to compete successfully in order to get access to a given resource unit. From this, it follows that for the calculation of $\bar{R}^{(u)}_{\mathrm{A,opt}}(\vartheta)$ it is not decisive which of the users are served adaptively inside a certain demand group $\mathcal{G}_i$, but only how many users $|\mathcal{G}_i|$ are served inside this group. Exploiting this fact, an algorithm with lowered complexity referred to as RedCom algorithm can be found which optimally solves (3.9). Like in an exhaustive search, all possible numbers U_A of adaptive users are tested. Assuming there are G different demand groups, for each possible number U_A of adaptive users there exist a G-tuple $\{\mu_{U_\mathrm{A},1},\mu_{U_\mathrm{A},2},..,\mu_{U_\mathrm{A},G}\}$ with $\mu_{U_\mathrm{A},i}$ denoting the number of adaptively served users inside demand group $\mathcal{G}_i$ where the following equation must hold:

$$\begin{aligned}\sum_{i=1}^{G}\mu_{U_\mathrm{A},i} &= U_\mathrm{A} \\ \text{with}\quad & \mu_{U_\mathrm{A},i} \le |\mathcal{G}_i|.\end{aligned} \tag{3.250}$$

Note that for G demand groups with $|\mathcal{G}_i|$ users in the i-th demand group $\mathcal{G}_i$, there exist

$$N_{\mathrm{tuple}} = \prod_{i=1}^{G}(|\mathcal{G}_i| + 1) \tag{3.251}$$

different G-tuples in total.

Since the data rate of each user u does not depend on the user serving vector, but on the number of adaptive users inside each demand group, it is enough to determine for each G-tuple $\{\mu_{U_\mathrm{A},1},\mu_{U_\mathrm{A},2},..,\mu_{U_\mathrm{A},G}\}$ the $\mu_{U_\mathrm{A},i}$ users in each demand group $\mathcal{G}_i$ which achieve the highest gain when served adaptively compared to the case when served non-adaptively, instead of testing all $\binom{U}{U_\mathrm{A}}$ possible user serving vectors. In the end, the system data rates of the best user serving vectors for all possible numbers U_A of adaptive users have to be compared to find the optimal user serving vector. Note that for the extreme case of $G = U$ with $|\mathcal{G}_i| = 1$, i.e., each user has a different weighting vector, the number of tuples to be checked equals

$$N_{\mathrm{tuple}} = \prod_{i=1}^{U}(1 + 1) = 2^U,$$

i.e., in this case the RedCom algorithm is equivalent to the ES algorithm. For the other extreme case of $G = 1$ with $|\mathcal{G}_1| = U$, i.e., all users have the same weighting factor, the number of tuples to be checked equals

$$N_{\text{tuple}} = \prod_{i=1}^{1}(U+1) = U+1.$$

The pseudo code of the RedCom algorithm is outlined as follows:

1) Determine $\bar{R}^{(u)}_{\text{N,opt}}$ for each user u.
2) Determine $\bar{R}^{(u)}_{\text{A,opt}}(U_{\text{A}}, \rho_u, \sigma^2_{\text{E},u})$ for each G-tuple $\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\}$ for $U_{\text{A}} = 1, .., U$ for each user u.
3) Determine $\bar{R}_{\text{sys}}(0)$ for the case of no adaptive user ($U_{\text{A}} = 0$), i.e. $\vartheta_u = 0 \; \forall \; u$.
4) Set the number of adaptive users to $U_{\text{A}} = 1$.
5) Determine the difference $\Delta_u(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\}) = \bar{R}^{(u)}_{\text{A,opt}}(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\}, \rho_u, \sigma^2_{\text{E},u}) - \bar{R}^{(u)}_{\text{N,opt}}$ for each G-tuple $(\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G})$ for each user u.
6) For each demand group $\mathcal{G}_i$ find the $\mu_{U_{\text{A}},i}$ users with the highest non-negative $\Delta_u(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\})$.
7) If there exist no $\mu_{U_{\text{A}},i}$ users with non-negative $\Delta_u(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\})$ for none of the G-tuples $\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\}$, store $\bar{R}_{\text{sys}}(U_{\text{A}}) = 0$ and go to 10), else set $\vartheta_u(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\}) = 1$ for these users.
8) For each G-tuple compute $\bar{R}_{\text{sys}}(\{\mu_{U_{\text{A}},1}, \mu_{U_{\text{A}},2}, .., \mu_{U_{\text{A}},G}\})$ and determine the G-tuple which achieves the highest system data rate for U_{A} adaptive users.
9) Store the user serving vector corresponding to the best G-tuple as $\vartheta(U_{\text{A}})$ and the corresponding system data rate as $\bar{R}_{\text{sys}}(U_{\text{A}})$.
10) If $U_{\text{A}} = U$, go to 11), else increase $U_{\text{A}} \rightarrow U_{\text{A}} + 1$ and go back to 5).
11) Find the optimal number of adaptive users $U_{\text{A,opt}}$ by determining the maximum system data rate $\bar{R}_{\text{sys}}(U_{\text{A,opt}}) = \max\limits_{U_{\text{A}}} \bar{R}_{\text{sys}}(U_{\text{A}})$ with $U_{\text{A}} = 0, .., U$. The optimal user serving vector is then given by $\vartheta(U_{\text{A,opt}})$.

3.7.3 Solutions for the special case of equal user channel access demands

3.7.3.1 Introduction

For the special case that all users have the same channel access demand, i.e., the number G of demand groups equals $G = 1$, it is possible to find an algorithm with even more reduced complexity than the RedCom algorithm presented in Section 3.7.2.3 referred to as RedCom2 algorithm. For the case of applying the Non-Adaptive First scheme, the RedCom2-NAF algorithm is still optimal compared to ES. However, for the case of applying the Adaptive First scheme, the RedCom2-AF algorithm is only suboptimal. In the following, the RedCom2-NAF algorithm is presented in Section 3.7.3.2 and the RedCom2-AF is presented in Section 3.7.3.3.

3.7.3.2 Non-Adaptive First

The complexity of the RedCom algorithm can be further reduced by taking into account the monotony of the function $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}})$ in case of applying the Non-Adaptive First scheme with $G = 1$. To illustrate this, the user data rate of an adaptive user is depicted in Fig. 3.12(a) as a function of the number U_{A} of adaptive users applying Non-Adaptive First assuming perfect CQI and an average SNR of $\bar{\gamma} = 10$ dB. Note that all adaptive users have the same weighting factor. One can see that $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}})$ increases monotonically with increasing value of U_{A} due to the inherent multi-user diversity. Hence, the user data rate of an adaptive user will always be the highest for a maximum number of adaptive users, i.e., it is always beneficial to adaptively serve as many users as possible. Note that for some users with fast varying channel conditions the side condition of (3.9) cannot be fulfilled even though all users are served adaptively. Thus, serving all users adaptively does not lead to the solution of the problem. However, it is enough to search for the user serving vector which fulfills the side condition of (3.9) and which contains the highest number $U_{\mathrm{A,max}}$ of adaptive users to optimally solve (3.9). Instead of testing all possible numbers U_{A} of adaptive users, one directly searches for $U_{\mathrm{A,max}}$ by determining for each user u the minimum number $U^{(u)}_{\mathrm{A,min}}$ of adaptive users where $\bar{R}^{(u)}_{\mathrm{A,opt}}(U^{(u)}_{\mathrm{A,min}}, \rho_u, \sigma^2_{\mathrm{E},u}) \geq \bar{R}^{(u)}_{\mathrm{N,opt}}$ is fulfilled, i.e., one searches for the smallest number $U^{(u)}_{\mathrm{A,min}}$ of adaptive users which user u requires to achieve at least the user data rate applying the non-adaptive transmission scheme. Then, one simply has to compare these numbers to find the $U_{\mathrm{A,max}}$ users which should be served adaptively to optimally solve (3.9). The following pseudo code outlines the steps of the RedCom2-NAF algorithm:

1) Determine $\bar{R}^{(u)}_{\mathrm{N,opt}}$ for each user u.

2) Determine $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, \rho_u, \sigma^2_{\mathrm{E},u})$ for $U_{\mathrm{A}} = 1, .., U$ for each user u.

3) Determine for each user u the minimum number $U^{(u)}_{\mathrm{A,min}}$ of adaptive users for which $\bar{R}^{(u)}_{\mathrm{A,opt}}(U^{(u)}_{\mathrm{A,min}}, \rho_u, \sigma^2_{\mathrm{E},u}) \geq \bar{R}^{(u)}_{\mathrm{N,opt}}$ holds. In case that $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, \rho_u, \sigma^2_{\mathrm{E},u}) < \bar{R}^{(u)}_{\mathrm{N,opt}}$ $\forall\ U_{\mathrm{A}} = 1, .., U$ for a given user u, set $U^{(u)}_{\mathrm{A,min}} = U + 1$.

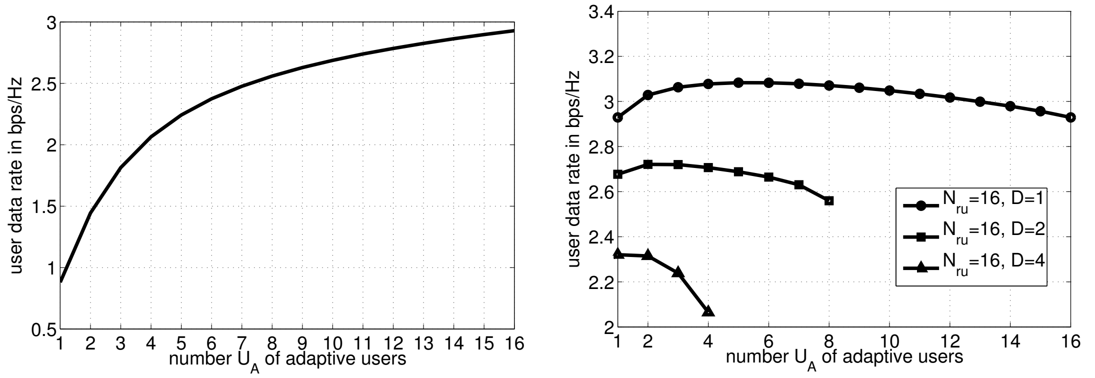
4) Sort the numbers $U^{(u)}_{\mathrm{A,min}}$ with $u = 1, .., U$ in descending order resulting in $U^{(\nu)}_{\mathrm{A,min,sort}}$ with $\nu = 1, .., U$.

5) Determine the smallest $\nu_{\min}$ for which $U^{(\nu_{\min})}_{\mathrm{A,min,sort}} \leq U - 1 + \nu_{\min}$ holds. The sought after highest number $U_{\mathrm{A,max}}$ of adaptive users is then given by $U_{\mathrm{A,max}} = U^{(\nu_{\min})}_{\mathrm{A,min,sort}}$. In case that $U^{(\nu)}_{\mathrm{A,min,sort}} > U - 1 + \nu\ \forall\ \nu = 1, .., U$, set $U_{\mathrm{A,max}} = U + 1$.

6) Set $\vartheta_u = 1$ if $U^{(u)}_{\mathrm{A,min}} \leq U_{\mathrm{A,max}}$, else set $\vartheta_u = 0$.

3.7.3.3 Adaptive First

In order to optimally solve (3.9) when applying Adaptive First, one can also use ES and the RedCom algorithm as shown in Section 3.7.2.3. In Fig. 3.12(b), the user data rate is depicted applying Adaptive First assuming $N_{\mathrm{ru}} = 16$ resource units in the system, where $D = \frac{N_{\mathrm{ru}}}{U_{\mathrm{A}}}$ resource units are allocated to each user. Increasing U_A, two opposed effects on the user data rate occur. On the one hand, an increasing U_A corresponds to an increased multi-user diversity leading to better user selection results. On the other hand, an increasing U_A leads to a less exclusive resource unit selection $W_{\mathrm{A}} = D \cdot U_{\mathrm{A}}$, i.e., the higher W_{A}, the higher the probability that a resource unit with a weak channel is selected. For the case that $U_{\mathrm{A}} = U$ and, thus, $W_{\mathrm{A}} = N_{\mathrm{ru}}$, no resource unit selection is performed, but only user selection resulting in the same performance as with Non-Adaptive First. The upper curve of Fig. 3.12(b) presents the user data rate in case that $D = 1$. For this case, it can be seen that for small U_{A}, the positive impact of an increased multi-user diversity on the user data rate is stronger than the negative impact of a less exclusive resource unit selection resulting in an increasing user data rate with increasing U_{A}. For $U_{\mathrm{A}} > 5$ the negative effect on the resource unit selection dominate the performance resulting in a decreasing user data rate for increasing U_{A}. From the other curves with $N_U = 2$ and $N_U = 4$, one can see that the characteristics of $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}})$ depend on the number N_{ru} of resource units and the number U of users in the system not being monotonic with respect to U_{A} anymore.



(a) (b)

Figure 3.12. User data rate as a function of the number U_{A} of adaptive users applying (a) Non-Adaptive First and (b) Adaptive First with $N_{\mathrm{ru}} = 16$.

Hence, the RedCom2-NAF algorithm is no longer reasonable since the monotonic increase of $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}})$ with increasing U_{A} no longer holds. Furthermore, adaptively serving as many users as possible does not automatically lead to the optimal solution.

In the following, a suboptimal algorithm which solves (3.9) when applying Adaptive First is presented referred to as RedCom2-AF algorithm. The main goal of this algorithm is to reduce the complexity compared to the optimal RedCom algorithm accepting some losses in system performance. As with the RedCom2-NAF algorithm, this algorithms aims at directly finding the user serving vector with a maximum number $U_{\mathrm{A,max}}$ of adaptive users such that the side condition of (3.9) is fullfilled while taking into account the fact that for large U_A the user data rate $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}})$ decreases with increasing U_A. The pseudo code of the RedCom2-AF algorithm is given as follows:

1) Determine $\bar{R}^{(u)}_{\mathrm{N,opt}}$ for each user u.

2) Determine $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, \rho_u, \sigma^2_{\mathrm{E},u})$ for $U_{\mathrm{A}} = 1, .., U$ for each user u.

3) If $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, \rho_u, \sigma^2_{\mathrm{E},u}) < \bar{R}^{(u)}_{\mathrm{N,opt}}\ \forall\ U_{\mathrm{A}} = 1, .., U$ for a given user u, set $U^{(u)}_{\mathrm{A,max}} = 0$, else determine for each user u the maximum number $U^{(u)}_{\mathrm{A,max}}$ of adaptive users for which $\bar{R}^{(u)}_{\mathrm{A,opt}}(U^{(u)}_{\mathrm{A,max}}, \rho_u, \sigma^2_{\mathrm{E},u}) \geq \bar{R}^{(u)}_{\mathrm{N,opt}}$ holds.

4) Sort the numbers $U^{(u)}_{\mathrm{A,max}}$ with $u = 1, .., U$ in ascending order resulting in $U^{(\nu)}_{\mathrm{A,max,sort}}$ with $\nu = 1, .., U$.

5) If $U^{(\nu)}_{\mathrm{A,max,sort}} < U - 1 + \nu\ \forall\ \nu = 1, .., U$, set $U_{\mathrm{A,max}} = 0$, else determine the highest ν_{max} for which $U^{(\nu_{\mathrm{max}})}_{\mathrm{A,max,sort}} \geq U - 1 + \nu_{\mathrm{max}}$ holds. The sought after highest number $U_{\mathrm{A,max}}$ of adaptive users is then given by $U_{\mathrm{A,max}} = U^{(\nu_{\mathrm{max}})}_{\mathrm{A,max,sort}}$.

6) Set $\vartheta_u = 1$ if $U_{\mathrm{A,max}}^{(u)} \geq U_{\mathrm{A,max}}$, else set $\vartheta_u = 0$.

3.7.4 Complexity analysis

3.7.4.1 Introduction

In this section, the complexity of the different algorithms is analyzed. As the user serving vector has to be updated online at regular intervals, it is important to know the complexity of the different algorithms. For the SNR threshold problem, the computational complexity is less critical, as the calculation of $\bar{R}_{\mathrm{N,opt}}^{(u)}$ and $\bar{R}_{\mathrm{A,opt}}^{(u)}(\{\mu_{U_{\mathrm{A}},1}, \mu_{U_{\mathrm{A}},2}, .., \mu_{U_{\mathrm{A}},G}\}, \rho_u, \sigma_{\mathrm{E},u}^2)$ for all G-tuples $\{\mu_{U_{\mathrm{A}},1}, \mu_{U_{\mathrm{A}},2}, .., \mu_{U_{\mathrm{A}},G}\}$ with $U_{\mathrm{A}} = 1, .., U$ for all users can be performed offline, so this computational complexity is not considered.

In the following, the number of operations of the different algorithms is determined as a function of the problem dimension, i.e., the number U of users. In order to do so, the number of operations for four different procedures are introduced in Table 3.1 [Knu97].

Table 3.1. Number of operations

Procedure	**Number of operations**
Access to U values from a look-up table	U
Addition of U values	U
Comparison of U values	U
Sorting of U unsorted values	U^2

Note that for the complexity considering the sorting of U unsorted values, the worst-case complexity [Knu97] is assumed. With the help of Table 3.1, the number of operations for the ES algorithm, the RedCom algorithm and the RedCom2 algorithm are derived in Section 3.7.4.2, 3.7.4.3, and 3.7.4.4, respectively.

3.7.4.2 ES algorithm

Applying the ES algorithm, U values have to be read out from the look-up table 2^U times. Further on, U values have to be added 2^U times. Finally, 2^U values have to be compared, resulting in a total number of

$$N_{\mathrm{O,ES}} = 2^U \cdot (2U + 1) \tag{3.252}$$

operations.

3.7.4.3 RedCom algorithm

Applying the RedCom algorithm, $N_{\text{tuple}} \cdot U$ times a value from the look-up table has to be read out. Furthermore, $N_{\text{tuple}} \cdot U$ substractions are performed. For each G-tuple, G sorting of $|\mathcal{G}_i|$ values with $i = 1, .., G$ have to be done. Further on, for each $G-$tuple, U additions have to be performed. Finally, N_{tuple} comparisons are made resulting in a total number of

$$N_{\text{O,RedCom}} = \left(\prod_{i=1}^{G} (|\mathcal{G}_i| + 1) \right) \cdot \left(2U + \sum_{i=1}^{G} |\mathcal{G}_i|^2 + 1 \right) \tag{3.253}$$

operations. For the extreme case $G = U$ with $|\mathcal{G}_i| = 1 \; \forall \; i = 1, .., G$, (3.253) is equivalent to (3.252) since $N_{\text{tuple}} = 2^U$ and the sorting can be neglected in this case as there is only one value per demand group. For the other extreme case of $G = 1$ with $|\mathcal{G}_1| = U$, the number of operations is given by

$$N_{\text{O,RedCom}}(G = 1) = (U + 1) \cdot (2U + U^2 + 1) = (U + 1)^3. \tag{3.254}$$

3.7.4.4 RedCom2 algorithms

Due to the similar structure of the RedCom2-NAF and RedCom2-AF algorithms, the complexity analysis is valid for both algorithms. For both algorithms, $(U+1) \cdot U$ times a value is read out from the look-up table. Furthermore, $(U + 1)$ comparisons of U values are performed. Further on, U values are sorted. Finally, at most U values are compared, resulting in a number of

$$N_{\text{O,RedCom2}} = 2 \cdot (U + 1) \cdot U + U^2 + U = 3 \cdot (U^2 + U). \tag{3.255}$$

In Fig. 3.13, the number of required operations is depicted as a function of the number U of users for the different algorithms for different numbers G of demand groups where it is assumed that $|\mathcal{G}_i| = \left\lfloor \frac{U}{G} \right\rfloor \; \forall \; i = 1, .., G$. It can be seen that the higher the number G of different demand groups, the higher the complexity. For the case $G = U$, the complexity of the RedCom algorithm is equivalent to the ES algorithm. However, for cases with $G < U$, the reduction of complexity of the RedCom-algorithm compared to the ES algorithm is tremendous, especially for large number U of users. Assuming that state of the art data processors are capable of executing 10^{10} operations per second, the time period T_{up} between updating the user serving vector must not be smaller than $T_{\text{up}} \leq 1$ ms to be able to execute the required operations for up to $G \leq 5$ demand groups and up to $U = 30$ users applying the RedCom algorithm. Note that one could

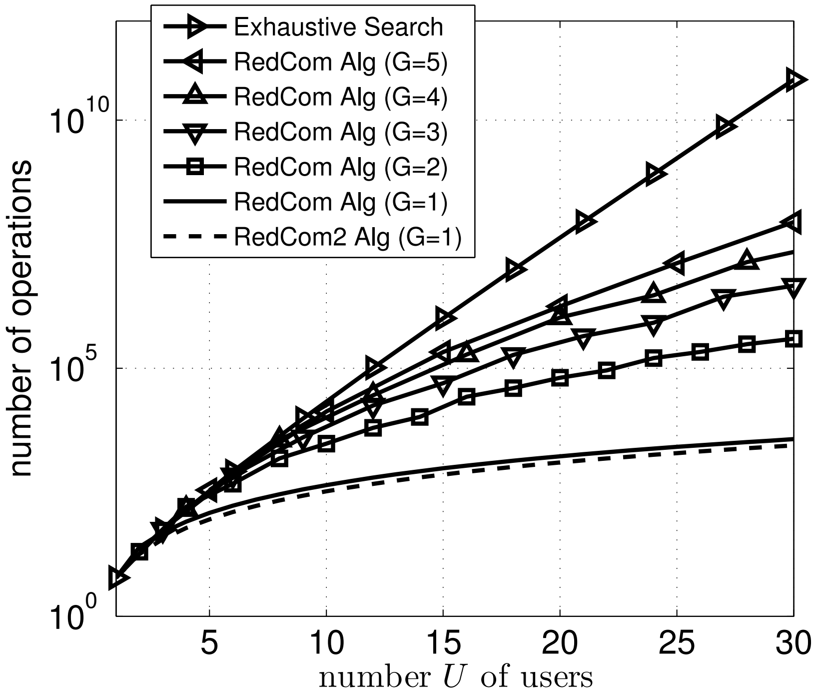


Figure 3.13. Number of operations vs. number U of users

also consider larger update time periods than $T_{\text{up}} = 1$ ms as the CQI impairment parameters which affect the solution of the user serving problem do not change so fast in a realistic scenario. Note that this aspect will be further discussed in Chapter 4.

Chapter 4

Pilot and signaling overhead

4.1 Introduction

In the consideration of combining adaptive and non-adaptive transmission modes, the effort in terms of pilot and signaling overhead which has to be spent in order to provide the BS with CQI and to perform adaptive schemes like adaptive modulation and resource allocation has not been taken into account so far. However, since the non-adaptive transmission mode requires much less overhead due to its property of working independent of any transmitter sided CQI, it is important to incorporate the overhead in the achievable user data rate applying either the adaptive transmission mode or the non-adaptive transmission mode to get a meaningful and realistic result. To do so, the effort in terms of pilot transmissions and signaling of side information have to be identified for both transmission modes. Since pilot and signaling overhead does not only effect the DL, also the UL has to be considered since in the UL, resources have to be spent such that the BS is able to acquire information about the UL and DL channel quality. These resources can no longer be used for data transmission reducing the overall system performance. Hence, considering overhead requires the introduction of a certain time frame structure of the transmission in both UL and DL direction which has to be done separately for TDD and FDD systems. It is assumed that the BS does all the computationally demanding calculations like the access scheme selection, resource allocation and modulation scheme selection and subsequently signals the results to the MSs. From this it follows that the MS can be kept rather simple which is reasonable from a practical point of view. Note that the transmission of control bits used for synchronization or other purposes not dealing with the channel estimation, resource allocation, modulation or antenna selection are not considered here since these control bits have to be spent for both adaptive and non-adaptive transmission.

This chapter is organized as follows. In Section 4.2, a frame structure is introduced to identify the amount of pilot and signaling overhead. Moreover, the effective user data rates taking into account the overhead of adaptively and non-adaptively served users are derived for both schemes Non-Adaptive First and Adaptive First. For comparison, the effective data rates for pure conventional non-adaptive and adaptive TDD systems are derived as well. In Section 4.3, the same is done for an FDD system, where one has to differentiate between Half Duplex and Full Duplex systems. Finally, it is shown in Section 4.4 how to maximize the effective system data rate. Several parts of this Chapter 4 have been originally published by the author in [KKWW09].

4.2 TDD systems

4.2.1 Superframe structure

In the following, a superframe structure is introduced. It is assumed that each DL time frame has the same time duration as an UL time frame, i.e. symmetric traffic is assumed. Assuming TDD, it is possible for the BS to exploit the reciprocity of the channel to estimate the DL channel during the pilot phase in the UL. The superframe structure of the considered TDD system is depicted in Fig. 4.1.

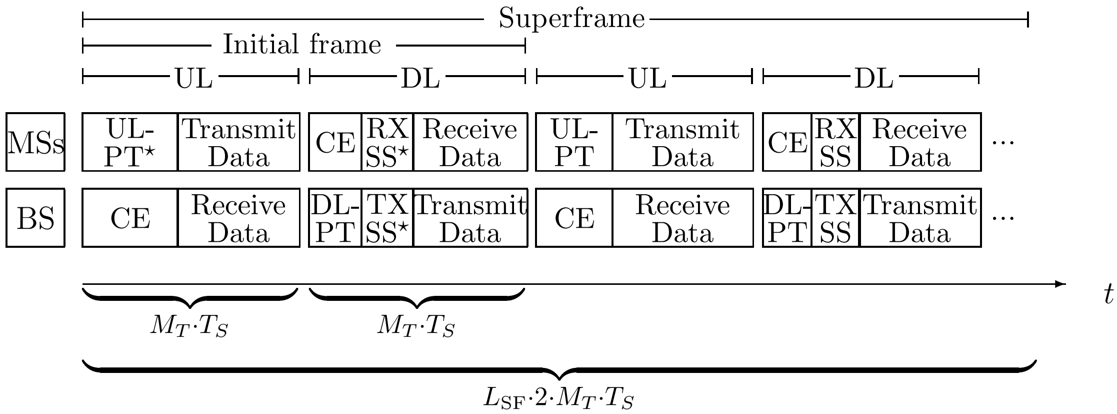


Figure 4.1. Superframe structure

Each subframe in UL and DL has a length of M_T OFDMA symbol durations T_S where it is assumed that the channel of each subframe is temporally correlated to the channel of the previous subframe and does not change significantly during one subframe as stated in Section 2.4. The total superframe has a length of $L_{\mathrm{SF}} \cdot 2 \cdot M_{\mathrm{T}} \cdot T_{\mathrm{S}}$ OFDMA symbols with L_{SF} denoting the superframe length. Each superframe starts with two special UL and DL subframes, the initial frame, which is required to perform the access scheme selection. Within these two initial subframes, the amount of pilot transmission and signaling differs compared to the remaining $L_{\mathrm{SF}} - 1$ UL and DL subframes.

The main idea of the superframe structure is to reduce the overhead due to signaling and pilot transmission by determining only once per superframe which user shall be served adaptively or non-adaptively instead of determining it frame by frame. This means, one has to assume that the user conditions which affect the impairment parameters describing the CQI imperfectness do *not* change significantly during the time duration $L_{\mathrm{SF}} \cdot 2 \cdot M_T \cdot T_S$ of a superframe. The main impairment parameter which can significantly chance over time is the correlation coefficient ρ_u, i.e., the velocity of user u. In a realistic scenario, a change in velocity happens in time regions of tenth seconds to seconds.

However, one has to differentiate between the two combining schemes Non-Adaptive First and Adaptive-First. With Non-Adaptive First, the resources units assigned to non-adaptive users and adaptive users do not chance for consecutive time frames assuming that the user serving vector ϑ remains the same as can be seen in Fig. 3.3, i.e., a resource unit which is allocated to a non-adaptive will always be allocated to a non-adaptive users within the superframe and a resource unit allocated to an adaptive user will always be allocated to an adaptive users with in the superframe even though the adaptive user can be different. Thus, the use of a superframe structure as presented above is possible when applying Non-Adaptive First. For the case of Adaptive First, the resource units which are assigned to non-adaptive users and adaptive users can differ compared to the previous frame even with a constant ϑ as can be seen in Fig. 3.3, i.e., a resource unit allocated to a non-adaptive user in the first frame can be allocated to an adaptive user in the second frame within the superframe. This means that when applying Adaptive First, it is not possible to use the presented superframe structure with $L_{\mathrm{SF}} > 1$, since in each frame, the BS has to know the CQI values of each user on each resource unit. From this, it follows that in this case the length of the superframe is limited to $L_{\mathrm{SF}} = 1$.

In the following, the frame structure of Fig. 4.1 is described in details. At the beginning of each superframe, UL Pilot Transmission (ULPT*) is performed at each MS. The star indicates that the PT is done on all N_{ru} resource units so that the BS is able to determine the CQI for the whole UL channel of each user u. Furthermore, the pilots are used for channel estimation (CE) to receive and equalize the data symbols which are transmitted in the remaining OFDMA symbols.

At the beginning of each superframe, the BS has to decide which user shall be served adaptively or non-adaptively using the updated information about the impairment parameters describing the CQI imperfectness which are calculated and updated during each superframe as shown in Section 2.9. Furthermore, the BS has to calculate which resource units are allocated to which user in the next DL subframe and which modulation scheme is applied on which resource unit.

At the beginning of the DL subframe within the initial frame, the Signaling of Side Information (SS*) concerning resource allocation, modulation scheme selection *and* user serving is performed which is additionally indicated by the star, i.e., the information concerning the applied multiple access scheme is only signalled once at the beginning of the superframe. Besides signaling SS*, DL Pilot Transmission (DLPT) is performed such that the MSs are able to perform CE. Note that TX indicates the transmission of the side information while RX indicates the receiption of the side information. In the remaining OFDMA symbols, the BS transmits data to the different MSs according to side information. At the beginning of the second UL subframe, adaptively served users perform ULPT for all resource units which assigned for adaptive users while non-adaptive users perform ULPT only for those resource units who are assigned to non-adaptive users. On the remaining OFDMA symbols, data symbols are transmitted based on the scheduling decisions of the previous DL subframe. In the next DL subframe, DLPT is performed for all resource units followed by the signaling of the Side Information (SS) concerning resource allocation and modulation scheme selection, i.e., the information concerning the access scheme selection does not have to be signalled as it remains unchanged. These last two UL and DL subframes are repeated $L_{\mathrm{SF}} - 1$ times until the beginning of the next superframe.

Note that this frame structure should not be seen as a transmission protocol. The main purpose is to identify the amount of pilot and side information which has to be transmitted. In this context, one can assume that instead of transmitting all pilots at the beginning of the frame, the pilots are rather distributed over the whole frame to track the channel so that the assumption of perfect CSI at the receiver is justifiable. However, since the resource allocation and adaptive modulation requires computational time, it is necessary to perform a CE right at the beginning of the UL subframe based on $M_{P,\mathrm{CQI}}$ pilots to determine the CQI values such that the BS can compute the side information for the next DL subframe in time. As each user has in total M_P pilots per resource units for the each subframe,

$$1 \leq M_{P,\mathrm{CQI}} \leq M_P. \tag{4.1}$$

4.2.2 Pilot and signaling overhead

4.2.2.1 Pilot overhead in the Downlink

In the following, the pilot and signaling overhead in terms of OFDMA symbols is determined in the DL and UL for both OSTBC-MRC and TAS-MRC based on the superframe structure introduced in Section 4.2.1.

Assuming that the channel does not significantly change within on resource unit consisting of a frequency block of Q_{sub} subcarriers in the frequency domain and M_T OFDMA symbols in the time domain as stated in Section 2.3, it is sufficient to transmit pilots on only one subcarrier per resource unit. The remaining subcarriers can be used for data transmission.

In case of OSTBC-MRC, the receiver requires pilots from each transmit antenna to estimate the channel of all possible transmit antenna - receive antenna pairs. This means that when pilots are sent on certain subcarriers from a given transmit antenna, these subcarriers have to remain unoccupied for the other transmit antennas so that the pilots symbols are only affected by the channel and not by other pilot or data symbols from other transmit antennas. Assuming that M_P pilots are transmitted per resource unit consisting of Q_{sub} subcarriers in the frequency domain and M_T OFDMA symbols in the time domain, the overhead for each user u results in

$$M_{\text{DLPT}-\text{STC}} = \frac{n_T \cdot M_P}{Q_{\text{sub}}} \tag{4.2}$$

OFDMA symbols, i.e., a given resource unit cannot be used for data transmission by a user for the duration of $M_{\text{DLPT}-\text{STC}}$ OFDMA symbols. Note that $M_{\text{DLPT}-\text{STC}}$ does not have to be an integer number as fractions of the resource unit can still be used for transmission. The same is true for all other pilot or signaling overhead values derived in the following.

Fig. 4.2 illustrates this for a system with $n_T = 2$ transmit antennas and $M_P = 1$ regarding a resource unit with $Q_{\text{sub}} = 3$ subcarriers with frequency spacing Δf and $M_T = 2$ OFDMA symbols with symbol duration T_{S}. On the first and third subcarriers of each antenna, the data symbols d_i with $i = 1, .., 4$ are transmitted according to the Alamouti Space-Time Coding. The second subcarrier is used for transmitting pilot symbols p_1 and p_2. Thus, two subcarriers cannot be used for transmission. Assuming, these two subcarriers are within one OFDMA symbol, $\frac{2}{3}$ of this OFDMA symbol would has to be spent for pilot transmission.

In case of TAS-MRC, the BS only transmits pilots and data on the selected transmit antenna leading to

$$M_{\text{DLPT}-\text{TAS}} = \frac{M_P}{Q_{\text{sub}}} \tag{4.3}$$

pilot overhead for each user u.

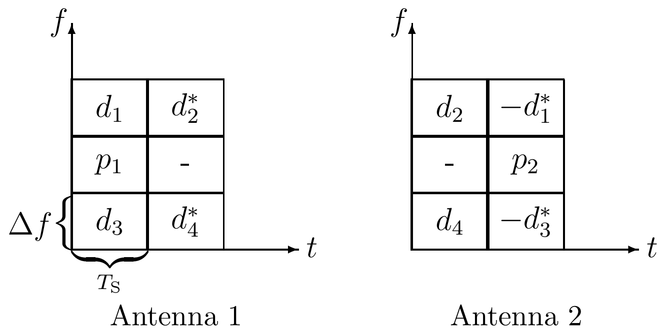


Figure 4.2. Example pilot overhead in the DL

4.2.2.2 Signaling overhead in the Downlink

In case of OSTBC-MRC, each resource unit has to carry the following information at the beginning of each superframe: First, the index of the user to which the corresponding resource unit is allocated. Second, the serving class of this user (either non-adaptive or adaptive). Third, the index of the applied modulation scheme for this DL subframe and the modulation scheme index for the next UL subframe. With U users and M available modulation schemes, the signaling overhead results in

$$M_{\mathrm{SS}^\star-\mathrm{STC}} = \frac{1 + \lceil \log_2(U) \rceil + 2 \cdot \lceil \log_2(M) \rceil}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.4}$$

for each user u with b_{SS} denoting the number of bits per symbol used for signaling and $\lceil \cdot \rceil$ denoting the nearest integer larger than or equal to the argument.

For the next DL subframes within the superframe, the resource units of the non-adaptive users are already allocated. Furthermore, the applied modulation schemes remain the same as the average SNR $\bar{\gamma}_u$ is assumed to be constant as stated in Section 2.4. Thus, for non-adaptive users, no signaling has to be performed. For the adaptive users, the resource units are allocated according to the CQI following the WPFS policy, i.e., the user index and the modulation scheme indices have to be signalled leading to

$$M_{\mathrm{SS-STC-A}} = \frac{\lceil \log_2(U) \rceil + 2 \cdot \lceil \log_2(M) \rceil}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.5}$$

signaling overhead.

In case of TAS-MRC, one has to differentiate between adaptive and non-adaptive users since transmit antenna selection is only performed for the adaptive users. Thus, at the beginning of the superframe, the resource units dedicated to adaptive users have to carry information about the user index, serving class, modulation scheme indices for the DL and the next UL subframe *and* the antenna index of the transmit antenna to be used in the next UL frame leading to

$$M_{\mathrm{SS}^\star-\mathrm{TAS-A}} = \frac{1 + \lceil \log_2(U) \rceil + \lceil \log_2(n_T) \rceil + 2 \cdot \lceil \log_2(M) \rceil}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.6}$$

signaling overhead.

Since non-adaptive users do not apply TAS in the UL, the overhead at the beginning of the superframe is given by

$$M_{\mathrm{SS}^\star-\mathrm{TAS-NA}} = \frac{1 + \lceil \log_2(U) \rceil + 2 \cdot \lceil \log_2(M) \rceil}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}}. \tag{4.7}$$

For the next DL subframes within the superframe, non-adaptive users need no signaling while for adaptive users, the signaling overhead in the next DL subframes within the superframe is given by

$$M_{\mathrm{SS-TAS}} = \frac{\lceil \log_2(U) \rceil + \lceil \log_2(n_T) \rceil + 2 \cdot \lceil \log_2(M) \rceil}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.8}$$

since the adaptive users need to know the user index, the modulation scheme indices and antenna index as these indices can change from frame to frame.

4.2.2.3 Pilot overhead in the Uplink

At the beginning of each superframe, all users have to transmit $M_{P,\mathrm{CQI}}$ pilots on each for the N_{ru} available resource units such that the BS is able to determine the CQI values for the whole DL channel of each user. Furthermore, this has to be done for each antenna separately. After that CQI pilot phase, all users transmit the remaining $M_P - M_{P,\mathrm{CQI}}$ pilots only on resource units that are allocated to them. Taking into account the number n_T of transmit antennas, the UL pilot overhead for both OSTBC-MRC and TAS-MRC results in

$$\begin{aligned} M_{\mathrm{ULPT}^\star} &= \frac{U \cdot n_T \cdot M_{P,\mathrm{CQI}}}{Q_{\mathrm{sub}}} + \frac{n_T \cdot (M_P - M_{P,\mathrm{CQI}})}{Q_{\mathrm{sub}}} \\ &= \frac{n_T \cdot (M_P + (U-1) \cdot M_{P,\mathrm{CQI}})}{Q_{\mathrm{sub}}} \end{aligned} \tag{4.9}$$

In other words, in each resource unit, $M_P + (U-1) \cdot M_{P,\mathrm{CQI}}$ pilot symbols have to be transmitted, M_P pilots from the user to which the resource is finally allocated and $(U-1) \cdot M_{P,\mathrm{CQI}}$ pilots from the other $(U-1)$ users in the CQI pilot phase.

For the next UL frame within the superframe, non-adaptive users only have to transmit pilots on resource units which are assigned to them leading to

$$M_{\mathrm{ULPT-NA}} = \frac{n_T \cdot M_P}{Q_{\mathrm{sub}}} \tag{4.10}$$

pilot overhead. For the U_{A} adaptive users, the pilot overhead in the UL within the superframe is given by

$$M_{\mathrm{ULPT-A}}(U_{\mathrm{A}}) = \frac{n_T \cdot (M_P + (U_{\mathrm{A}} - 1) \cdot M_{P,\mathrm{CQI}})}{Q_{\mathrm{sub}}} \tag{4.11}$$

since the BS needs to determine the CQI values of each adaptive user from all resource units assigned to adaptive users as the resource allocation changes frame by frame. Comparing the pilot overhead of non-adaptive users and adaptive users, one has to spend $1 + \frac{(U_{\mathrm{A}}-1)M_{P,\mathrm{CQI}}}{M_P}$ times more resources for the adaptive users. This is the price one has to pay exploiting multi-user diversity in the adaptive transmission mode. Note that $M_{\mathrm{ULPT-A}}(U_{\mathrm{A}})$ is a function of the number U_{A} of adaptive users.

4.2.3 Effective user data rate applying Non-Adaptive First

4.2.3.1 Non-adaptive users

In the following, the effective user data rate for non-adaptive users taking into account the overhead in UL and DL is derived when applying the Non-Adaptive First combining scheme.

During one superframe, each non-adaptive user can transmit DL data on a total number of $L_{\mathrm{SF}} \cdot M_T$ OFDMA symbols. However, during the first DL subframe at the beginning of the superframe, $M_{\mathrm{SS}^\star-\mathrm{NA}} + M_{\mathrm{DLPT}}$ OFDMA symbols are needed for signaling and PT. For the remaining $(L_{\mathrm{SF}}-1)$ DL subframes within the superframe, $(L_{\mathrm{SF}}-1)\cdot M_{\mathrm{DLPT}}$ OFDMA symbols have to be spent for PT. From this, it follows that the actual number M_{DLT} of OFDMA symbols available for DL data transmission for each non-adaptive user is given by

$$\begin{aligned} M_{\mathrm{DLT-NA}} &= L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS}^\star-\mathrm{NA}} - M_{\mathrm{DLPT}} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{DLPT}} \\ &= L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS}^\star-\mathrm{NA}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}}. \end{aligned} \tag{4.12}$$

On these $M_{\mathrm{DLT-NA}}$ OFDMA symbols, each non-adaptive user u can achieve a data rate of $\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u)$ in the DL direction assuming optimized SNR thresholds as shown in Section 3.6 where $\bar{\gamma}_u$ denotes the average SNR of user u in the DL.

For the UL, also a total number of $L_{\text{SF}} \cdot M_T$ OFDMA symbols are available within one superframe. However, $M_{\text{ULPT}^\star}$ OFDMA symbols are used for PT at the beginning of the superframe and $(L_{\text{SF}} - 1) \cdot M_{\text{ULPT-NA}}$ OFDMA symbols are spent for PT in the remaining $(L_{\text{SF}} - 1)$ UL frames within the superframe. Thus, the actual number M_{ULT} of OFDMA symbols available for UL data transmission for each non-adaptive user is given by

$$M_{\text{ULT-NA}} = L_{\text{SF}} \cdot M_T - M_{\text{ULPT}^\star} - (L_{\text{SF}} - 1) \cdot M_{\text{ULPT-NA}}. \tag{4.13}$$

On these $M_{\text{ULT-NA}}$ OFDMA symbols, each non-adaptive user u can achieve a data rate of $\bar{R}^{(u)}_{\text{N,opt}}(\kappa_{\text{UL}} \cdot \bar{\gamma}_u)$ in the UL direction, where $\bar{\gamma}_u$ denotes the average SNR of user u in the DL and κ_{UL} the UL factor as introduced in Eq. (2.8).

The achievable effective user data rate $\bar{R}^{(u)}_{\text{N,eff,opt}}$ of a non-adaptive users assuming optimized SNR thresholds and taking into account UL and DL is then given by

$$\begin{aligned}
\bar{R}^{(u)}_{\text{N,eff,opt}} &= \frac{1}{L_{\text{SF}} \cdot 2 \cdot M_T} \\
&\quad \cdot \Big[\bar{R}^{(u)}_{\text{N,opt}}(\bar{\gamma}_u) \cdot (L_{\text{SF}} \cdot M_T - M_{\text{SS}^\star\text{-NA}} - L_{\text{SF}} \cdot M_{\text{DLPT}}) \\
&\quad + \bar{R}^{(u)}_{\text{N,opt}}(\kappa_{\text{UL}} \cdot \bar{\gamma}_u) \cdot (L_{\text{SF}} \cdot M_T - M_{\text{ULPT}^\star} - (L_{\text{SF}} - 1) \cdot M_{\text{ULPT-NA}})\Big]
\end{aligned} \tag{4.14}$$

4.2.3.2 Adaptive users

In the following, the effective user data rate for non-adaptive users taking into account the overhead in UL and DL is derived when applying the Non-Adaptive First combining scheme.

Like with non-adaptive users, one can determine the actual number of OFDMA symbols which are available for adaptive users in DL and UL transmission. As can be seen from the superframe structure, there are

$$\begin{aligned}
M_{\text{DLT-A}} &= L_{\text{SF}} \cdot M_T - M_{\text{SS}^\star\text{-A}} - M_{\text{DLPT}} \\
&\quad -(L_{\text{SF}} - 1) \cdot (M_{\text{DLPT}} + M_{\text{SS-A}}) \\
&= L_{\text{SF}} \cdot M_T - M_{\text{SS}^\star\text{-A}} - L_{\text{SF}} \cdot M_{\text{DLPT}} - (L_{\text{SF}} - 1) \cdot M_{\text{SS-A}}
\end{aligned} \tag{4.15}$$

OFDMA symbols available for DL data transmission which the adaptive users can use for adaptive transmission employing WPFS scheduling and adaptive modulation scheme selection based on CQI. From the superframe structure it can be seen that the time delay between measuring the CQI in the UL subframe and the actual data transmission is $T = M_T \cdot T_S$, i.e., the corresponding correlation coefficient is given by

$$\rho_u(M_T) = J_0(2\pi \cdot f_{D,u} \cdot M_T \cdot T_S). \tag{4.16}$$

Furthermore, the CQI is only an estimate based on a CE performed at the beginning of the subframe using $M_{P,\mathrm{CQI}}$ pilots leading to an error estimation variance

$$\sigma^2_{E,u}(M_{P,\mathrm{CQI}}) = \frac{1}{\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u \cdot M_{P,\mathrm{CQI}}}. \tag{4.17}$$

From this, it follows that on these $M_{\mathrm{ULT-A-NAF}}$ OFDMA symbols, each adaptive user can achieve a data rate of $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ assuming optimized SNR thresholds with respect to the impairment parameters $\rho_u(M_T)$ and $\sigma^2_{E,u}(M_{P,\mathrm{CQI}})$ and the number U_{A} of adaptive users as shown in Section 3.6.2.

For the UL, there are

$$M_{\mathrm{ULT-A}} = L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT}^\star} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{ULPT-A}}(U_{\mathrm{A}}) \tag{4.18}$$

OFDMA symbols available for actual UL data transmission. As stated in Section 4.2.1, each MS uses the resource allocation and modulation scheme selection which was signalled by the BS in the previous DL frame. Thus, the time delay between the CQI measuring on which the resource allocation and modulation scheme selection is based and the actual data transmission in the UL equals $2 \cdot M_T \cdot T_S$. Since the BS is aware of this time delay, the SNR thresholds can be chosen accordingly, meaning that the modulation schemes have to be selected more conservatively leading to an achievable user data rate of $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, 2 \cdot M_T, M_{P,\mathrm{CQI}})$.

The achievable effective user data rate $\bar{R}^{(u)}_{\mathrm{A,eff,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ of an adaptive users assuming optimized SNR thresholds and taking into account UL and DL is then given by

$$\begin{aligned}
\bar{R}^{(u)}_{\mathrm{A,eff,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \quad = \quad & \frac{1}{L_{\mathrm{SF}} \cdot 2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \\
& \cdot (L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS}^\star\mathrm{-A}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}} \\
& - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{SS-A}}) \\
& + \bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, 2 \cdot M_T, M_{P,\mathrm{CQI}}) \\
& \cdot (L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT}^\star} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{ULPT-A}}(U_{\mathrm{A}})) \Big] .
\end{aligned} \tag{4.19}$$

4.2.4 Effective user data rate applying Adaptive First

4.2.4.1 Non-adaptive users

As mentioned in Section 4.2.1, the length of the superframe is limited to $L_{\mathrm{SF}} = 1$ when applying Adaptive First. From this it follows that the effective user data rate applying the Adaptive First scheme is a special case of the effective user data rate applying Non-Adaptive First with $\text{Ł}_{\text{SF}} = 1$. Hence, for non-adaptive users the achievable effective user data rate assuming optimized SNR thresholds is given by

$$\begin{aligned}\bar{R}^{(u)}_{\text{N,eff,opt}} &= \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\text{N,opt}}(\bar{\gamma}_u) \cdot (M_T - M_{\text{SS}^\star-\text{NA}} - M_{\text{DLPT}}) \\ &\quad + \bar{R}^{(u)}_{\text{N,opt}}(\kappa_{\text{UL}} \cdot \bar{\gamma}_u) \cdot (M_T - M_{\text{ULPT}^\star})\Big] .\end{aligned} \tag{4.20}$$

4.2.4.2 Adaptive users

For adaptive users, the achievable effective user data rate assuming optimized SNR thresholds is given by

$$\begin{aligned}\bar{R}^{(u)}_{\text{A,eff,opt}}(U_{\text{A}}, M_T, M_{P,\text{CQI}}) &= \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\text{A,opt}}(U_{\text{A}}, M_T, M_{P,\text{CQI}}) \\ &\quad \cdot (M_T - M_{\text{SS}^\star-\text{A}} - M_{\text{DLPT}}) \\ &\quad + \bar{R}^{(u)}_{\text{A,opt}}(U_{\text{A}}, 2 \cdot M_T, M_{P,\text{CQI}}) \cdot (M_T - M_{\text{ULPT}^\star})\Big] .\end{aligned} \tag{4.21}$$

Note that applying the Adaptive First scheme, $\bar{R}^{(u)}_{\text{A,opt}}(U_{\text{A}}, M_T, M_{P,\text{CQI}})$ is calculated differently compared to the case of applying the Non-Adaptive First scheme as shown in Section 3.6.2.

4.2.5 Effective user data rate for pure non-adaptive transmission

In the following, also the effective user data rate of a conventional pure non-adaptive transmission scheme is derived. In this case, the superframe structure as presented in Section 4.2.1 is not necessary since no access scheme selection is performed, i.e., all users are served non-adaptively all the time. This means that at the beginning of the each DL subframe, the BS only has to transmit pilots on each resource unit corresponding to $M_{\text{DLPT}-\text{STC}}$ pilot overhead assuming that the signaling of the user and modulation scheme indices can be neglected since this has to be done only once. In the UL, each MS only has to transmit pilots on the resource units assigned to user u leading to $M_{\text{ULPT}-\text{NA}}$ OFDMA symbols pilot overhead. Thus, the achievable effective user data rate for a pure non-adaptive transmission scheme assuming optimized SNR thresholds is given by

$$\begin{aligned}\bar{R}^{(u)}_{\text{pureN,eff,opt}} &= \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\text{N,opt}}(\bar{\gamma}_u) \cdot (M_T - M_{\text{DLPT}-\text{STC}}) \\ &\quad + \bar{R}^{(u)}_{\text{N,opt}}(\kappa_{\text{UL}} \cdot \bar{\gamma}_u) \cdot (M_T - M_{\text{ULPT}-\text{NA}})\Big] .\end{aligned} \tag{4.22}$$

4.2.6 Effective user data rate for pure adaptive transmission

For a conventional pure adaptive transmission, the superframe structure presented in Section 4.2.1 is also not required, since all users are served adaptively all the time. At the beginning of each DL subframe, for each resource unit the BS has to signal the user index, the modulation scheme indices and in case of TAS the antenna index of the transmit antenna to be used in the next UL subframe. This results in $M_{\mathrm{SS-STC-A}}$ and $M_{\mathrm{SS-TAS-A}}$ OFDMA symbols signaling overhead, respectively. In the DL, the BS transmits pilots on each resource unit leading to $M_{\mathrm{DLPT-STC}}$ OFDMA symbols pilot overhead in case of OSTBC and $M_{\mathrm{DLPT-TAS}}$ in case of TAS. In the UL, each MS has to transmit pilots on all resource units since the BS needs to know the channel quality of the whole DL channel of each user. This leads to $M_{\mathrm{ULPT}^\star}$ OFDMA symbols overhead. Thus, the achievable effective user data rate for a pure adaptive transmission scheme assuming optimized SNR thresholds and applying OSTBC-MRC is given by

$$\begin{aligned} \bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \;=\; & \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \\ & \cdot (M_T - M_{\mathrm{SS-STC-A}} - M_{\mathrm{DLPT-STC}}) \\ & + \bar{R}^{(u)}_{\mathrm{A,opt}}(U, 2 \cdot M_T, M_{P,\mathrm{CQI}}) \cdot (M_T - M_{\mathrm{ULPT}^\star}) \Big] \end{aligned} \tag{4.23}$$

and

$$\begin{aligned} \bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \;=\; & \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \\ & \cdot (M_T - M_{\mathrm{SS-TAS-A}} - M_{\mathrm{DLPT-TAS}}) \\ & + \bar{R}^{(u)}_{\mathrm{A,opt}}(U, 2 \cdot M_T, M_{P,\mathrm{CQI}}) \cdot (M_T - M_{\mathrm{ULPT}^\star}) \Big] \end{aligned} \tag{4.24}$$

applying TAS-MRC.

4.3 FDD systems

4.3.1 Superframe structure with Half Duplex

In the following, it is assumed that the bandwidth in UL and DL is identical and that the DL subframe has the same time duration as the UL subframe.

Assuming an FDD Half Duplex system, UL and DL data transmissions are performed in different frequency bands, i.e., the BS cannot exploit the reciprocity of the channel to estimate the DL channel during the pilot phase in the UL to utilize it for adaptive resource allocation and modulation scheme selection in the DL. Instead, the MSs have to measure the DL channel during the pilot phase in the DL and feed back the CQI to the BS in the next UL. However, for resource allocation in the UL, the BS can estimate the UL channel during the pilot phase in the UL as done in TDD system. Since Half Duplex is assumed, UL and DL are carried out consecutively in time.

In Fig. 4.3, the frame structure of the considered FDD Half Duplex system is depicted. Like in the TDD system introduced in Section 4.2.1, a superframe structure consisting of L_{SF} UL and DL subframes is considered where each UL and DL frame consists of M_T OFDMA symbols. Again, the first UL and DL subframes form the initial frame which is required to perform the access scheme selection. The main target of the superframe is to reduce pilot and signaling overhead assuming that the impairment parameters on which the user serving classification is based do not change frame by frame as explained in Section 4.2.1. Again, in case of applying the Adaptive First combining scheme, the length of the superframe is limited to $L_{\mathrm{SF}} = 1$.

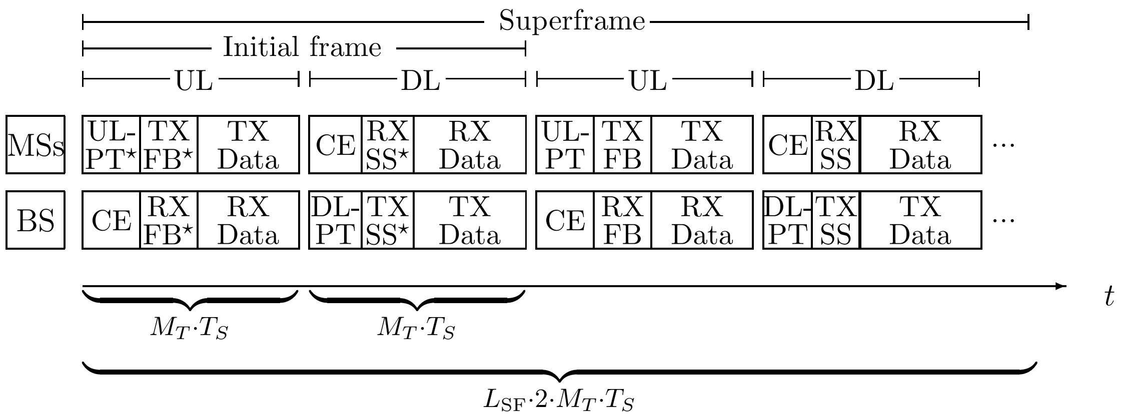


Figure 4.3. Superframe structure Half Duplex

At the beginning of the superframe, all MSs transmit pilots such that the BS can estimate the UL channel for data equalization. Further on, based on this CE, the BS performs the resource allocation and modulation scheme selection for the next UL frame. Moreover, the MSs feed back the quantized CQI values measured in the previous DL frame, indicated by Transmit Feedback (TX FB) in Fig. 4.3. The BS receives the fed back CQI values (RX FB) and performs the resource allocation and modulation scheme selection for the next DL frame based o these values. In the remaining OFDMA symbols, each MS transmits UL data.

Based on the impairment parameters measured during the last superframe, the BS decides which user shall be served adaptively or non-adaptively for the rest of this superframe. In the first DL subframe of the superframe, the BS informs the user about the access scheme selection. Furthermore, it signals the results of the resource allocation and modulation scheme selection for the current DL subframe and the next UL subframe. Besides signaling, the BS transmits pilots on each resource unit such that the MSs can estimate the DL channel for the data equalization and quantize the CQI values which then are fed back in the next UL subframe.

In the first UL subframe after the initial frame, pilot transmission is only performed on resource units which are assigned to the adaptive users and non-adaptive users. Furthermore, only adaptive users feed back the CQI values. On the remaining OFDMA symbols, data symbols are transmitted based on the scheduling decisions signaled in the previous DL subframe. In the next DL subframe, again pilots are transmitted on all resource units. However, only for the adaptive users, it is signalled which resource unit and modulation scheme to be used since the resource allocation and modulation scheme selection remains the same for the non-adaptive users within the superframe. The next $L_{\mathrm{SF}} - 2$ UL and DL subframes are carried out the same way until the beginning of the next superframe.

4.3.2 Superframe structure with Full Duplex

In an FDD Full Duplex system, UL and DL can be performed simultaneously as shown in Fig. 4.4, i.e., in the UL and DL, the BS and the MSs can transmit and receive at the same time with UL and DL one different frequency bands. Again, a superframe structure is utilized to save signaling and pilot transmissions where one superframe consists of L_{SF} UL-DL subframes each consisting of M_T OFDMA symbols. The first two UL-DL subframes of the superframe form the initial frame required for the access scheme selection. These subframes are mandatory, i.e., $L_{\mathrm{SF}} \geq 2$. The amount of pilot and signaling overhead in the remaining $L_{\mathrm{SF}} - 2$ UL-DL frames of the superframe is less as the selection of the access schemes is kept fix for the remaining subframes.

In the first UL subframe of the initial frame, all MSs send pilots on all resource units. In the first DL subframe of the initial frame, the BS also transmits pilots on each resource unit. Each MS performs CE and quantizes the CQI values. The BS also performs CE and, based on that, performs resource allocation and modulation scheme selection for the next UL subframe. Further on, each MS signals the quantized CQI values which the BS uses to perform resource allocation and modulation scheme selection for the next DL subframe. The BS signals the results of the resource allocation and modulation scheme selection for the current DL subframe and the next UL subframe. In the remaining

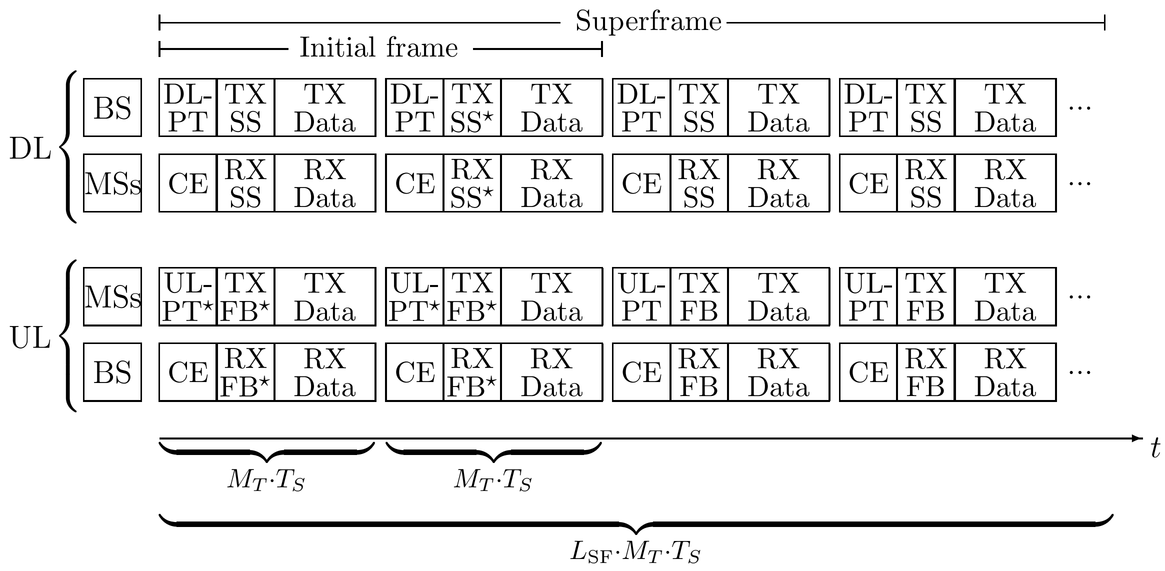


Figure 4.4. Superframe structure Full Duplex

OFDMA symbols of the DL frame, the BS transmits data according to the scheduling decisions signaled in this DL subframe while the MSs transmit data according to the scheduling decisions signaled from the BS in the previous DL subframe.

Based on the impairment parameters, the BS now decides which user shall be served adaptively or non-adaptively for the rest of the superframe. Thus, in the second UL-DL subframe of the initial frame, the same is done as in the first UL-DL subframe with one exception: the BS has to additionally signal which user is served adaptively or non-adaptively.

In the remaining $L_{\mathrm{SF}} - 2$ UL-DL subframes of the superframe, the result of the access scheme selection does not have to be signalled anymore. Further on, only adaptive users feed back CQI values and transmit pilots on each resource unit.

4.3.3 Pilot and signaling overhead

4.3.3.1 Pilot overhead in the Downlink

Concerning the pilot transmissions in the DL in an FDD system, one has to differentiate between OSTBC-MRC and TAS-MRC. Applying OSTBC-MRC, the overhead in terms of OFDMA symbols is the same as in a TDD system applying OSTBC-MRC given by (4.2). Applying TAS-MRC in a TDD system, one can use the CQI pilot phase at the beginning of the UL frame to select the transmit antenna for both UL and DL direction utilizing the reciprocity of the UL and DL channel. However, in an FDD system, this reciprocity no longer exists, i.e., at the beginning of each DL frame, a CQI pilot phase has to be introduced such that each MS can estimate the CQI value for each transmit antenna of the BS. For this CQI phase, $M_{\mathrm{P,CQI}}$ pilots per resource unit and per transmit antenna are used. Note that when pilots are sent on certain subcarriers from a given transmit antenna, these subcarriers have to remain unoccupied for the other transmit antennas so that the pilots symbols are only effected by the channel and not by other pilots from other transmit antennas. After the CQI pilot phase, only the selected transmit antenna transmits pilots and data leading to

$$M_{\mathrm{DLPT-TAS}} = \frac{(n_T - 1) \cdot M_{\mathrm{P,CQI}} + M_P}{Q_{\mathrm{sub}}} \tag{4.25}$$

OFDMA symbols overhead.

4.3.3.2 Signaling overhead in the Downlink

In the following, it is assumed that each MS has knowledge about the number N_Q of quantization bits and the fixed SNR thresholds for the normalized SNR values. From this, it follows that the amount of signaling in the DL for the considered FDD system remains the same as in a TDD system. Thus, the signaling overhead for an OSTBC-MRC system in the initial frame and for subframes within the remaining superframe is given by the equations (4.4) and (4.5). For a TAS-MR system, the signaling overhead in the initial frame is given by (4.7) and (4.6) while for subframes within the remaining superframes, the signaling overhead is given by (4.8).

4.3.3.3 Pilot overhead in the Uplink

As for the pilot transmission in the DL, the amount of pilot transmissions in the UL of the considered FDD is equivalent to the amount of a TDD system. Thus, the pilot overhead in the initial frame is given by (4.9) for both OSTBC-MRC and TAS-MRC systems. For subframes within the remaining superframe, the pilot overhead is given by (4.10) and (4.11).

4.3.3.4 Signaling overhead in the Uplink

In the uplink of an OSTBC-MRC system, each MS has to feed back the N_Q bits quantized CQI values of the different resource units to the BS. In the initial frame, all

U MSs have to signal their CQI values leading to

$$M_{\mathrm{FB}^{\star}-\mathrm{STC}} = \frac{U \cdot N_Q}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.26}$$

OFDMA symbols signaling overhead. In other words, on each resource unit one has to signal the quantized CQI values from all U users. For subframes within the remaining superframe, non-adaptive users do not have to feed back CQI values as they are served non-adaptively independent from any CQI. For the U_{A} adaptive users, the signaling overhead reduces to

$$M_{\mathrm{FB}-\mathrm{STC}} = \frac{U_{\mathrm{A}} \cdot N_Q}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}}. \tag{4.27}$$

For a TAS-MRC system, there are two possible feedback schemes as introduced in Section 2.5.3, namely the TAS Feedback-All (TAS-FA) scheme where simply all n_T CQI values per resource unit per user are fed back to the BS and the TAS Feedback-Best (TAS-FB) scheme where only the CQI value of the best antenna plus the antenna index is fed back to the BS. Thus, in the initial frame, the signaling overhead for TAS-FA is given by

$$M_{\mathrm{FB}^{\star}-\mathrm{TAS}-\mathrm{FA}} = \frac{U \cdot n_T \cdot N_Q}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.28}$$

while for TAS-FB it is given by

$$M_{\mathrm{FB}^{\star}-\mathrm{TAS}-\mathrm{FB}} = \frac{U \cdot (\lceil \log_2(n_T) \rceil + N_Q)}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}}. \tag{4.29}$$

For subframes within the remaining superframe, the signaling overhead for the U_{A} adaptive user is given by

$$M_{\mathrm{FB}-\mathrm{TAS}-\mathrm{FA}} = \frac{U_{\mathrm{A}} \cdot n_T \cdot N_Q}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.30}$$

for TAS-FA and

$$M_{\mathrm{FB}-\mathrm{TAS}-\mathrm{FB}} = \frac{U_{\mathrm{A}} \cdot (\lceil \log_2(n_T) \rceil + N_Q)}{Q_{\mathrm{sub}} \cdot b_{\mathrm{SS}}} \tag{4.31}$$

for TAS-FB.

4.3.4 Effective user data rate applying Non-Adaptive First

4.3.4.1 Non-adaptive users with Half Duplex

In the following, the effective user data rate of non-adaptive users taking into account the overhead in UL and DL is derived when applying the Non-Adaptive First combining scheme in an FDD system with Half Duplex.

In each superframe, non-adaptive users can transmit data on a total number of $L_{\mathrm{SF}} \cdot M_T$ OFDMA symbols in the DL. Due to pilot transmissions and signaling during the first DL sub in the initial frame and pilot transmissions during the $(L_{\mathrm{SF}} - 1)$ remaining DL subframes, the actual number $M_{\mathrm{DLT-NA}}$ of OFDMA symbols which can be used for DL data transmission by the non-adaptive users is given by

$$\begin{aligned} M_{\mathrm{DLT-NA}} &= M_T - M_{\mathrm{SS^\star-NA}} - M_{\mathrm{DLPT}} + (L_{\mathrm{SF}} - 1) \cdot (M_T - M_{\mathrm{DLPT}}) \\ &= L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS^\star-NA}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}}. \end{aligned} \tag{4.32}$$

On these $M_{\mathrm{DLT-NA}}$ OFDMA symbols, each non-adaptive user u can achieve a data rate of $\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u)$ in DL direction assuming optimized SNR thresholds as shown in Section 3.6 where $\bar{\gamma}_u$ denotes the average SNR of user u in the DL.

For the UL, also a total number of $L_{\mathrm{SF}} \cdot M_T$ OFDMA symbols are available within one superframe. However, due to pilot transmissions and signaling of quantized CQI values during the first UL frame in the initial frame and pilots transmissions during the $(L_{\mathrm{SF}} - 1)$ remaining UL subframes, the actual number $M_{\mathrm{ULT-NA}}$ of OFDMA symbols available for UL data transmission for each non-adaptive user is given by

$$M_{\mathrm{ULT-NA}} = L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT^\star}} - M_{\mathrm{FB^\star}} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{ULPT-NA}}. \tag{4.33}$$

On these $M_{\mathrm{ULT-NA}}$ OFDMA symbols, each non-adaptive user u can achieve a data rate of $\bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u)$ in UL direction where $\bar{\gamma}_u$ denotes the average SNR of user u in the DL and κ_{UL} the UL factor as introduced in Section 2.4.

The achievable effective user data $\bar{R}^{(u)}_{\mathrm{N,eff,opt}}$ of a non-adaptive user assuming optimized SNR thresholds and taking into account UL and DL is then given by

$$\begin{aligned} \bar{R}^{(u)}_{\mathrm{N,eff,opt}} &= \frac{1}{L_{\mathrm{SF}} \cdot 2 \cdot M_T} \\ &\cdot \Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u) \cdot (L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS^\star-NA}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}}) \\ &+ \bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u) \cdot (L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT^\star}} - M_{\mathrm{FB^\star}} \\ &\quad -(L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{ULPT-NA}})\big]. \end{aligned} \tag{4.34}$$

4.3.4.2 Adaptive users with Half Duplex

For adaptive users in an Half Duplex FDD system, the actual number $M_{\mathrm{DLT-A}}$ of OFDMA symbols which are available for DL transmission is the same as in a TDD system since the superframe structure is similar and, thus, given by

$$\begin{aligned} M_{\mathrm{DLT-A}} &= L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS^\star-A}} - M_{\mathrm{DLPT}} \\ &\quad -(L_{\mathrm{SF}} - 1) \cdot (M_{\mathrm{DLPT}} + M_{\mathrm{SS-A}}) \\ &= L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS^\star-A}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{SS-A}}. \end{aligned} \tag{4.35}$$

However, one major difference to the mentioned TDD system is that the time delay between measuring the CQI values at the MSs and the actual data transmission in the DL is $T = 2 \cdot M_T \cdot T_S$, i.e., the corresponding correlation coefficient is given by

$$\rho_u(M_T) = J_0(2\pi \cdot f_{D,u} \cdot 2 \cdot M_T \cdot T_S). \tag{4.36}$$

That means that the quantized CQI values on which the resource allocation is based are outdated by T. Furhermore, the CQI values are only estimates measured at the MSs with the help of $M_{P,\mathrm{CQI}}$ pilots leading to an error estimation variance of

$$\sigma^2_{E,u}(M_{P,\mathrm{CQI}}) = \frac{1}{\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u \cdot M_{P,\mathrm{CQI}}}. \tag{4.37}$$

From this, it follows that on these $M_{\mathrm{ULT-A-NAF}}$ OFDMA symbols, each adaptive user can achieve a data rate of $\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, 2 \cdot M_T, M_{P,\mathrm{CQI}})$ assuming optimized modulation schemes applied for the fixed SNR thresholds with respect to the impairment parameters $\rho_u(M_T)$ and $\sigma^2_{E,u}(M_{P,\mathrm{CQI}})$ and the number U_{A} of adaptive users as shown in Section 3.6.3. Note that the achievable user data rate $\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q)$ applying quantized CQI is a function of the number N_Q of quantization bits for the CQI feedback and should not be confused with the achievable user data rate $\bar{R}^{(u)}_{\mathrm{A,opt}}$ applying continuous CQI as with the mentioned TDD system.

For the UL, there are

$$M_{\mathrm{ULT-A}} = L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT^\star}} - M_{\mathrm{FB^\star}} - (L_{\mathrm{SF}} - 1) \cdot (M_{\mathrm{ULPT-A}}(U_{\mathrm{A}}) + M_{\mathrm{FB}}) \tag{4.38}$$

OFDMA symbols available for actual UL data transmission. Since each MS uses the resource allocation and modulation scheme selection which was signalled by the BS in the previous DL frame, the time delay between the CQI measuring on which the resource allocation and modulation scheme selection is based and the actual data transmission in the UL equals $2 \cdot M_T \cdot T_S$. However, these CQI values are continuous, i.e., not quantized. That means, the SNR thresholds can be chosen according to the impairment parameters as shown in Section 3.6.2. The achievable effective user data rate of adaptive users taking into account DL and UL in then given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{A,eff,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{L_{\mathrm{SF}} \cdot 2M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, 2M_T, M_{P,\mathrm{CQI}}) \\ & \cdot(L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{SS}^\star-\mathrm{A}} - \\ & L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}} - (L_{\mathrm{SF}} - 1) \cdot M_{\mathrm{SS-A}}) \\ & +\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, 2M_T, M_{P,\mathrm{CQI}}) \\ & \cdot(L_{\mathrm{SF}} \cdot M_T - M_{\mathrm{ULPT}^\star} - M_{\mathrm{FB}^\star} \\ & -(L_{\mathrm{SF}} - 1) \cdot (M_{\mathrm{ULPT-A}}(U_{\mathrm{A}}) + M_{\mathrm{FB}}))\Big] . \end{aligned} \tag{4.39}$$

4.3.4.3 Non-adaptive users with Full Duplex

Compared to the case of Half Duplex, factor 2 in the denominator of the achievable effective user data rate in case of Full Duplex vanishes since UL and DL is performed simultaneously. Furthermore, the time delay between measuring the CQI values and the actual data transmission of the adaptive users is $T = M_T \cdot T_S$ for both UL and DL as can be seen from the superframe structure in Fig. 4.4.

The achievable effective user data $\bar{R}^{(u)}_{\mathrm{N,eff,opt}}$ of a non-adaptive user assuming optimized SNR thresholds in an FDD Full Duplex system is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{N,eff,opt}} &= \frac{1}{L_{\mathrm{SF}} \cdot M_T} \\ & \cdot \Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u) \cdot (L_{\mathrm{SF}} \cdot M_T - 2 \cdot M_{\mathrm{SS}^\star-\mathrm{NA}} - L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}}) \\ & +\bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u) \cdot (L_{\mathrm{SF}} \cdot M_T - 2 \cdot M_{\mathrm{ULPT}^\star} - 2 \cdot M_{\mathrm{FB}^\star} \\ & -(L_{\mathrm{SF}} - 2) \cdot M_{\mathrm{ULPT-NA}})\Big] . \end{aligned} \tag{4.40}$$

4.3.4.4 Adaptive users with Full Duplex

For adaptive users, the achievable effective user data $\bar{R}^{(u)}_{\mathrm{A,eff,opt}}$ assuming optimized SNR thresholds in an FDD Full Duplex system is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{A,eff,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{L_{\mathrm{SF}} \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \\ & \cdot(L_{\mathrm{SF}} \cdot M_T - 2 \cdot M_{\mathrm{SS}^\star-\mathrm{A}} \\ & -L_{\mathrm{SF}} \cdot M_{\mathrm{DLPT}} - (L_{\mathrm{SF}} - 2) \cdot M_{\mathrm{SS-A}}) \\ & +\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \\ & \cdot(L_{\mathrm{SF}} \cdot M_T - 2 \cdot M_{\mathrm{ULPT}^\star} - 2 \cdot M_{\mathrm{FB}^\star} - \\ & (L_{\mathrm{SF}} - 2) \cdot (M_{\mathrm{ULPT-A}}(U_{\mathrm{A}}) + M_{\mathrm{FB}}))\Big] . \end{aligned} \tag{4.41}$$

4.3.5 Effective user data rate applying Adaptive First

4.3.5.1 Non-adaptive users with Half Duplex

Applying the Adaptive First combining scheme, one cannot save pilot transmissions and signaling by introducing a superframe structure since the resource allocation of the non-adaptive users has to be performed frame by frame. Thus, in each DL subframe, the result of the access scheme selection, the resource allocation and the modulation scheme selection for UL and DL have to be signalled. In each UL subframe, each MS has to transmit pilots on all resource units and each MS has to feed back the quantized CQI values for each resource unit.

In case of an FDD Half Duplex system, applying Adaptive First can be seen as a special case of applying Non-Adaptive First with a superframe length of $L_{\mathrm{SF}} = 1$. Thus, the achievable effective user data rate of non-adaptive users is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{N,eff,opt}} &= \frac{1}{2 \cdot M_T} \\ &\cdot \Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u) \cdot (M_T - M_{\mathrm{SS^\star-NA}} - M_{\mathrm{DLPT}}) \\ &+ \bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u) \cdot (M_T - M_{\mathrm{ULPT^\star}} - M_{\mathrm{FB^\star}})\Big]. \end{aligned} \tag{4.42}$$

4.3.5.2 Adaptive users with Half Duplex

For adaptive users, applying Adaptive First can be also seen as a special case of applying Non-Adaptive First with a superframe length of $L_{\mathrm{SF}} = 1$. The achievable effective user data rate is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{A,eff,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) = \frac{1}{2 \cdot M_T} \cdot \Big[&\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, 2M_T, M_{P,\mathrm{CQI}}) \\ &\cdot (M_T - M_{\mathrm{SS^\star-A}} - M_{\mathrm{DLPT}}) \\ &+ \bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, 2M_T, M_{P,\mathrm{CQI}}) \\ &\cdot (M_T - M_{\mathrm{ULPT^\star}} - M_{\mathrm{FB^\star}})\Big]. \end{aligned} \tag{4.43}$$

Note that applying the Adaptive First scheme, $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ and $\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ are calculated differently compared to the case of applying the Non-Adaptive First scheme as shown in Section 3.6.2 and 3.6.3.

4.3.5.3 Non-adaptive users with Full Duplex

In case of an FDD Full Duplex system, applying Adaptive First can be seen as a special case of applying Non-Adaptive First with a superframe length of $L_{\mathrm{SF}} = 2$ as seen in Fig. 4.4.

For non-adaptive users, the achievable effective user data rate is given by

$$\begin{aligned} \bar{R}^{(u)}_{\mathrm{N,eff,opt}} &= \frac{1}{M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u) \cdot (M_T - M_{\mathrm{SS}^\star-\mathrm{NA}} - M_{\mathrm{DLPT}}) \\ &\quad + \bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}} \cdot \bar{\gamma}_u) \cdot (M_T - M_{\mathrm{ULPT}^\star} - M_{\mathrm{FB}^\star}) \Big] . \end{aligned} \tag{4.44}$$

4.3.5.4 Adaptive users with Full Duplex

Again, for adaptive users applying Adaptive First can be seen as a special case of applying Non-Adaptive First with a superframe length of $L_{\mathrm{SF}} = 2$ which results in an achievable effective user data rate given by

$$\begin{aligned} \bar{R}^{(u)}_{\mathrm{A,eff,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{SS}^\star-\mathrm{A}} - M_{\mathrm{DLPT}}) \\ &\quad + \bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{ULPT}^\star} - M_{\mathrm{FB}^\star}) \Big] . \end{aligned} \tag{4.45}$$

Also in this case, $\bar{R}^{(u)}_{\mathrm{A,opt}}(U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ and $\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U_{\mathrm{A}}, M_T, M_{P,\mathrm{CQI}})$ are calculated differently compared to the case of applying the Non-Adaptive First scheme as shown in Section 3.6.2 and 3.6.3.

4.3.6 Effective user data rate for pure non-adaptive transmission

4.3.6.1 Half Duplex

Like in TDD systems, also the effective user data rate for a conventional pure adaptive FDD transmission scheme is derived for both Half and Full Duplex. Since all users are served non-adaptive all the time, the BS only has to transmit pilots on each resource unit assuming that the signaling of the user and modulation scheme indices can be neglected since this has to be done only once. In the UL subframe, each MS only has to transmit pilots on the resource units assigned to user u.

For a pure non-adaptive FDD Half Duplex system, this corresponds to an achievable effective user data rate assuming optimized SNR thresholds given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureN,eff,opt}} &= \frac{1}{2\cdot M_T}\cdot\Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u)\cdot(M_T - M_{\mathrm{DLPT-STC}}) \\ &\quad +\bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}}\cdot\bar{\gamma}_u)\cdot(M_T - M_{\mathrm{ULPT-NA}})\Big]\,. \end{aligned} \tag{4.46}$$

4.3.6.2 Full Duplex

For a pure non-adaptive FDD Full Duplex system, factor 2 in the denominator vanishes due the simultaneous transmission of UL and DL data resulting in

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureN,eff,opt}} &= \frac{1}{M_T}\cdot\Big[\bar{R}^{(u)}_{\mathrm{N,opt}}(\bar{\gamma}_u)\cdot(M_T - M_{\mathrm{DLPT-STC}}) \\ &\quad +\bar{R}^{(u)}_{\mathrm{N,opt}}(\kappa_{\mathrm{UL}}\cdot\bar{\gamma}_u)\cdot(M_T - M_{\mathrm{ULPT-NA}})\Big]\,, \end{aligned} \tag{4.47}$$

i.e., the achievable effective user data rate applying Full Duplex is twice the achievable effective user data rate applying Half Duplex.

4.3.7 Effective user data rate for pure adaptive transmission

4.3.7.1 Half Duplex

Also, for a conventional pure adaptive FDD system, the superframe structure is not required since all users are served adaptively all the time. Concerning pilot transmissions, the BS and each MS have to transmit pilots on each resource unit. Concerning signaling, the BS has to signal the user index, the modulation scheme indices and in case of TAS the antenna index of the transmit antenna to be used in the next UL frame. The MSs have to feed back the CQI of each resource unit.

Thus, in an Half Duplex FDD system, the achievable effective user data rate for a pure adaptive transmission scheme assuming optimized SNR thresholds and applying OSTBC-MRC is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(N_Q,U,M_T,M_{P,\mathrm{CQI}}) &= \frac{1}{2\cdot M_T}\cdot\Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q,U,2M_T,M_{P,\mathrm{CQI}}) \\ &\quad \cdot(M_T - M_{\mathrm{SS-STC-A}} - M_{\mathrm{DLPT-STC}}) \\ &\quad +\bar{R}^{(u)}_{\mathrm{A,opt}}(U,2M_T,M_{P,\mathrm{CQI}}) \\ &\quad \cdot(M_T - M_{\mathrm{FB^\star-STC}} - M_{\mathrm{ULPT^\star}})\Big]\,. \end{aligned} \tag{4.48}$$

Applying TAS-MRC, the achievable effective user data rate is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(N_Q, U, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{2 \cdot M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U, 2M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{SS-TAS-A}} - M_{\mathrm{DLPT-TAS}}) \\ &\quad + \bar{R}^{(u)}_{\mathrm{A,opt}}(U, 2M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{FB^\star-TAS}} - M_{\mathrm{ULPT^\star}})\Big] .\end{aligned} \tag{4.49}$$

4.3.7.2 Full Duplex

For a Full Duplex FDD system, the time delay between measuring the CQI and the actual data transmission for both UL and DL is $M_T \cdot T_S$. Furthermore, taking into account the the simultaneous transmission of UL and DL data, the achievable effective user data rate applying OSTBC-MRC is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(N_Q, U, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{SS-STC-A}} - M_{\mathrm{DLPT-STC}}) \\ &\quad + \bar{R}^{(u)}_{\mathrm{A,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{FB^\star-STC}} - M_{\mathrm{ULPT^\star}})\Big] .\end{aligned} \tag{4.50}$$

Applying TAS-MRC, the achievable effective user data rate is given by

$$\begin{aligned}\bar{R}^{(u)}_{\mathrm{pureA,eff,opt}}(N_Q, U, M_T, M_{P,\mathrm{CQI}}) &= \frac{1}{M_T} \cdot \Big[\bar{R}^{(u)}_{\mathrm{A,opt}}(N_Q, U, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{SS-TAS-A}} - M_{\mathrm{DLPT-TAS}}) \\ &\quad + \bar{R}^{(u)}_{\mathrm{A,opt}}(U, M_T, M_{P,\mathrm{CQI}}) \\ &\quad \cdot (M_T - M_{\mathrm{FB^\star-TAS}} - M_{\mathrm{ULPT^\star}})\Big] .\end{aligned} \tag{4.51}$$

4.4 Maximizing effective system data rate

Until now, the achievable effective user data rates for adaptive and non-adaptive users were derived assuming optimized SNR thresholds to fulfill the BER requirements where the effective user data rate of adaptive users was given as a function of the number U_{A} of the adaptive users. Next, the maximized effective system data rate $\bar{R}_{\mathrm{sys,eff,opt}}$ has to be found subject to a minimum data rate requirement by searching for the optimal user serving vector as done in the optimization problem of (3.9). This results in the following problem given by

$$\begin{aligned}\bar{R}_{\text{sys,eff,opt}} &= \max_{\vartheta} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\text{ru}}}\right) \cdot \left[\vartheta_u \bar{R}^{(u)}_{\text{A,eff,opt}}(\vartheta) + (1-\vartheta_u) \cdot \bar{R}^{(u)}_{\text{N,eff,opt}}\right] \qquad (4.52)\\ &\quad \text{subject to}\\ &\quad \vartheta_u \bar{R}^{(u)}_{\text{A,eff,opt}}(\vartheta) + (1-\vartheta_u) \cdot \bar{R}^{(u)}_{\text{N,eff,opt}} \geq \bar{R}^{(u)}_{\text{N,eff,opt}}.\end{aligned}$$

replacing $\bar{R}^{(u)}_{\text{A,opt}}(\vartheta)$ with $\bar{R}^{(u)}_{\text{A,eff,opt}}(\vartheta)$ and $\bar{R}^{(u)}_{\text{N,opt}}$ with $\bar{R}^{(u)}_{\text{N,eff,opt}}$.

For the case of the Non-Adaptive First scheme, one has to consider in the expression $\bar{R}^{(u)}_{\text{A,eff,opt}}(\vartheta)$ that $M_{\text{ULPT}-\text{A}}$, i.e., the pilot overhead in the UL, is a function of the number U_{A} of adaptive users. From (4.11) it can be seen that this overhead increases with the number of adaptively served users, i.e., in contrast to the case neglecting the overhead, the maximum achievable user data rate $\bar{R}^{(u)}_{\text{A,eff,opt}}(\vartheta)$ no longer increases monotonically with increasing number U_{A} of users. Thus, the RedCom2-NAF algorithm can no longer be applied for solving the user serving problem since monotony was a requirement. From this, it follows that the more complex RedCom algorithm has to be applied. For the case of the Adaptive First scheme, the same methods for solving the user serving problem shown in Section 3.7 can be applied to maximize the effective system data rate according to (4.52) for both TDD and FDD systems.

Chapter 5

Performance evaluation

5.1 Introduction

In this chapter, the performances of adaptive, non-adaptive and hybrid OFDMA transmission schemes for both TDD systems and FDD systems are evaluated taking account user-dependent imperfect CQI.

The chapter is organized as follows. In Section 5.2, the system under evaluation operates in the TDD mode, i.e., the BS is able to measure both the UL and DL channel. First, the impact of different user demands on the system performance is analyzed followed by an investigation of the joint impact of user demand and outdated CQI. Furthermore, the two different hybrid schemes Non-Adaptive First and Adaptive First are compared with conventional pure adaptive and pure non-adaptive transmission schemes in the presence of user-dependent imperfect CQI for a fixed total number U of users in the cell. Moreover, also the impact of pilot and signaling overhead is taken into account when comparing the performances of hybrid and conventional schemes. Finally, the impact of the number of active users in the cell is analyzed. In Section 5.3, the system under consideration operates in the FDD mode, i.e., quantized CQI is applied. First, the impact of the number N_Q of quantization bits for the CQI feedback is investigated. Furthermore, the impact of feedback bit errors on the system performance is discussed. As done in the TDD case, the two hybrid schemes are compared to conventional pure adaptive and non-adaptive schemes with and without considering pilot and signaling overhead for a fixed number U of users in the cell. Finally, also the impact of the cell load is discussed. Section 5.4 summarizes the main conclusions of the performance evaluation.

5.2 TDD systems

5.2.1 Impact of user demand

In the following, it is evaluated how the average system data rate and the average user data rate behave when the user demand for certain users is increased, i.e., when certain users are favored regarding access to resource units. This investigation is carried out for both adaptive and non-adaptive users.

For the adaptive users, a system applying a pure adaptive OFDMA transmission scheme is considered. It is assumed that the system consists of a total number of $N_{\mathrm{ru}} = 30$ resource units serving in total $U = 10$ users. The number of transmit and receive antennas is set to $n_{\mathrm{T}} = 2$ and $n_{\mathrm{R}} = 2$, respectively. As antenna technique, Alamouti OSTBC in combination with MRC is applied. Furthermore, it is assumed that there are $G = 2$ demand groups, where the first demand group comprises one user ($|\mathcal{G}_1| = 1$) and the second demand group contains the remaining $U - 1 = 9$ users. From this, it follows that the user demand vector $\mathbf{D}$ of (2.36) is given by

$$\mathbf{D} = \left[D_{\mathrm{H}},\ \frac{N_{\mathrm{ru}} - D_{\mathrm{H}}}{U - 1},\ ...,\ \frac{N_{\mathrm{ru}} - D_{\mathrm{H}}}{U - 1} \right], \tag{5.1}$$

i.e., there is only one variable, namely the user demand D_{H} of the high demand user.

For simplicity, it is assumed that each user has the same average SNR $\bar{\gamma}_u = 8$ dB leading to a estimation error variance $\sigma^2_{\mathrm{E},u} = 0.16$ for each user user u with $u = 1, .., U$ assuming the number $M_{\mathrm{P,CQI}}$ of pilots in the CQI phase to be $M_{\mathrm{P,CQI}}$. Furthermore, no time delay is assumed, i.e., $\rho_u = 1$. The target BER is set to $BER_{\mathrm{T}} = 10^{-3}$, i.e., the SNR thresholds are optimized to meet this requirement considering $\sigma^2_{\mathrm{E},u} = 0.16$ and $\rho_u = 1$.

In Fig. 5.1, the average number of transmitted bits per allocated subcarrier is depicted as a function of the user demand D_{H} ranging from $D_{\mathrm{H}} = 3$ which corresponds to a totally fair system to $D_{\mathrm{H}} = 30$ where all $N_{\mathrm{ru}} = 30$ resource units are allocated to one user. The solid curve represents the average number of allocated bits in the overall system where the dashed curves represent the average number of allocated bits for the high demand user and for a low demand user, respectively. In Fig. 5.2, the user data rates of the high demand user (green curve) and a low demand user (red curve) are depicted as a function of the user demand D_{H}.

From Fig. 5.1 it can be seen that the average number of allocated bits in the overall system decreases when increasing the user demand D_{H}, since favoring the high demand user even if he is in bad channel condition results in a performance degradation. From the upper dashed line in Fig. 5.1, representing the number of transmitted bits per subcarrier when allocated to the high demand user, one can see that the number of bits decreases with increasing user demand D_{H}, i.e., the quality of the allocated channels gets worse and, thus, the modulation schemes have to be selected more conservatively. Concerning the user data rate of the high demand user represented by the lower dashed

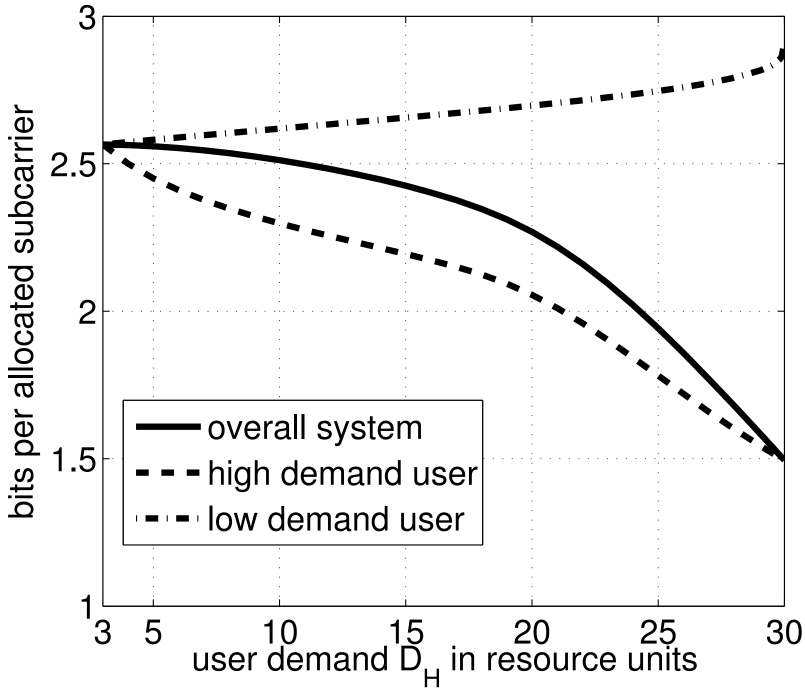


Figure 5.1. Average number of transmitted bits per allocated subcarrier vs. user demand D_H

curve in Fig. 5.2, one can see that the user data rate increases due to the increased access to the channel. It can be seen that for user demands $D_\mathrm{H} > 10$, the user data rate no longer increases linearly with D_H, but goes into saturation. This is due to the fact that when increasing D_H, the probability that the channel quality of selected resource units is bad also increases since there is no competition as the high demand user is favored.

For the low demand users it is vice versa, i.e., the number of bits per subcarrier, when allocated to a low demand user, increases with increasing D_H, see Fig. 5.1, since only strong channels of low demand users can compete successfully with the favored channels of high demand users. However, the user data rate of a low demand user decreases due to the reduced channel access, see Fig. 5.2.

Next, the impact of user demand is investigated for non-adaptive users applying a pure non-adaptive OFDMA scheme with an average SNR of $\bar{\gamma}_u = 10$ dB for each user. Again, 2×2 Alamouti OSTBC in combination with MRC is applied and the target BER is set to $BER_\mathrm{T} = 10^{-3}$. The user demand vector $\mathbf{D}$ is again given by (5.1).

Fig. 5.3(a) depicts the number per bits allocated subcarrier as a function of the user demand D_H for both high and low demand user and the overall system. It can be

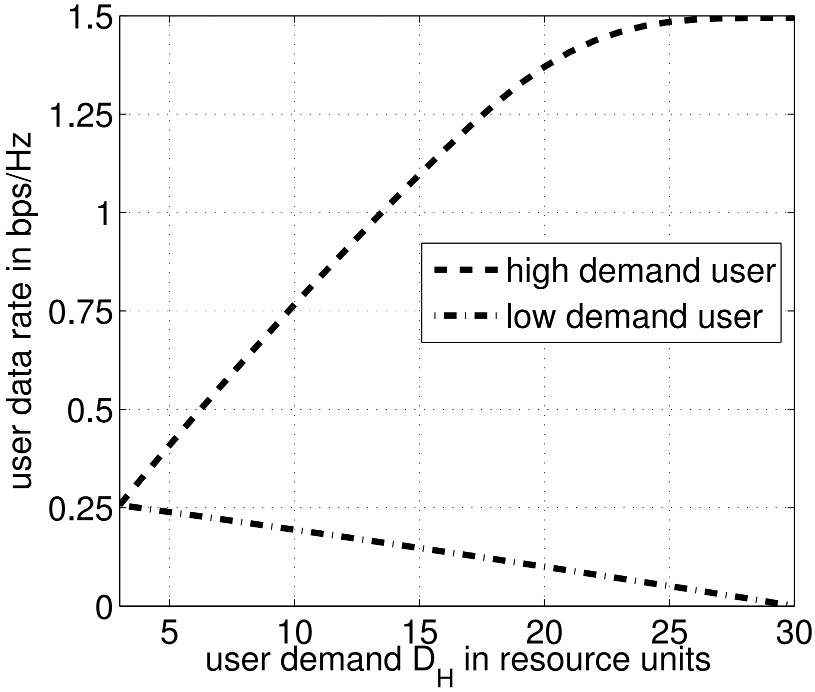
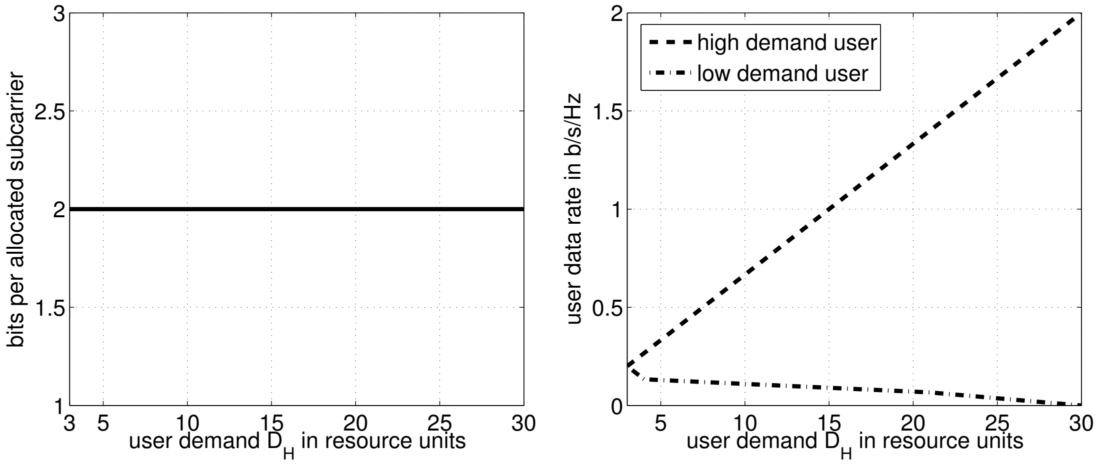


Figure 5.2. User data rate vs. user demand D_{H}

seen that the number of bits is 2 for all values of D_{H}, i.e., as no adaptive modulation is performed in the non-adaptive OFDMA scheme, the chosen modulation scheme, in this case QPSK, is applied for all subcarriers. In Fig. 5.3(b), the user data rates of the high demand and the low demand users are depicted as a function of D_{H}. From the upper dashed curve representing the high demand user, one can see that the user data rate linearly increases with D_{H} due to the increased channel access. For the low demand users, the user data rate decreases due to the reduced channel access.

5.2.2 Joint impact of user demand and outdated CQI

In the following, the joint impact of outdated CQI and user demand on the performance of the system is investigated only for adaptive users as non-adaptive users do not apply any CQI. The system assumptions remain the same as in Section 5.2.1 but now, the CQI is assumed to be outdated expressed by the normalized time delay $f_D T$, where the Doppler frequency f_D is assumed to be the same for each user. Note that the CQI is also assumed to be noisy with $\sigma^2_{\mathrm{E},u} = 0.16$. However, this value is kept fixed as it only changes with the average SNR as shown in Section 2.9.2 which in this case is assumed to be constant and the same for each user. Furthermore, not only Alamouti OSTBC-MRC is used as antenna technique but also TAS-MRC.



(a) (b)

Figure 5.3. (a) Number of transmitted bits per allocated subcarrier vs. user demand D_{H}; (b) User data rate vs. user demand D_{H}

In Fig. 5.4, the average system data rate, indicated by different levels of gray, applying TAS at the BS and MRC at the MSs is depicted as a function of the normalized time delay $f_D T$ and the channel demand gain D_{H}. As one can see, the achievable data rate

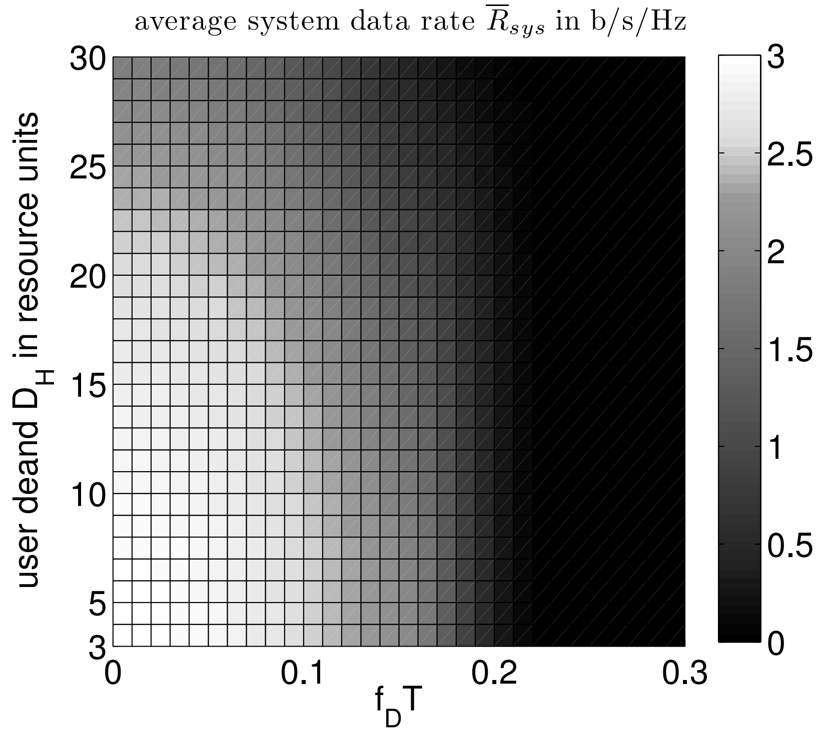


Figure 5.4. 2×2 TAS-MRC system data rate vs. normalized time delay $f_D T$ and user demand D_{H}

is high for small time delays and low user demands. When increasing D_{H} for a given $f_D T$, the system data rate decreases since favoring high priority users even if they are in bad channel conditions results in a performance degradation. When increasing $f_D T$ for a given D_{H}, the data rate also decreases, since a more robust modulation scheme is required to cope with the outdated CQI. It can be seen that for higher user demands D_{H}, the transmission becomes more vulnerable to outdated CQI. For example in Fig. 5.4, if $D_{\mathrm{H}} = 3$, a system data rate of $\bar{R}_{sys} = 2.5$ b/s/Hz can be achieved up to a delay of $f_D T = 0.1$. If $D_{\mathrm{H}} = 20$, $\bar{R}_{sys} = 2.5$ b/s/Hz can only be achieved up to a delay of $f_D T = 0.06$.

In Fig. 5.5, the same analysis is shown applying the OSTBC scheme at the BS and MRC at the MSs. Comparing the system performance applying OSTBC and TAS, TAS clearly outperforms OSTBC in the region of small time delays $f_D T$. The reason why TAS outperforms OSTBC for small time delays $f_D T$ is the averaging effect of OSTBC on the SNR values, i.e., applying OSTBC, the probability for high SNR values decreases. However, when increasing the time delay $f_D T$, OSTBC outperforms TAS since now OSTBC is more robust against outdated CQI due to the exploitation of spatial diversity.

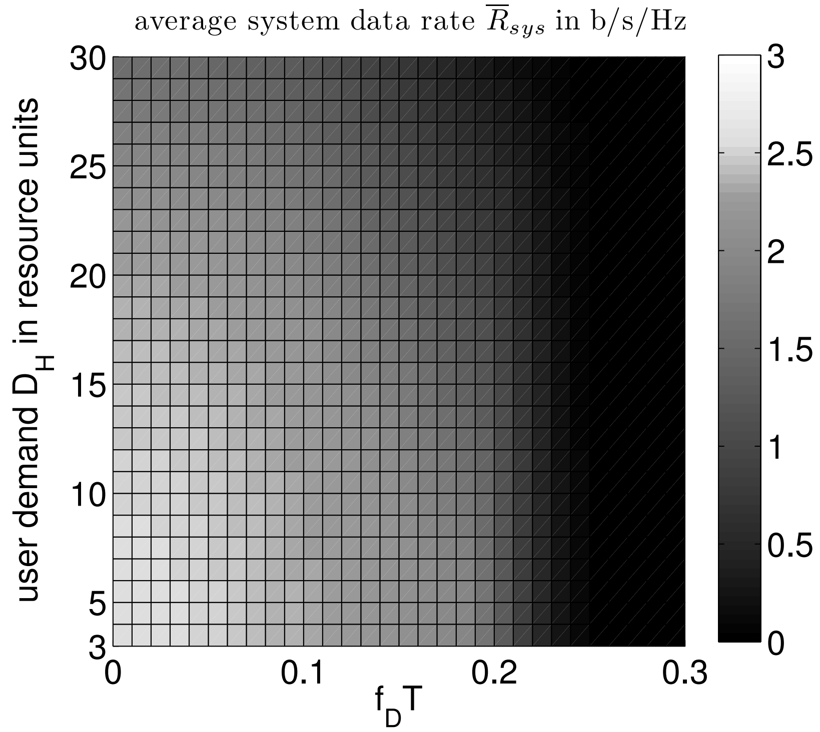


Figure 5.5. 2×2 OSTBC-MRC system data rate vs. time delay $f_D T$ and user demand D_{H}

5.2.3 Comparison of hybrid transmission schemes with conventional transmissions scheme in the presence of imperfect CQI

In this section, the performance of the hybrid schemes is compared to the performance of the conventional schemes in the presence of imperfect CQI in the DL without considering any pilot or signaling overhead.

For the investigation performed in Section 5.2.1 and 5.2.2 it was enough to assume a simplified system with equal channel conditions for all users to show the effects of user demand and outdated CQI. However, for a reasonable DL performance evaluation of hybrid and conventional pure adaptive and pure non-adaptive transmission schemes, a more realistic setting is required as described in Chapter 2. In the following, an OFDMA system with the system parameters given in Table 5.1 is assumed.

Table 5.1. System parameters

Bandwidth B	10 MHz
Number N of subcarriers	240
Frequency block size Q_{sub}	8
Number N_{ru} of resource units	30
Number U of users	15
Number n_{T} of transmit antennas	2
Number n_{R} of receive antennas	2
Carrier frequency f_0	2 GHz
Target BER BER_{T}	10^{-3}
Cell radius R	300 m
Minimum distance BS-MS d_0	10 m
Pathloss exponent α	2.6

Thus, the frequency spacing between the subcarriers is $\Delta f = 41.67$ kHz, i.e., a frequency block of Q_{sub} subcarriers occupies $Q_{\text{sub}} \cdot \Delta f = 333.3$ kHz bandwidth. Assuming a maximum time delay of $\tau_{\max} = 3\mu s$, this corresponds to the coherence bandwidth B_{C}, i.e., the assumption that adjacent frequency blocks are uncorrelated is justified.

The transmit power $P_{\text{T,sub}}$ at the transmitter is adjusted in such a way that a user at the cell border with no reliable CQI can achieve the target BER applying the non-adaptive transmission scheme. Furthermore, the time delay between the CQI updates is assumed to be $T = 2$ ms and the CQI values are noisy estimates based on $M_{P,\text{CQI}} = 1$ pilot. Moreover, the applied modulation schemes range from QPSK for users at the cell edge up to 512-QAM for users near the BS.

Furthermore, only one user demand group is assumed, i.e., $G = 1$ and the user demand vector is set to $\mathbf{D} = [2, 2, .., 2]$ meaning that each of the $U = 15$ users demands two out of the $N_{\mathrm{ru}} = 30$ resource units.

In the following, the hybrid transmission schemes are compared with conventional pure adaptive and the pure non-adaptive OFDMA schemes in the presence of imperfect user-dependent CQI. The two parameters describing the CQI impairment are the estimation error variance $\sigma^2_{\mathrm{E},u}$ and the correlation coefficient ρ_u. As $\sigma^2_{\mathrm{E},u}$ is directly linked with the average SNR of user u and, thus, determined by the scenario, only ρ_u is the remaining CQI impairment parameter which is used as variable to analyze the system performance. As ρ_u is directly linked with the MS velocity of each user and each user has a different velocity as stated in Section 2.2, the average MS velocity $\bar{v}$ is the variable which indicates in the following how much outdated the CQI is in the cell. To evaluate the performance, 500 different user positions in the cell are generated assuming uniformly distributed users as stated in Section 2.2. For each of these realizations, 500 different MS velocities $\mathbf{v}_u = [v_x, v_y]^{\mathrm{T}}$, i.e., 500 different angles and velocity magnitudes, are generated where the radial components of the MS velocities are half-normally distributed as shown in Section 2.2. Fig. 5.6 shows an example of the distribution of the magnitude of the radial velocity for an average MS velocity of $\bar{v} = 20$ km/h. The dashed curve represents the PDF of the magnitude of the radial MS velocity calculated analytically according to (2.1, the solid lines represent the PDF generated simulative.

From this, it follows that for each value of $\bar{v}$, in total 250000 realizations of MS positions and MS velocities are generated. To determine the average system data rate, for each of these realizations, the system data rate is calculated according to the equations and algorithms derived in Chapter 3 and 4. The average system data rate is then determined by averaging over these 250000 realizations.

For the pure adaptive system, two types of schemes are considered: Firstly, a naive approach where the BS always assumes perfect CQI, i.e., the SNR threshold vector is calculated assuming perfect CQI for all users. Secondly, a pure adaptive scheme which is aware of the CQI imperfectness of each user and which adapts the SNR threshold vectors correspondingly, i.e., in case of imperfect CQI, the selection of the applied modulation schemes is performed more conservatively compared to the naive approach in order to fulfill the BER requirements. In case that the target BER is not fulfilled, the data rate of a user u is defined to be zero, i.e., $\bar{R}^{(u)} = 0$.

In Fig. 5.7, the average system data rate is depicted as a function of the average MS velocity $\bar{v}$ in the cell for the different transmission schemes applying OSTBC-MRC. Fig. 5.8 shows the same for a TAS-MRC system.

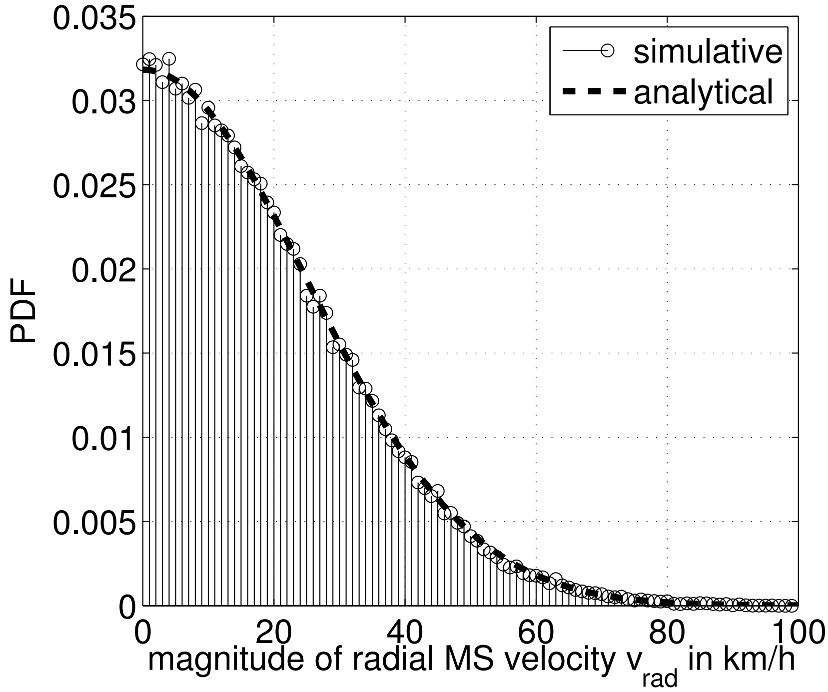


Figure 5.6. PDF magnitude radial MS velocity v_{rad} for an average MS velocity of $\bar{v} = 20$ km/h

As one can see in both figures, the pure non-adaptive scheme achieves a constant system data rate, since it does not depend on the reliability of the CQI, neglecting the effect of intercarrier interference due to Doppler shifts. In case of $\bar{v} = 0$ km/h, the pure adaptive transmission scheme and the hybrid transmission schemes achieve the same system data rate and outperform the non-adaptive scheme. However, when increasing the average MS velocity in the cell and, thus, the unreliability of the CQI, the performances of the pure adaptive scheme dramatically decrease, especially for the naive approach since now, due to the imperfect CQI, wrong users and modulation schemes are selected for transmission. This results in a BER which no longer fulfills the target BER requirements. For the pure adaptive scheme which is aware of the imperfect CQI, the decrease is less dramatic. However, at some point the system performance is worse than for pure non-adaptive transmission schemes. Applying the hybrid schemes Non-Adaptive First and Adaptive First for an increasing MS velocity in the cell, the system performance is always equal to or better than both the pure adaptive and pure non-adaptive scheme. Adaptive First (RedCom-AF, RedCom2-AF) outperforms Non-Adaptive First (RedCom2-NAF) due to the more exclusive resource selection. Note that there is hardly a difference in the performance comparing the optimal Adaptive First algorithm (RedCom-AF) and the optimal Adaptive First algorithm (RedCom2-

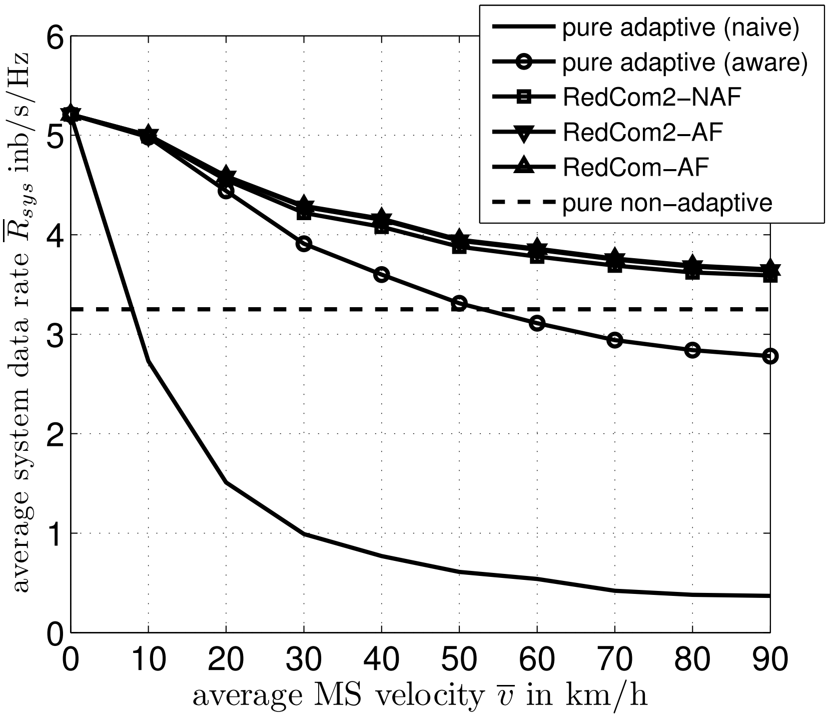


Figure 5.7. System data rate versus average MS velocity $\bar{v}$ applying OSTBC-MRC

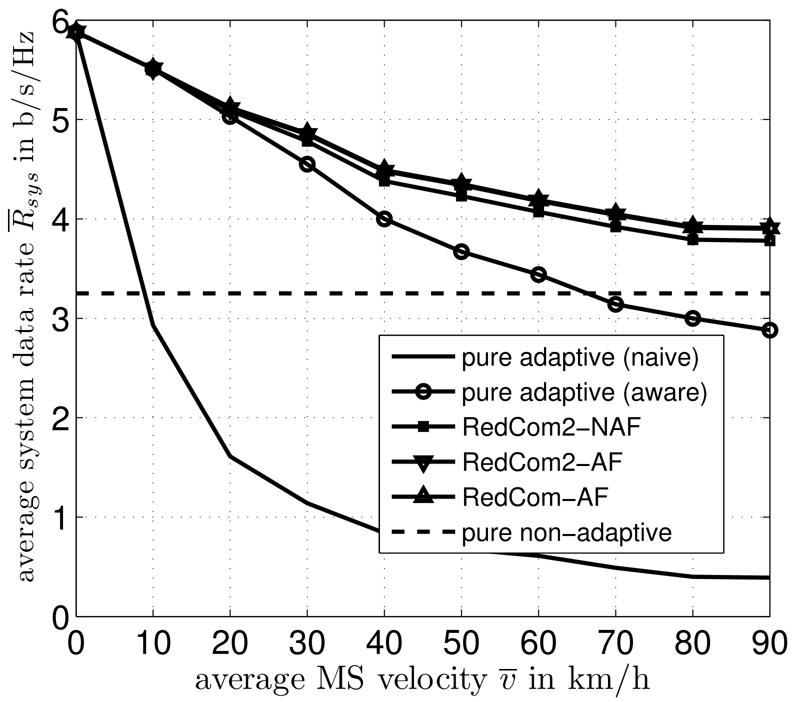


Figure 5.8. System data rate versus average MS velocity $\bar{v}$ applying TAS-MRC

AF). Comparing hybrid TAS-MRC with hybrid OSTBC-MRC, it can be observed that hybrid TAS-MRC outperforms hybrid OSTBC-MRC. This matches with the observation of previous investigations where for accurate up to medium accurate channel knowledge, TAS was better than OSTBC. For rather bad channel knowledge, OSTBC is better. However, in these regions it is better to use the non-adaptive transmission scheme anyway which is automatically done by the hybrid schemes as for large velocities, more and more of the users are served applying the non-adaptive scheme due to the totally outdated CQI.

The effect of serving the users non-adaptively for an increasing $\bar{v}$ is also shown in Fig. 5.9, where the average number U_A of adaptively served users is depicted as a function of the MS velocity $\bar{v}$ for an OSTBC-MRC system. One can see that for low velocities,

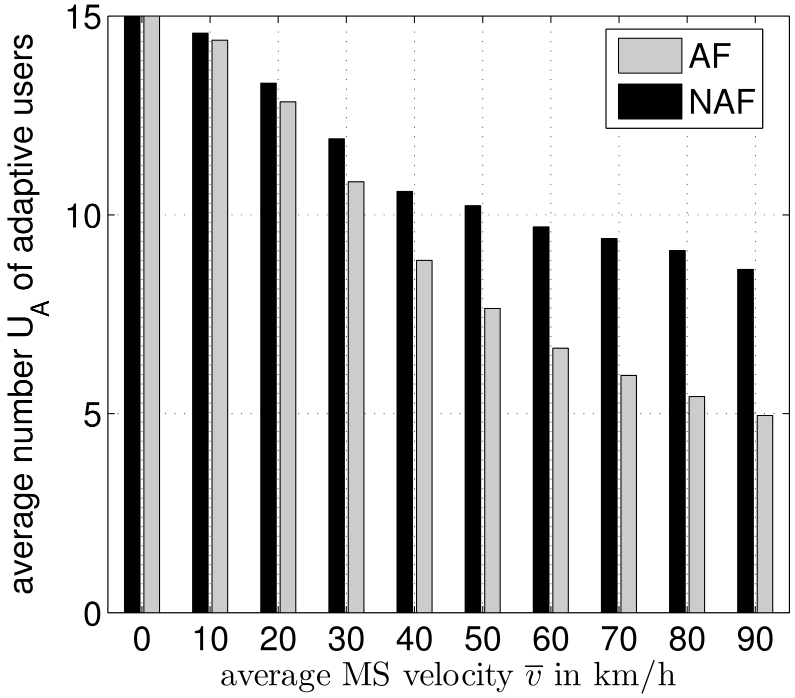


Figure 5.9. Number U_A of adaptive users versus average MS velocity $\bar{v}$ applying OSTBC-MRC

almost all of the $U = 15$ users are served adaptively. When increasing $\bar{v}$, more and more users are served non-adaptively. Comparing the average number U_A of adaptively served users applying the Adaptive First and the Non-Adaptive First scheme, one can see that with the Adaptive-First scheme it is beneficial to serve less user adaptively compared to the Non-Adaptive First scheme. This is due to the interdependency between user data rate and U_A as shown in Section 3.7.3. Note that for TAS-MRC, one gets similar results concerning the average number U_A of adaptively served users.

To further compare the hybrid schemes with the conventional ones, another metric is introduced, namely the user satisfaction S which is defined as the percentage of users for which the minimum rate requirement is fulfilled. With the variable s_u given by

$$s_u = \begin{cases} 1 & \bar{R}^{(u)} \geq \bar{R}^{(u)}_{\min} \\ 0 & \text{else} \end{cases}, \tag{5.2}$$

the user satisfaction S is given by

$$S = \frac{\sum_{u=1}^{U} s_u}{U}. \tag{5.3}$$

In Fig. 5.10 the user satisfaction S is depicted as a function of the MS velocity $\bar{v}$ for an OSTBC-MRC system. While applying the pure non-adaptive scheme and all the hybrid schemes, each user always achieves at least the minimum data rate, the user satisfaction decreases dramatically applying the pure adaptive schemes. Hence, the hybrid schemes outperform the pure adaptive schemes also in terms of user satisfaction. This can also be observed applying TAS-MRC.

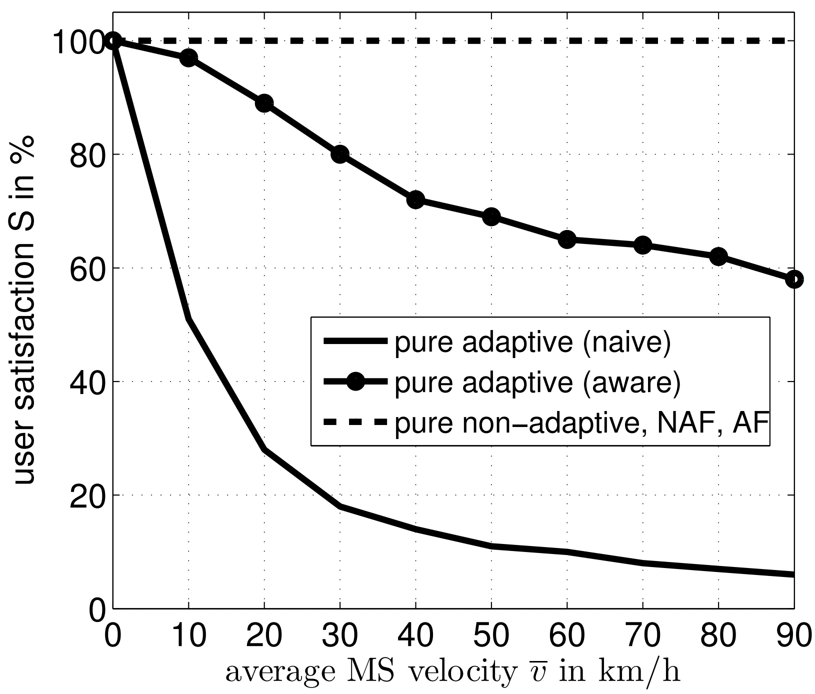


Figure 5.10. User satisfaction S versus average MS velocity $\bar{v}$ applying OSTBC-MRC

5.2.4 Comparison of hybrid transmission schemes with conventional transmission schemes considering pilot and signaling overhead

In this section, the pilot and signaling overhead is taken into account when comparing the performances of the hybrid and conventional transmission schemes. However, this requires the consideration of the UL as well since resources have to be spent for pilot transmissions in the UL in order to update the CQI at the BS. A superframe structure is assumed as shown in Section 4.2.1 with the following parameters given in Table 5.2.

Table 5.2. Superframe system parameters

Bandwidth B for DL and UL each	10 MHz
Number N of subcarriers	240
Frequency block size Q_{sub}	8
Time frame size M_T in OFDMA symbols	28
Number N_{ru} of resource units	30
Number U of users	15
UL factor κ_{UL}	1
Number M_P of pilots per resource unit	5
Number $M_{P,\text{CQI}}$ of pilots in the CQI pilot phase	1
Number b_{SS} of bits per symbol (signaling)	1
Number n_{T} of transmit antennas	2
Number n_{R} of receive antennas	2
Carrier frequency f_0	2 GHz
Target BER BER_{T}	10^{-3}
Cell radius R	300 m
Minimum distance BS-MS d_0	10 m
Pathloss exponent α	2.6

With a symbol duration of $T_S = \frac{N}{B} = 24\mu$s the total time duration of a resource unit is given by $M_T \cdot T_S = 0.672$ ms which corresponds to a third of the coherence time T_{C} as defined in Section 5.2.3, i.e., the assumption that the channel remains almost constant within a resource unit is justified. Furthermore, the delay between UL and DL is kept low which is desirable for communication systems.

The user demand vector remains the same as in Section 5.2.3. The UL factor is set to $\kappa_{\text{UL}} = 1$. For the Non-Adaptive First scheme, the superframe length is set to $L_{\text{SF}} = 74$, i.e., the time duration of a superframe is given by $2 \cdot M_T \cdot L_{\text{SF}} = 0.1$ s as stated in Section 4.2.1. For the Adaptive first scheme, the time duration of a superframe is $2 \cdot M_T = 1.344$ ms. That means that the time period between updating the user serving vector is larger than 1 ms for both the Non-Adaptive First scheme and Adaptive First scheme, i.e., for the given system parameters, the computation of the user serving vector is feasible as shown in Fig. 3.13.

In Fig. 5.11, the effective system data rate is depicted as function of the average MS velocity $\bar{v}$ for an OSTBC-MRC system. It can be seen that when the signaling and pilot overhead is considered, the Non-Adaptive First scheme slightly outperforms the Adaptive First scheme, i.e., although the Adaptive First scheme can achieve higher data rates in the DL as shown in Section 5.2.3 compared to the Non-Adaptive First scheme, the latter requires less overhead due to the exploitation of the superframe structure. This is also true for a TAS-MRC system as can be seen in Fig. 5.12.

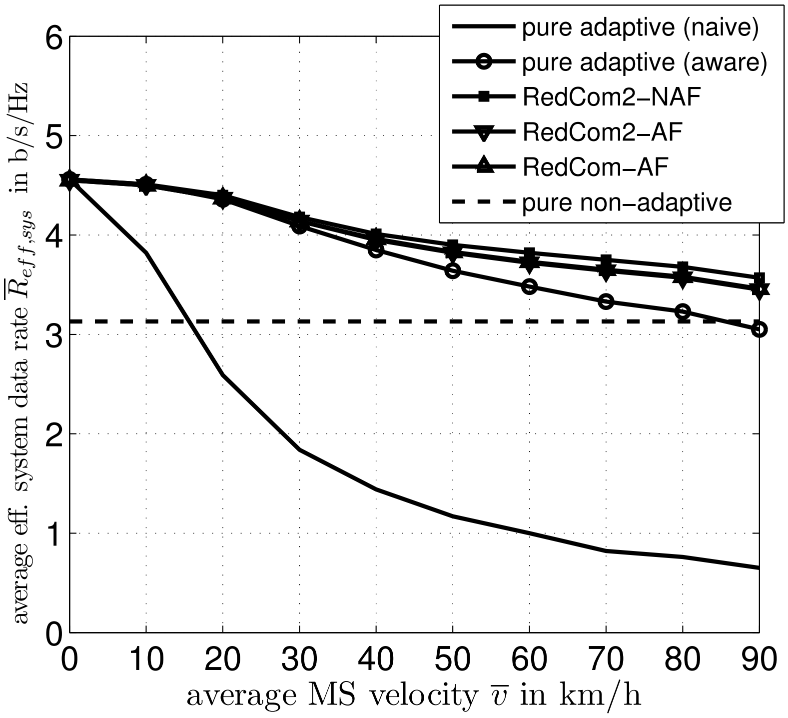


Figure 5.11. Effective system data rate versus average MS velocity $\bar{v}$ applying OSTBC-MRC

Note that although the difference in terms of effective system data rate between the hybrid schemes and the aware pure adaptive scheme is rather small, only the hybrid schemes and the pure non-adaptive schemes fulfill the minimum user data rate requirement as can be seen in Fig. 5.13 and Fig. 5.14 which show the user satisfaction S as a function of the MS velocity $\bar{v}$ for both OSTBC-MRC and TAS-MRC systems. Applying the pure adaptive scheme, full multi-user diversity can always be exploited which is beneficial for users with accurate CQI. However, for users with rather unreliable CQI it is possible that the minimum rate requirement cannot be fulfilled, i.e., at the cost of users with unreliable CQI, the users with accurate CQI are favored. With the

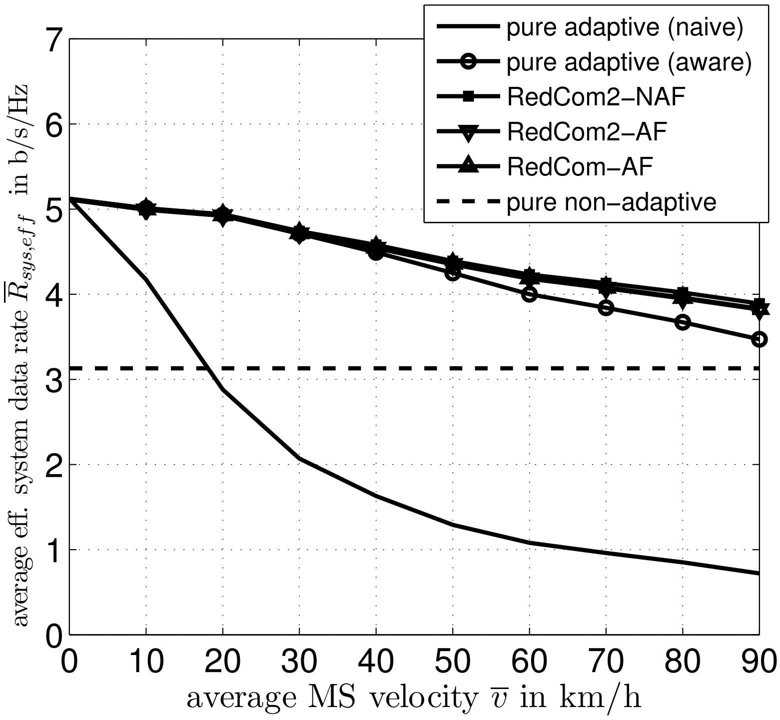


Figure 5.12. Effective system data rate versus average MS velocity $\bar{v}$ applying TAS-MRC

hybrid schemes this situation is avoided as all users achieve at least the minimum user data rate achievable with the non-adaptive scheme even though some of the users with accurate CQI would achieve a higher data rate applying the pure adaptive scheme.

5.2.5 Impact of number of active users in the cell

In the previous examples, the number of active users in the cell was set to $U = 15$. One could see from the simulation results that for this number of users, the use of adaptive schemes is beneficial. In the following, it is investigated if one can expect the same for different numbers U of users. To do so, one assumes the ideal case of perfect CQI for the adaptive transmission scheme for different numbers of users. Using an adaptive scheme, the resulting effective system data rate should be considerably larger than the resulting effective system data rate of the non-adaptive scheme assuming perfect CQI, otherwise it would be pointless to apply a hybrid system with an adaptive access scheme in the presence of imperfect CQI.

Fig. 5.15 shows the effective system data rates of a 2×2 TAS-MRC system for different numbers U of users for both the adaptive and non-adaptive scheme assuming equal user

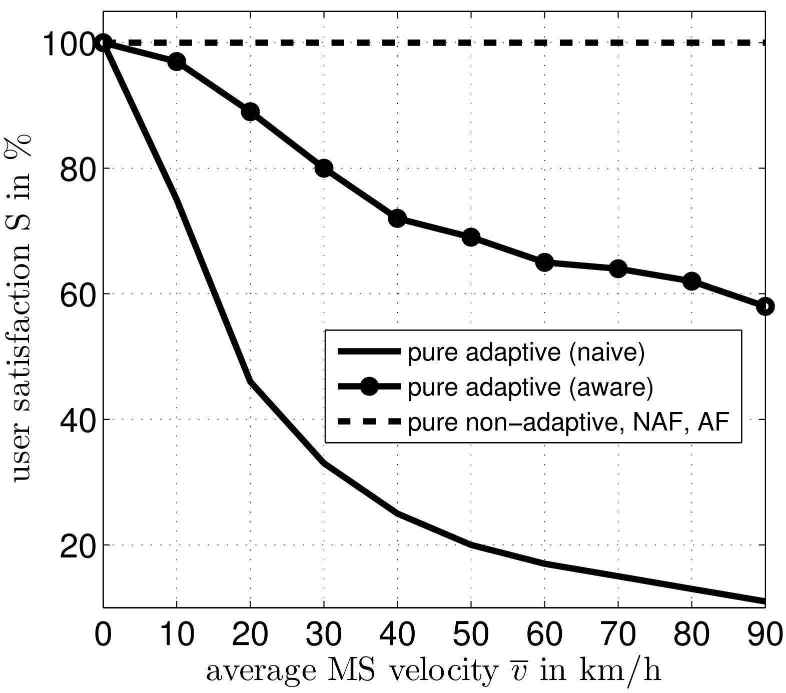


Figure 5.13. User satisfaction S versus average MS velocity $\bar{v}$ applying OSTBC-MRC considering pilot and signaling overhead

demands. As one can see, the effective system data rate of the non-adaptive scheme monotonically decreases since for an increasing number of users, less frequency diversity can be exploited by each user. For larger number of users, each user is allocated to only one frequency block, i.e., no further frequency diversity can be exploited and the effective system data rate almost remains constant. For the adaptive scheme, it can be observed that the effective system data rate increases for increasing U as long as $U < 15$. For larger U, the effective system data decreases. The reason for that is the increasing overhead which at some point compensates the multi-user diversity gains. For the given system parameters, it would be meaningless to apply an adaptive scheme if there are more than $U = 100$ users in the cell. However, as the difference in effective system data rates between the adaptive and non-adaptive access scheme should be considerably larger than zero to justify the effort in terms of computational complexity of a hybrid OFDMA system and keeping in mind that the presence of imperfect CQI in a real system leads to performance degradations, a hybrid system should only be operated for $U < 80$ users in the cell. For a larger number of users, it is beneficial to operate only in the non-adaptive mode. Fig. 5.16 shows the difference in effective system data rates between the adaptive and non-adaptive scheme as a function of U. It can be seen that the largest difference can be achieved for $U = 15$.

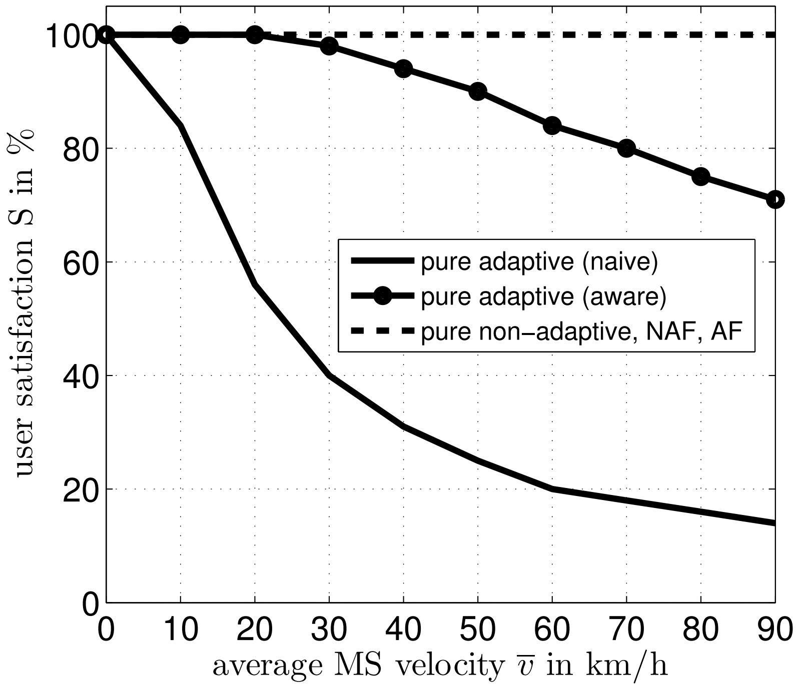


Figure 5.14. User satisfaction S versus average MS velocity $\bar{v}$ applying TAS-MRC considering pilot and signaling overhead

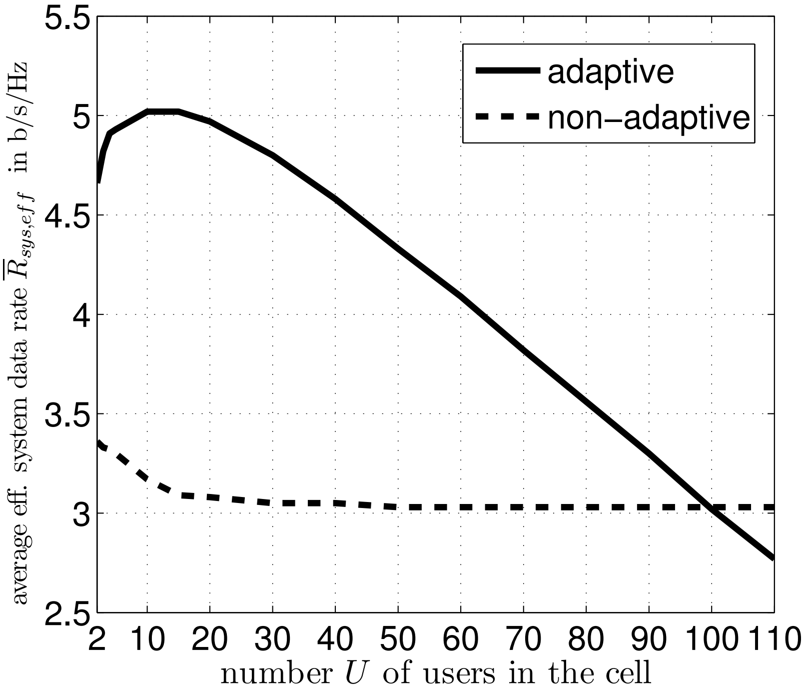


Figure 5.15. Effective system data rate versus number U of users in the cell

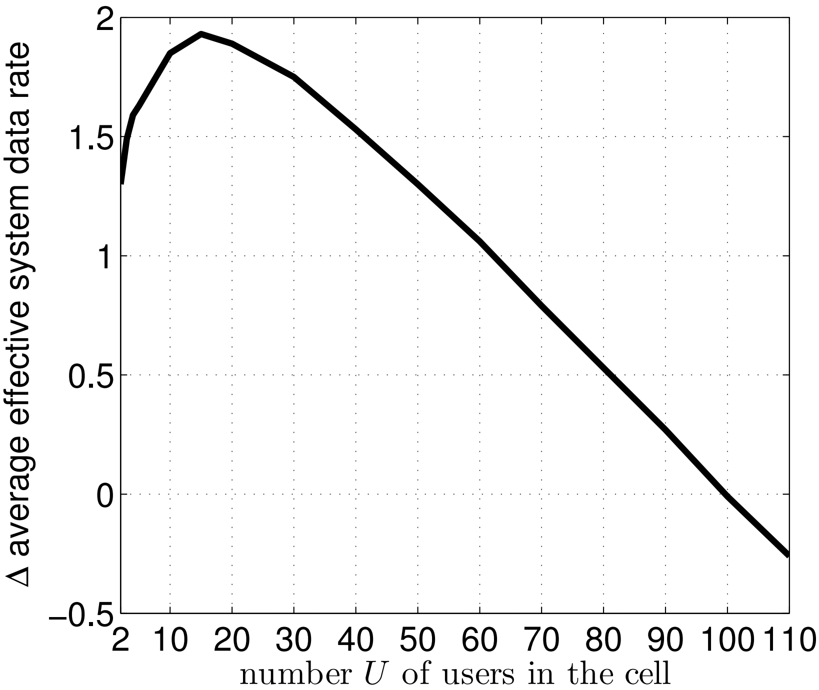


Figure 5.16. Difference in effective system data rate between the adaptive and non-adaptive access scheme versus number U of users in the cell

5.3 FDD systems

5.3.1 Impact of number N_Q of feedback quantization bits on the system performance

Since in an FDD system, the CQI for the DL scheduling has to be fed back to the BS, it is important to investigate the impact of the number N_Q of quantization bits on the system performance. On the one hand, the more quantization bits one spends for the CQI feedback, the better the resolution of the different CQI values of the different users leading to a better performance since the scheduler can distinguish between different users in a better way. One the other hand, N_Q directly effects the amount of signaling overhead, i.e., the more quantization bits one uses, the higher the overhead. Thus, a trade-off has to be found.

In the following, the impact of the number N_Q of feedback quantization bits on the system performance applying a pure adaptive transmission scheme is investigated using either OSTBC or TAS at the transmitter and MRC at the receiver. First, only the DL is considered employing the same system parameters as described in Table 5.1 with equal user demand for each user. The feedback BER is set to $p_b = 0$, i.e., it is assumed that the signaling can be regarded as quasi error-free for each user.

In Fig. 5.17, the average system data rate of an OSTBC-MRC is depicted as a function of the average MS velocity $\bar{v}$ for different numbers N_Q of quantization bits. As one can see, the higher the number of quantization bits, the better the achievable data rate due to the fact that the scheduler can distinguish much better between the CQI values of different users. Furthermore, the SNR range of a quantization interval is much smaller leading to a better fitting modulation scheme selection.

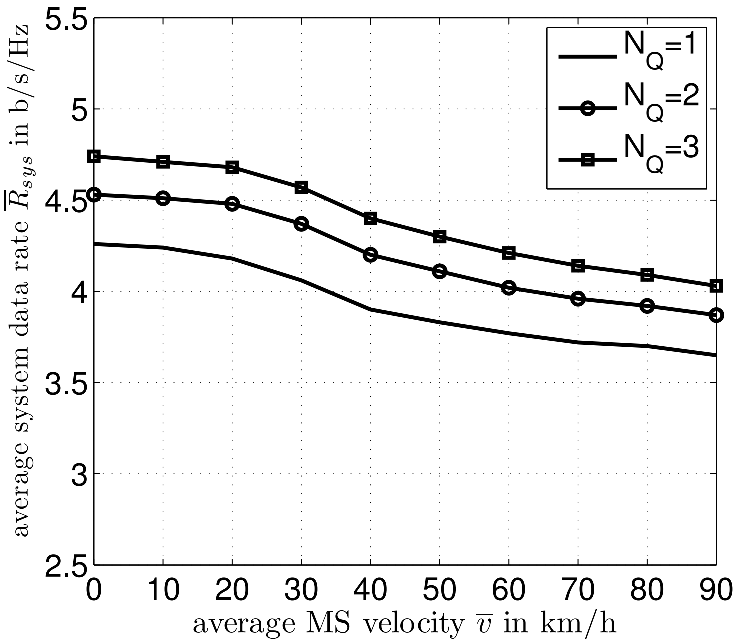


Figure 5.17. 2×2 OSTBC-MRC system data rate vs. average MS velocity $\bar{v}$

In Fig. 5.18 and Fig. 5.19, the same investigation is shown for a system applying TAS-MRC using the Feedback All (FA) scheme and the Feedback Best (FB) scheme, respectively. From both figures, one can see that increasing the number N_Q of quantization bits, the system data rate also increases, i.e., it is beneficial to use as much quantization bits as possible.

When considering the overhead, the superframe structure introduced in Section 4.3.1 is applied with the system parameters given in Table 5.2. Again, the feedback BER p_b is assumed to be $p_b = 0$ for each user.

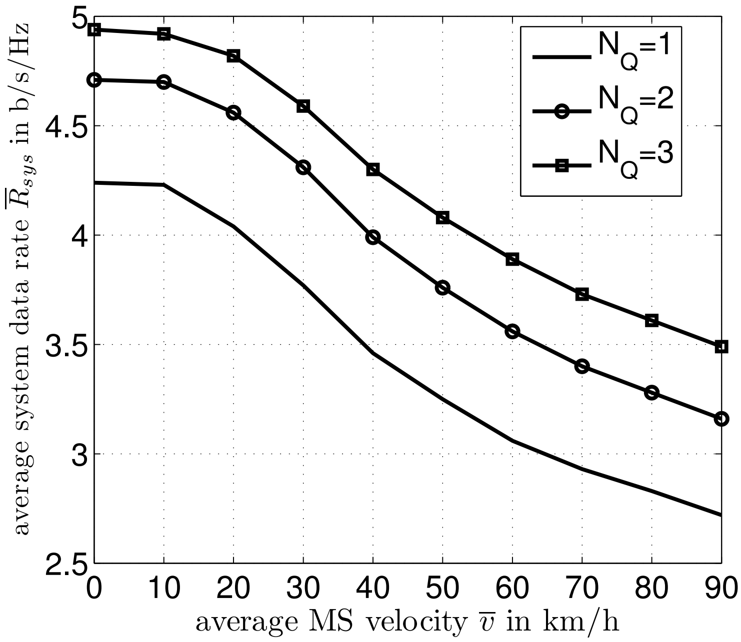


Figure 5.18. 2×2 TAS-FA-MRC system data rate vs. average MS velocity $\overline{v}$

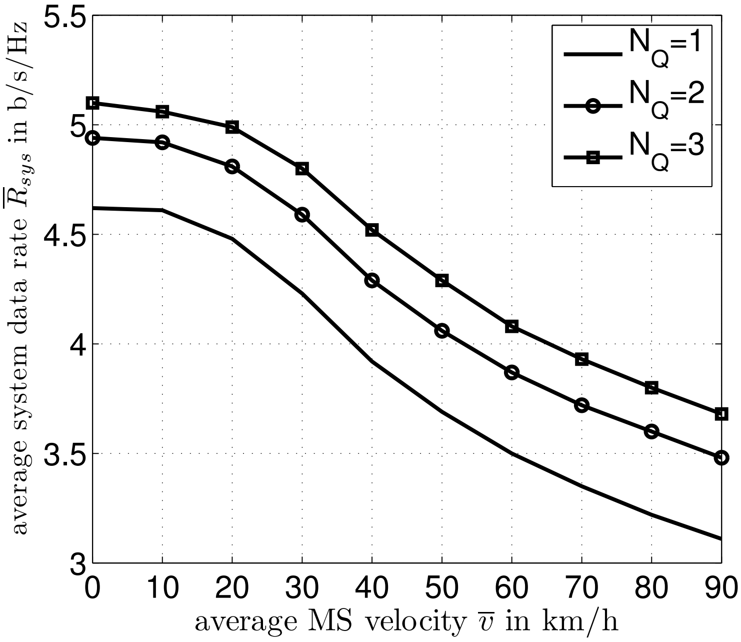


Figure 5.19. 2×2 TAS-FB-MRC system data rate vs. average MS velocity $\overline{v}$

In Fig. 5.20, the effective system data rate is depicted as a function of the average MS velocity for $N_Q = 1, 2, 3$. Now, it can be observed that due to the overhead resulting from feeding back the quantized CQI values which linearly increases with N_Q, the system data rate is no longer the best applying $N_Q = 3$ quantization bits. Instead, for the considered scenario it is beneficial to use only $N_Q = 2$ quantization bits to provide the highest effective system data rate, i.e., with $N_Q = 2$, a good trade-off between effort and gain is found. For $N_Q = 3$, the advantages of higher achievable data rates in the DL are eroded by the the overhead which has to be spent to feed back the CQI in the UL. For $N_Q = 1$, the overhead is the smallest, but also the achievable data rates are lowest.

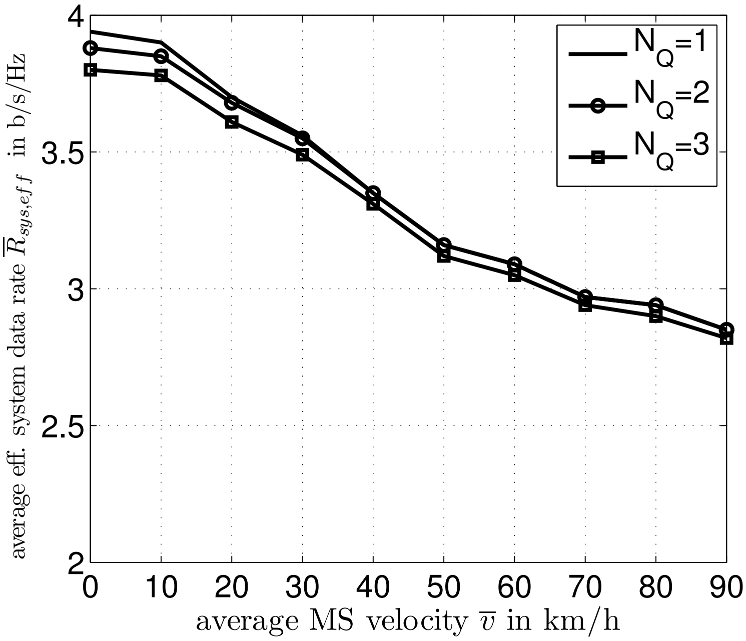


Figure 5.20. 2×2 OSTBC-MRC effective system data rate vs. average MS velocity $\bar{v}$

In Fig. 5.21 and Fig. 5.22, also the effective system data rate for a TAS-FA-MRC and a TAS-FB-MRC system are depicted. For TAS-FA-MRC, it can be seen that the best effective system data rate is achieved using only $N_Q = 1$ quantization bit. The reason for that lies in the Feedback All scheme. Since this scheme requires a high amount of feedback as the CQI values of all transmit antennas are fed back (factor n_T), it is beneficial to use only $N_Q = 1$ bit even though the data rates in this case are smaller.

Applying the Feedback Best scheme, $N_Q = 2$ provides the best effective system data rate since only the CQI values of the best transmit antenna plus the antenna index are fed back.

Comparing the performances of OSTBC-MRC, TAS-FA-MRC and TAS-FB-MRC for $N_Q = 2$ as shown in Fig. 5.23, one can see that for the considered scenario, TAS-FB-MRC always outperforms TAS-FA-MRC considering overhead. However, even without considering signaling overhead, TAS-FB-MRC also outperforms TAS-FA-MRC,

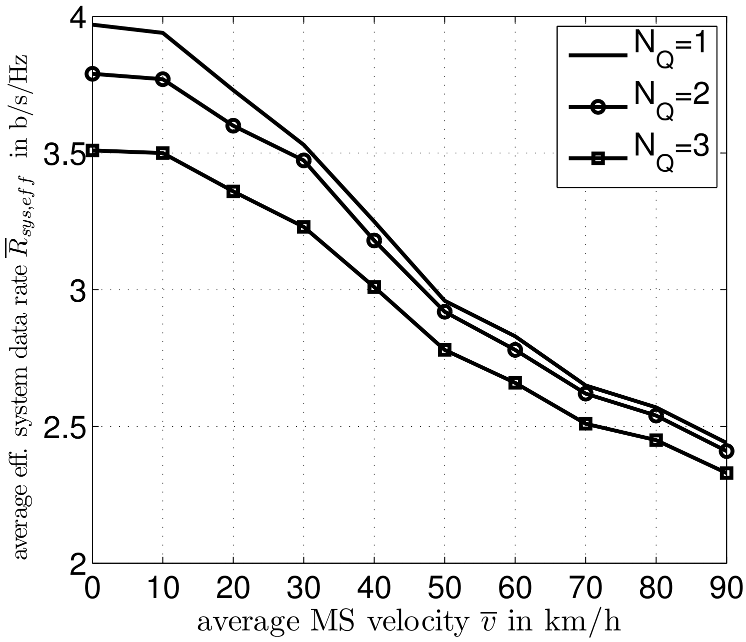


Figure 5.21. 2×2 TAS-FA-MRC effective system data rate vs. average MS velocity $\bar{v}$

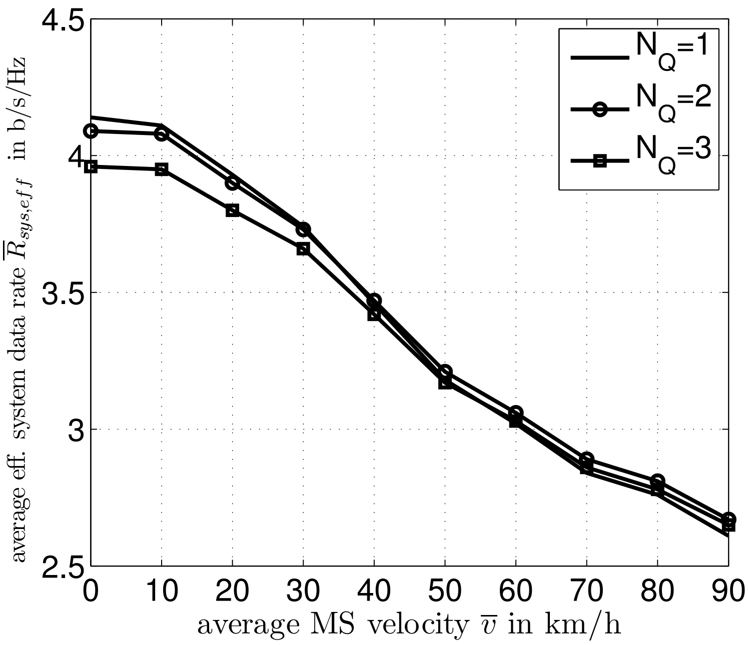


Figure 5.22. 2×2 TAS-FB-MRC effective system data rate vs. average MS velocity $\bar{v}$

although one could have expected that more feedback would automatically lead to a better performance. In fact, the probability that the SNR value of the channel of the selected user applying TAS-FB is above a certain value is higher than applying TAS-FA as with TAS-FB the transmit antenna selection is done at the MSs with continuous SNR values. From this, it follows that when U_q users in a TAS-FB system have the same quantized CQI value and the scheduler has to perform a random user selection as described in Section 2.8.4.3, one can be sure that the resulting SNR of the channel of the selected user randomly chosen from the U_q users is the best out of n_{T} channel realizations as shown in Chapter 3. When U_q users in a TAS-FA system have the same quantized CQI value and the scheduler must randomly select one user, the resulting SNR of the channel of the selected user does not arise from a selection of the best out n_{T} channel realizations. Instead, the resulting SNR arises from a random selection out of U_q channel realizations. That means that although the scheduler has n_{T} times more CQI values from which it can choose from when applying TAS-FA, the outcome of the selection on average is worse compared to TAS-FB due to the limited differentiation of the quantized CQI values.

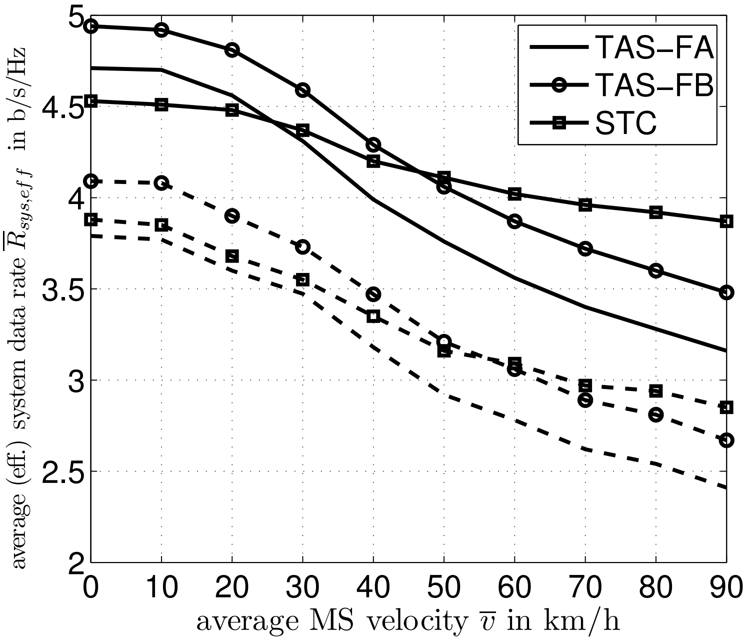


Figure 5.23. Comparison OSTBC-MRC with TAS-FA-MRC and TAS-FB-MRC for $N_Q = 2$; solid lines: without considering overhead, dashed lines: considering overhead

Comparing OSTBC-MRC with TAS-MRC, one can see in Fig. 5.23 that for small MS velocities, i.e., accurate CQI, TAS-MRC outperforms OSTBC-MRC due to the averaging effect of the spatial diversity which inhibits the occurrence of high SNR values applying OSTBC-MRC. However, if the MS velocity and, thus, the level of CQI imperfectness increases, OSTBC-MRC outperforms TAS-MRC due to its more robust exploitation of spatial diversity. This can also be observed when considering the overhead as shown with the dashed lines in Fig. 5.23.

5.3.2 Impact of feedback BER p_b on the system performance

In the following, the impact of an erroneous feedback channel and the applied bit coding scheme is investigated assuming the number of quantization bits is set to $N_Q = 2$. This investigation is carried out in a pure adaptive OFDMA system with the setting given by Table 5.1 assuming the average velocity $\bar{v}$ to be zero, i.e., the quantized CQI is assumed to be perfectly up to date. Furthermore, equal user demand is assumed. As antenna techniques, OSTBC-MRC and TAS-MRC with Feedback Best are applied. As bit coding, either binary coding or binary-reflected Gray coding is used as introduced in Section 2.9.5. For $N_Q = 2$, the corresponding Hamming distance matrices are given by

$$\mathbf{B}_{\text{bin}} = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B}_{\text{gray}} = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix}. \tag{5.4}$$

In the following, it is assumed that besides $\sigma^2_{\mathrm{E},u}$ and ρ_u, also p_b is known to the BS where p_b is assumed to be equal for all users, i.e., the applied modulation schemes are selected in such a way that the target BER is met while the system data rate is maximized.

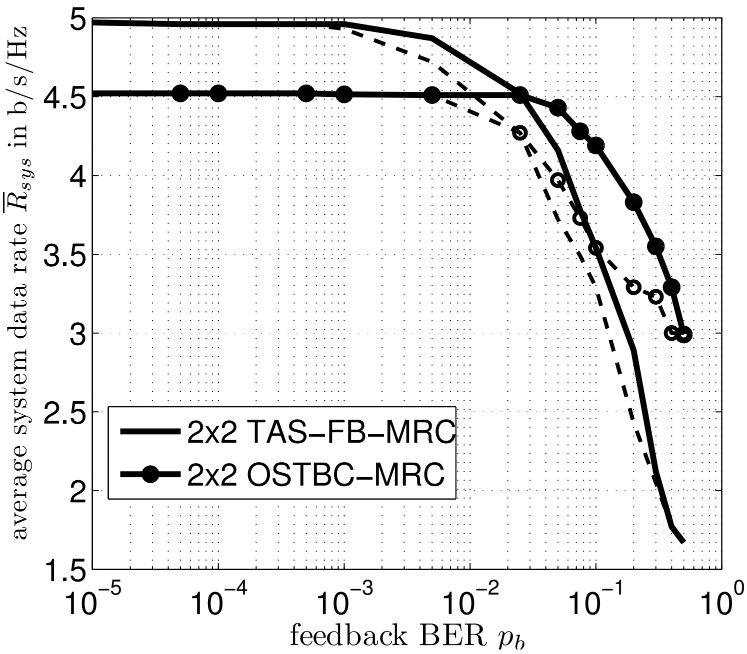


Figure 5.24. 2×2 OSTBC-MRC and TAS-FB-MRC system data rate vs. feedback BER p_b; solid lines: binary coding, dashed lines: binary-reflected Gray coding

In Fig. 5.24, the average system data rate is depicted as a function of the feedback BER p_b applying TAS-FB-MRC and OSTBC-MRC. The solid lines represent the system data rate using binary encoding for the quantized CQI values while the dashed lines represent the system data rate using binary-reflected Gray encoding. For both antenna techniques, one can see that the impact of the imperfect feedback channel can be neglected up to a BER of $p_b < 10^{-3}$. Applying OSTBC-MRC, the BER can be even higher up to $p_b < 3 \cdot 10^{-2}$ without a significant impact on the system performance due to its more robust exploitation of spatial diversity. However, in the region $p_b < 10^{-3}$, TAS outperforms OSTBC as already seen before in other investigations. When further increasing the feedback BER p_b, the system data rate decreases since the applied modulation schemes have to be selected more robust to cope with the fact that the CQI values are possibly received incorrectly at the BS. For high feedback error rates, OSTBC provides a better performance for two reasons. Firstly, the more robust spatial diversity of OSTBC, i.e., even if the scheduler chooses the wrong user for transmission, the resulting channel quality will never be that bad due to the averaging effect of the spatial diversity. Secondly, TAS-FB additionally suffers from the fact that besides the CQI values also the antenna label is possibly received incorrectly. Comparing the two bit encoding schemes, binary encoding clearly outperforms the Gray encoding for feedback BER $p_b > 10^{-3}$. The reason for that lies in the Hamming distance between the smallest and the highest quantization level. For $N_Q = 2$, the Hamming distance between the first and the fourth quantization level is 2 when applying binary encoding, while using Gray encoding, the Hamming distance is only 1, i.e., the probability that an actually weak channel is assumed to be a strong channel at the BS is much higher for Gray encoding than for binary encoding. Hence, when applying Gray encoding, the modulation schemes have to be chosen more conservatively compared to the case when applying binary encoding. For the case of $p_b = 0.5$, there is no difference between both encoding schemes since the CQI values are totally random.

5.3.3 Comparison of hybrid transmission schemes with conventional transmission schemes in the presence of imperfect CQI

In the following, the performance of the hybrid transmission schemes is compared with the performance of conventional transmission schemes in an FDD system assuming user-dependent imperfect CQI. The number N_Q of quantization bits is set to $N_Q = 2$ as this number of quantization bits for the CQI feedback turned out to provide the best trade-off between gain and effort as shown in Section 5.3.1. Furthermore, it is assumed that $p_b < 10^{-3}$, i.e., the impact of the erroneous feedback channel can be neglected as shown in Section 5.3.2. Note that if this assumption would not hold true for certain users in a real hybrid system, these users would only be served by the non-adaptive access scheme, as the CQI feed back for the adaptive users would be too erroneous. The remaining system parameters are listed in Table 5.1. As TAS-FB has been shown to outperform TAS-FA, only TAS-FB is considered where the abbreviation FB is omitted in the following.

In Fig. 5.25, the average system data rate applying OSTBC-MRC is depicted as function of the average MS velocity $\bar{v}$ as also done in Section 5.2.

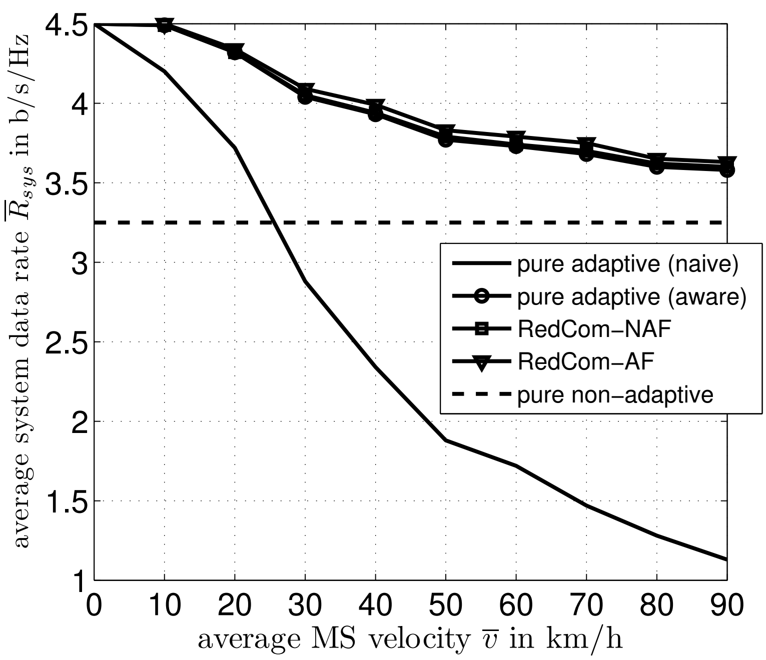


Figure 5.25. 2×2 OSTBC-MRC system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$

It can be seen that the different schemes behave similarly as shown in the Fig. 5.7 for a TDD system. Nevertheless, the achievable data rates for small MS velocities are smaller compared to the TDD system due to the fact that the CQI feedback is quantized. For higher MS velocities, the performances are comparable. The reason for that lies in the limited possibility to adapt to the current channel condition having only four different CQI values the scheduler must select from. Thus, there is an increased degree of uncertainty which forces the BS to select the modulation schemes rather conservatively to fulfill the target BER. Furthermore, only four modulation schemes can be applied per user. In situations with accurate CQI, this cautiousness results in a reduced system data rates while in situations with rather inaccurate CQI, a conservative modulation scheme selection has to be done in any event. This also explains the small difference between the Non-Adaptive First and Adaptive-First scheme, as a better SNR due to a more exclusive resource allocation does not automatically lead to a better data rate due to the limited number of modulation schemes and the rather high safety margin which is essential to fulfill the BER requirements in this case. In this context, it has to be mentioned that although the system data rate of the aware pure adaptive scheme is almost the same as the hybrid schemes, only the hybrid schemes and the pure non-adaptive scheme fulfill the minimum rate requirement which can be seen in Fig. 5.26 depicting the user satisfaction S as a function of the MS velocity $\bar{v}$.

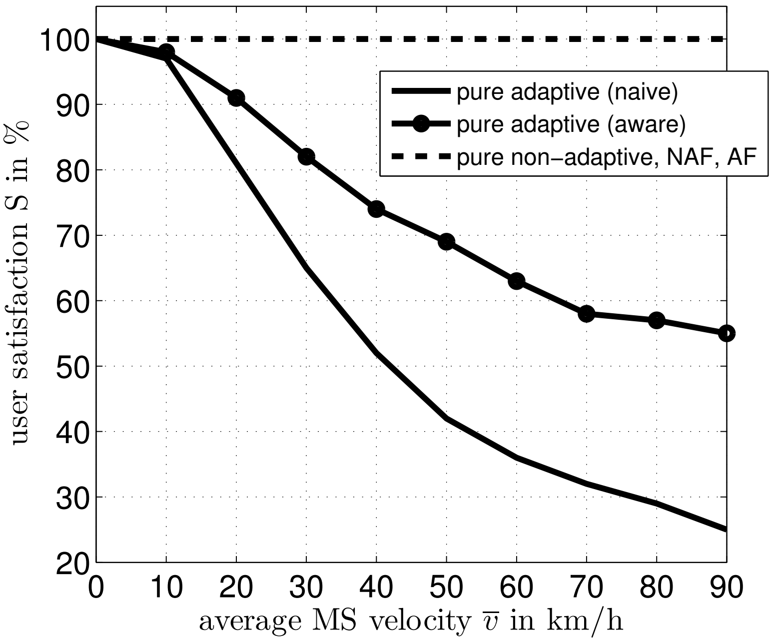


Figure 5.26. User satisfaction S versus average MS velocity $\bar{v}$ applying OSTBC-MRC

For the case of a TAS-MRC system given in Fig. 5.27, it can be observed that the pure adaptive scheme which is aware of the CQI impairments is much more vulnerable to inaccurate CQI as TAS is less robust compared to the spatial diversity exploiting OS-TBC scheme, i.e., the average system data rate of the pure adaptive scheme decreases much faster with increasing $\bar{v}$ as in the case of OSTBC. Also, the difference between the two hybrid schemes is rather small for the same reason as explained for the OSTBC case. Comparing the performance of the TAS-MRC system with the performance of the OSTBC-MRC system, TAS-MRC outperforms OSTBC-MRC for the same reason as shown in Section 5.2.3.

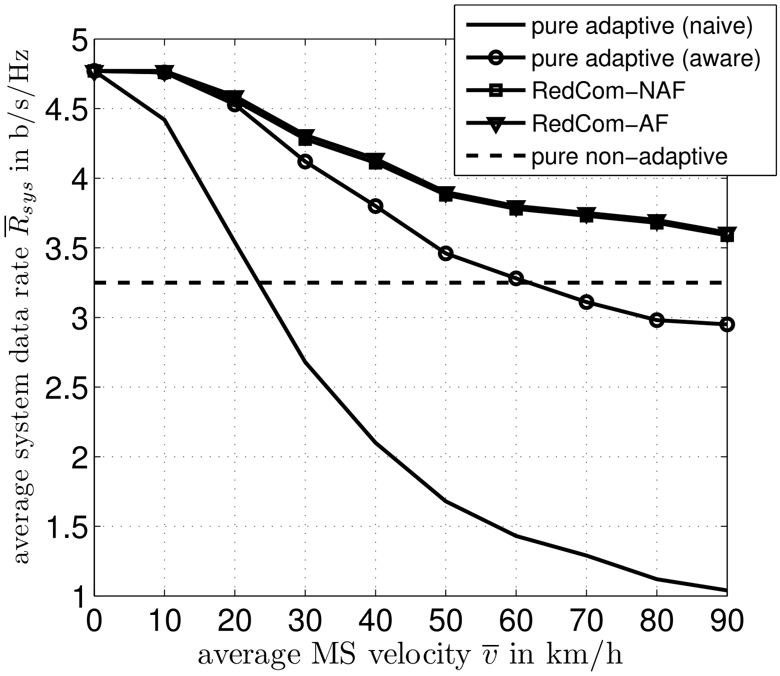


Figure 5.27. 2×2 TAS-MRC system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$

5.3.4 Comparison of hybrid transmission schemes with conventional transmission schemes considering pilot and signaling overhead

5.3.4.1 Half Duplex

Considering the pilot and signaling overhead in an FDD system, two possible duplex schemes have to be taken into account Half Duplex and Full Duplex. For both schemes the superframe structures as presented in Section 4.3.1 and 4.3.2 are assumed. The superframe length in case of the Non-Adaptive First scheme is set to $L_{\mathrm{SF}} = 74$. Furthermore, it is assumed that the signaling in both DL and UL direction is assumed to be error-free, i.e., $p_b = 0$. The remaining system parameters are listed in Table 5.2.

Fig. 5.28 shows the system data rate of an OSTBC system for the different hybrid and conventional transmission schemes as a function of the MS velocity $\bar{v}$. Like in the TDD case, the Non-Adaptive First scheme outperforms the Adaptive-First scheme due to the overhead saving use of the superframe structure. Moreover, it can be observed that the gain between the hybrid schemes and the pure non-adaptive scheme is smaller compared to the case when the overhead is not considered as the hybrid schemes require much more overhead. For high MS velocities it is even possible that the Adaptive First scheme is worse than the pure non-adaptive scheme since this hybrid scheme always requires more overhead than the pure non-adaptive one even if ultimately all users are served non-adaptively, i.e., in this case the use of the Adaptive First scheme would not be reasonable.

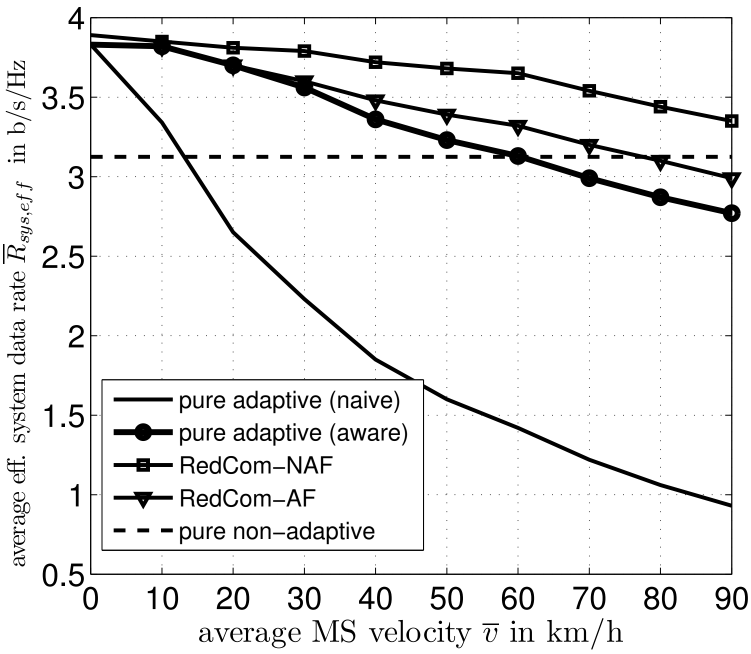


Figure 5.28. 2×2 OSTBC-MRC effective system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$ and half duplex

In Fig. 5.29 the same is shown for a TAS-MRC system where the hybrid schemes in a TAS-MRC system slightly provide better results compared to the OSTBC-MRC system due to the reasons explained in Section 5.2.3.

5.3.4.2 Full Duplex

In Fig. 5.30, the effective system data rate is depicted when applying OSTBC-MRC in a full duplex FDD system. Due to the simultaneous transmitting and receiving of data, the achievable data rates are almost twice as high compared to the half duplex case. However, in principle, the progression of the system data rates for the different schemes with increasing MS velocity is similar to the half duplex case with the Non-Adaptive First scheme outperforming all other schemes. This can also be observed for the case of a TAS-MRC full duplex system as shown in Fig. 5.31. Again, it can be seen that the hybrid schemes applying TAS-MRC outperform the hybrid schemes applying OSTBC-MRC.

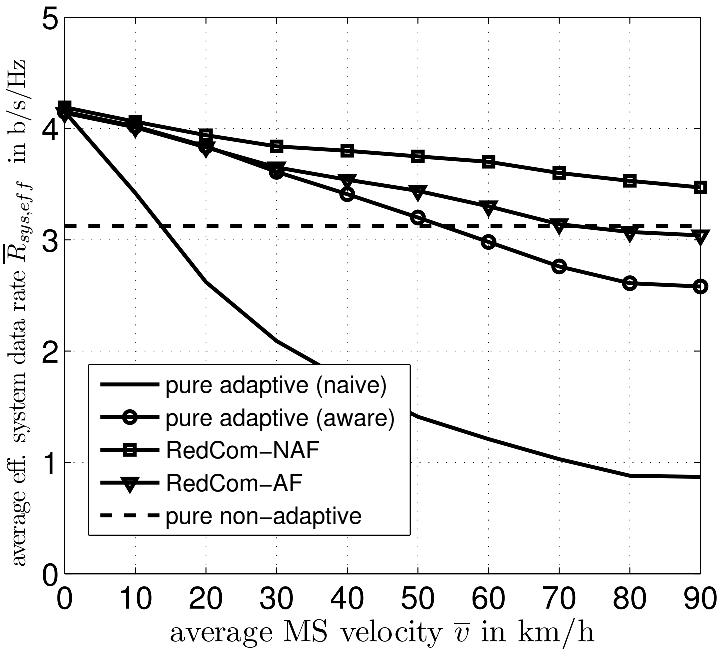


Figure 5.29. 2×2 TAS-MRC effective system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$ and half duplex

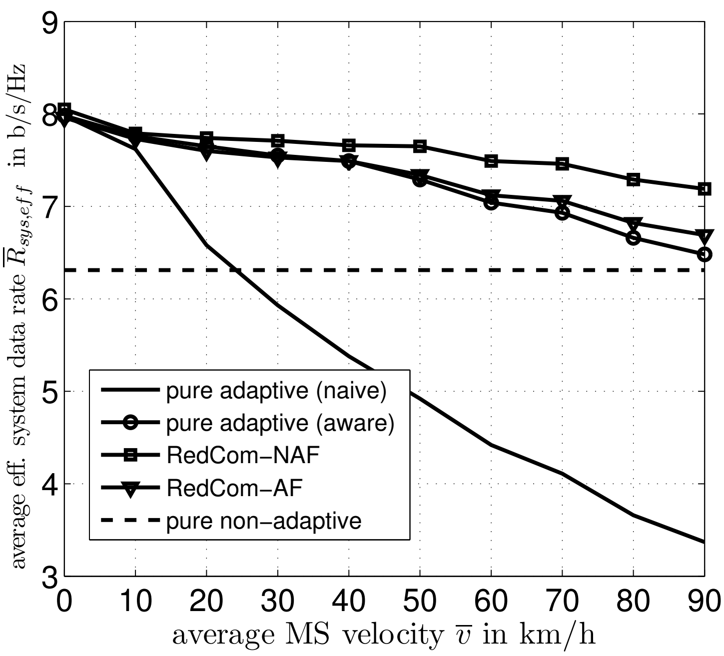


Figure 5.30. 2×2 OSTBC-MRC effective system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$ and full duplex

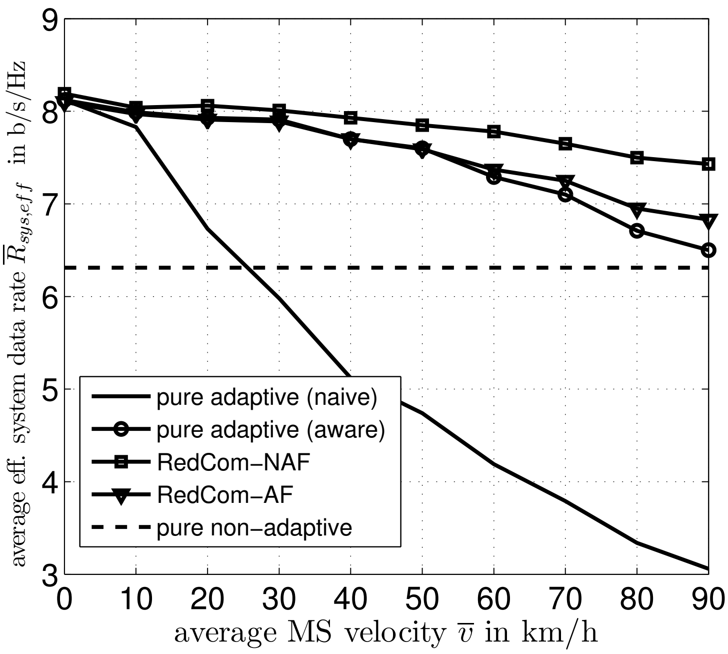


Figure 5.31. 2×2 TAS-MRC effective system data rate vs. average MS velocity $\bar{v}$ with $N_Q = 2$ and full duplex

5.3.5 Impact of the number of active users in the cell

As done in the TDD case, the impact of the number of active users in the cell is investigated. In Fig. 5.32, the effective system data rate of a 2×2 TAS-MRC system for both the adaptive and non-adaptive access scheme is depicted as function of the total number U of users in the cell assuming perfect CQI ($\bar{v} = 0$ km/h). In this ideal case, the effective system data rate of an FDD system in half duplex is half the effective system data rate of a full duplex FDD system as can be seen in Section 4.3, i.e., it is enough to only consider half duplex. Like in the TDD system, the effective system data rate increases for an increasing U for small number of users. However, if $U > 5$, the effective system data rate decreases with increasing U due to the increasing overhead. As in the UL of FDD systems both pilot transmissions and CQI feedback have to be performed which both linearly increase with U, the multi-user diversity gains are already compensated for a smaller number U of users compared to a TDD system where no additional CQI feedback has to be signaled. Hence, the use of adaptive schemes in hybrid FDD systems is only reasonable if the number of users does not exceed $U = 25$. For a higher number of active users in the cell, it is better to apply only the non-adaptive scheme. Fig. 5.33 shows the difference in effective system data rates between the adaptive and non-adaptive access scheme as a function of U. It can be seen that the largest difference can be achieved for $U = 4$.

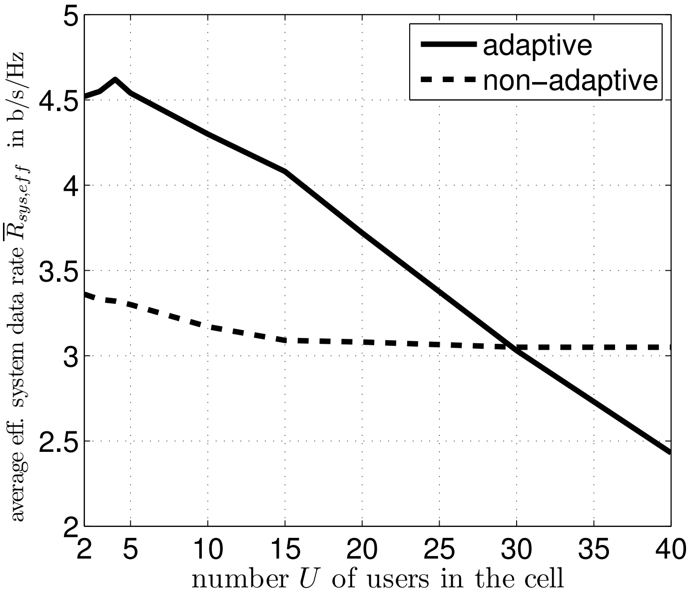


Figure 5.32. Effective system data rate versus number U of users in the cell

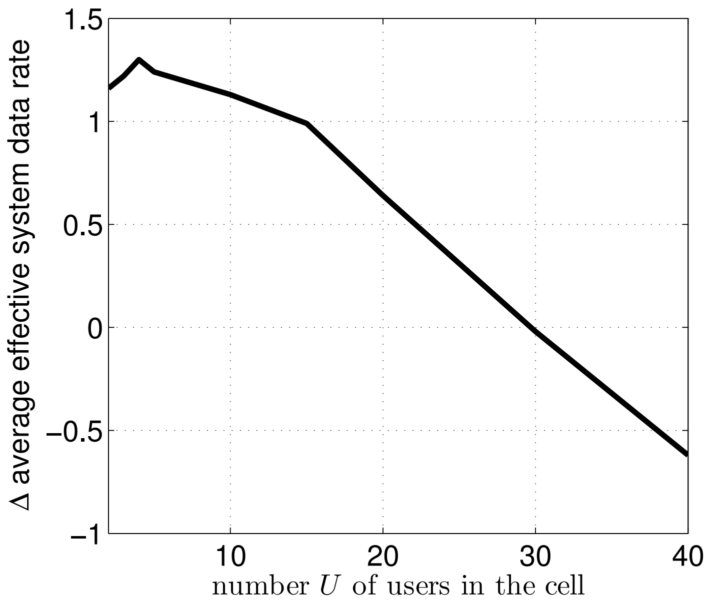


Figure 5.33. Difference in effective system data rate between the adaptive and non-adaptive access scheme versus number U of users in the cell

5.4 Conclusions

In this chapter, the performance of hybrid schemes and conventional pure adaptive and non-adaptive schemes has been evaluated for both TDD and FDD systems assuming user-dependent imperfect CQI. The main conclusions can be summarized as follows:

- Serving adaptive users with high user demand comes at the expense of a reduced system data rate compared to a system with equal user demands.
- Systems which serve adaptive users with different user demands are more sensitive to outdated CQI compared to systems with equal user demands.
- For pure adaptive transmission schemes in scenarios with accurate CQI, TAS systems outperform OSTBC systems while for rather unreliable CQI, OSTBC provides a better performance.
- Considering the CQI feedback signaling overhead in an FDD system, $N_Q = 1$ up to $N_Q = 2$ feedback quantization bits provide the best trade-off between data rate gain and signaling effort.
- Using TAS in an FDD system, it is better to apply the Feedback Best (FB) scheme rather than the Feedback All (FA) scheme in terms of achievable data rate with and without considering signaling overhead.
- For both TDD and FDD systems, hybrid transmission schemes outperform pure adaptive and pure non-adaptive schemes in terms of system data rate and user satisfaction neglecting the pilot and signaling overhead which has to be spent to conduct these transmission schemes.
- For both TDD and FDD systems, the Adaptive First scheme provides the best performance due to its superior resource selection when neglecting the pilot and signaling overhead.
- Considering pilot and signaling overhead in TDD systems, the hybrid schemes still outperform the conventional pure adaptive and non-adaptive schemes. However, now the Non-Adaptive First scheme provides the best performance as this scheme requires less signaling.
- When considering the overhead in FDD systems, it is possible for a high level of CQI imperfectness in the cell that the Adaptive First hybrid scheme delivers an effective system data rate which is actually smaller than the one of the pure non-adaptive scheme due to the large amount of signaling especially the CQI feedback in the UL which only occurs in FDD systems.

- The use of hybrid systems is only beneficial for a low to medium number of active users in the cell due to the increasing pilot and signaling overhead where for FDD systems the supportable number of active users in the cell is smaller compared to TDD systems. For a high number of active users in the cell, it is better to operate only in the non-adaptive mode.

Chapter 6
Conclusions

6.1 Summary

This thesis deals with the analytical description and evaluation of a hybrid multi-user OFDMA transmission scheme with different channel access user demands assuming user-dependent imperfect CQI. The considered hybrid transmission scheme offers two possible modes to serve the user: Firstly, via a non-adaptive OFDMA mode which applies a DFT precoding to exploit frequency diversity and, thus, does not require any channel knowledge at the transmitter. Secondly, via an adaptive OFDMA mode which performs an adaptive resource allocation and modulation scheme selection based on CQI to adjust to the current channel conditions. Assuming perfect CQI at the transmitter, the adaptive mode outperforms the non-adaptive mode due to a better adaptation to the channel. However, as the system performance of the adaptive mode suffers from CQI impairments such as estimation errors and time delays which could probably lead to a worse performance compared to the non-adaptive mode, the question arises which user shall be served adaptively or non-adaptively and which resource shall be allocated to which user such that the total system data rate is maximized while each user achieves a certain target BER and minimum user data rate. To answer this question, analytical expressions of the performances of the adaptive and non-adaptive transmission schemes as function of the parameters describing the CQI impairments and the user demands have been derived. Based on these expressions, algorithms which determine which user is served adaptively or non-adaptively subject to the BER and minimum data rate constraints have been developed.

In Chapter 1, the concept of hybrid OFDMA is introduced and an overview of current state-of-the-art is presented. Based on that, the open issues are identified and formulated. Finally, the main contributions and an overview of the thesis is provided.

In Chapter 2, first the OFDMA system model with the underlying channel model and system assumptions is presented. Furthermore, the considered multiple antenna techniques OSTBC and TAS in combination with MRC at the receiver are introduced as well as the adaptive and non-adaptive multi-user OFDMA transmission modes. Finally, the modelling of imperfect CQI is presented.

In Chapter 3, the hybrid OFDMA scheme is introduced where two resource allocation schemes are considered which differ in the order of allocation. With the Non-Adaptive

First scheme, the resources assigned to the non-adaptive users are allocated in a first step followed by the resources assigned to the adaptive users which are allocated following a WPFS approach. With the Adaptive First scheme, the order of allocation is vice-versa. To fulfill certain user demands, it shown how to adjust the weighting factors for the WPFS for both the Non-Adaptive First and the Adaptive First scheme considering both antenna techniques OSTBC and TAS in combination with MRC and considering both continuous and quantized CQI. To do so, analytical derivations of the channel access probability have been carried out for the various cases. Furthermore, the main problem formulation of this thesis is introduced in this chapter. It is shown that the main problem can be divided in two smaller problems, namely the SNR threshold and the user serving problem without simplifying the main problem. The SNR threshold problem deals with the question of which modulation scheme shall be applied such that the user data rate is maximized while the target BER is fulfilled. In order to solve this problem, complex derivations of analytical expressions of the user data rate and BER as function of the CQI impairment parameters and the number of adaptive users have been performed in this work. This also includes the derivation of the post-scheduling SNR distribution assuming continuous and quantized CQI for both the Non-Adaptive First and Adaptive First scheme. These expressions are then used to solve the SNR threshold problem via a Lagrange multiplier approach in case of a TDD system and via a 2^{N_Q}-dimensional search with reduced solution space in case of an FDD system which applies N_Q bits for CQI quantization. Being able to determine the maximum achievable user data of each possible number of adaptive users, the combinatorial user serving problem can be solved, i.e., different from approaches in the literature, the applied access scheme is selected based on analytical calculations of the expected performance taking into account imperfect CQI and the number of adaptively served users, where it has been shown that it is not necessary to check all possible 2^U user serving combinations in order to find the best solution. The chapter is concluded by a complexity analysis of the proposed user serving algorithms.

In Chapter 4, also the overhead in terms of pilot transmissions and signaling is taken into account, since the non-adaptive transmission modes requires much less overhead due to its property of working independently from any transmitter sided CQI. Thus, it is important to incorporate the overhead in the achievable user data rate applying either the adaptive transmission mode or the non-adaptive transmission mode to get a meaningful and realistic result. For both the adaptive and non-adaptive transmission mode, the effort in terms of pilot transmissions and signaling of side information are identified. Since pilot and signaling overhead does not only effect the DL, also the UL is considered since in the UL, resources have to be spent such that the BS is able to acquire information about the UL and DL channel quality. To do so, a super frame structure for the transmission in both UL and DL direction for both TDD and FDD systems is introduced and analytical expressions for the effective system data rates using the hybrid scheme, the pure adaptive and the pure non-adaptive scheme are derived. Furthermore, it is shown that the same user serving algorithms which has been developed in Chapter 3 can be used to solve the user serving problem taking into account the pilot and signaling overhead.

Chapter 5 provides performances evaluations for both TDD and FDD systems showing that for a low to medium number of active users in the cell, the hybrid scheme outperforms conventional pure adaptive and pure non-adaptive schemes in the presence of user-dependent imperfect CQI with or without considering pilot and signaling overhead. When neglecting the overhead, the Adaptive First scheme outperforms the Non-Adaptive First scheme due to its more exclusive resource selection. In case of considering the overhead, the Non-Adaptive First scheme provides the best performance as this scheme requires less signaling compared to the Adaptive First scheme. For a high number of active users in the cell, it is better to operate only in the non-adaptive mode as the increasing effort of acquiring transmitter sided CQI for the adaptive users undoes the multi-user diversity gains.

6.2 Outlook

In this thesis, only uncoded transmission has been considered as a general analytical description of the performance of coded transmission is unfeasible. However, one could approximate the achievable BER as a function of the applied modulation scheme and the instantaneous SNR for certain classes of codes and code rates using, e.g., curve fitting approaches. Applying an exponential function or a sum of exponential functions to approximate the BER curve, the analytical expressions derived in this work could be used again as can be seen from Eq. (3.60).

Furthermore, in this thesis, only multiple antennas techniques have been considered which use the transmitter sided channel knowledge solely for adaptive resource allocation and link adaption purposes, like Space-Time Coding and Transmit Antenna Selection with Maximum Ratio Combining. Future work could also consider multiple antennas techniques which use the channel knowledge to spatially multiplex several data streams in order to increase the data rate of the adaptive transmissions as done with Singular Value Decomposition (SVD), adaptive Beamforming or Zero-Forcing approaches. In this case, imperfect channel knowledge would not only effect the resource allocation but also the spatial separation of the data streams.

Another possibility that multiple antennas offer is the spatial multiplexing of the resources for the adaptive and non-adaptive transmissions, i.e., for a given resource the data transmissions for adaptively served users and non-adaptively served users are spatially separated which increases the bandwidth efficiency. Hereby, it has to be noted that the usefulness of spatial multiplexing in general MIMO scenarios is less clear compared to time and frequency multiplexing as the spatial channels may lose their orthogonality over time which requires further studies.

Moreover, hybrid OFDMA schemes which apply multi-hop relay transmissions to cope with coverage limitations could be considered. In this case, the signaling overhead in such relay networks is larger compared to conventional schemes as the relay transmission schemes known in the literature strongly rely on accurate channel knowledge. Since the effort of providing channel knowledge for the different relay nodes linearly increases with the number of relays, one has to assume partial or imperfect channel knowledge in a realistic scenario especially for the hop from the relay to the MS as this link is the most unreliable due to the users' mobility. This could imply that the applied multiple access scheme does not only change from user to user, but also from hop to hop.

Finally, also multi-cell scenarios could be considered. Assuming partial or full cooperation between neighboring cells offers the possibility of interference cancellation by means of Joint Detection/Joint Transmission approaches resulting in significant system performance enhancements [WMSL02]. However, transmitter sided channel knowledge is required leading to a significant amount of overhead. Moreover, the channel knowledge might be imperfect leading to performance degradations which has been studied in [WWKK09]. On that account, other interference avoiding approaches which do not rely on channel knowledge such as interference averaging techniques might be more suitable in certain scenarios. Analogue to the single cell scenario, one could think of a hybrid solution where both techniques are applicable.

Appendix

A.3 Derivation upper bound $G_{\max}$ of number of demand groups of (2.38)

In the following, the maximum number $G_{\max}$ of demand groups, i.e., groups of users with the same channel access demand D_u, is derived as a function of the number of resource units N_{ru} and number of users U. This number is important for the providers of mobile radio systems as it defines the maximum number of different demand groups a system with N_{ru} resource units and U users can offer.

In a system with N_{ru} resource units and U users with $N_{\mathrm{ru}} \geq U$ it is obvious that the maximum number of possible demand groups is at most U. However, not for every constellation of N_{ru} and U, it is possible to have U different channel access demands D_u while fulfilling the side condition

$$\sum_{u=1}^{U} D_u = N_{\mathrm{ru}} \ \text{ with } \ 1 \leq D_u \leq N_{\mathrm{ru}} - (U-1) \tag{A.1}$$

introduced in Eq. 2.37. In order to find the maximum number $G_{\max}$ of demand groups, it is assumed that with out loss of generality the user with the lowest demand requests one resource unit and that the demand request of the next $G_{\max} - 1$ users differs by just one resource unit. Thus, the sum over the demands of these $G_{\max}$ users is given by $\frac{1}{2} \cdot G_{\max} \cdot (G_{\max} + 1)$. If the remaining $U - G_{\max}$ users request just 1 resource unit, i.e., there are $G_{\max}$ different demands in total, the inequality

$$\frac{1}{2} \cdot G_{\max} \cdot (G_{\max} + 1) + (U - G_{\max}) \leq N_{\mathrm{ru}}, \tag{A.2}$$

must hold, otherwise (A.1) is not fulfilled. (A.2) can be rewritten to

$$G_{\max}^2 - G_{\max} - 2 \cdot (N_{\mathrm{ru}} - U) \leq 0 \tag{A.3}$$

which can be solved resulting in

$$G_{\max} \leq \frac{1}{2} \cdot \left(1 + \sqrt{1 + 8 \cdot (N_{\mathrm{ru}} - U)}\right), \tag{A.4}$$

ignoring the negative solution. Since $G_{\max}$ has to be an integer number smaller or equal to U, $G_{\max}$ is finally given by Eq. (2.38).

A.4 Proof of construction of Hamming distance matrices of (2.90) and (2.91)

A.4.1 Hamming distance matrix for binary coding of (2.90)

In the following, it is proven by mathematical induction that the $2^{N_Q} \times 2^{N_Q}$ Hamming distance matrix $\mathbf{B}_{N_Q}$ for binary coding applying N_Q bits is given by Eq. (2.90).

1) Basis with $N_Q = 1$.
   Applying one bit, there are two different binary codes X_i with $i = 1, 2$, i.e., $X_1 = 0$ and $X_2 = 1$. Form this it follows that the Hamming distance matrix $\mathbf{B}_1$ is given by

$$\mathbf{B}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

   Applying (2.90) with $N_Q = 1$ yields

$$\mathbf{B}_1 = \begin{pmatrix} \mathbf{B}_0 & 1+\mathbf{B}_0 \\ 1+\mathbf{B}_0 & \mathbf{B}_0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   what was to be shown.

2) Induction hypothesis.
   If

$$\mathbf{B}_{N_Q} = \begin{pmatrix} \mathbf{B}_{N_Q-1} & 1+\mathbf{B}_{N_Q-1} \\ 1+\mathbf{B}_{N_Q-1} & \mathbf{B}_{N_Q-1} \end{pmatrix}$$

   holds, then

$$\mathbf{B}_{N_Q+1} = \begin{pmatrix} \mathbf{B}_{N_Q} & 1+\mathbf{B}_{N_Q} \\ 1+\mathbf{B}_{N_Q} & \mathbf{B}_{N_Q} \end{pmatrix} \tag{A.5}$$

   also must hold for any N_Q.

3) Inductive step.
   Applying $N_Q + 1$ bits to binarily encode $M = 2^{N_Q+1}$ quantization levels results

in

$$\begin{array}{rcl} X_0 & = & [0, \overbrace{0, \overbrace{0, ..., 0}^{N_Q}}^{N_Q+1}] \\ X_1 & = & [0, 0, ..., 1] \\ & \vdots & \\ X_{M/2-1} & = & [0, 1, ..., 1] \\ X_{M/2} & = & [1, 0, ..., 0] \\ & \vdots & \\ X_{M-1} & = & [1, 1, ..., 1]. \end{array} \tag{A.6}$$

As one can see, the Hamming distance between the binary codes X_i with $i = 0, .., M/2 - 1$ do not change compared to the case of N_Q bits since there is only a 0 added at the beginning. Thus, the Hamming distance between the binary codes X_i with $i = 0, .., M/2 - 1$ and the binary codes X_j with $j = 0, .., M/2 - 1$ are expressed by $\mathbf{B}_{N_Q+1}(1, .., M/2; 1, .., M/2) = \mathbf{B}_{N_Q}$. Comparing the Hamming distance between the binary codes X_i with $i = 0, .., M/2-1$ and the binary codes X_j with $j = M/2, .., M-1$, we only have to increase the Hamming distance by one due to the additional 1 at the beginning. Hence, the Hamming distance between the binary codes X_i with $i = 0, .., M/2 - 1$ and the binary codes X_j with $j = M/2, .., M-1$ are expressed by $\mathbf{B}_{N_Q+1}(M/2+1, .., M; M/2+1, .., M) = 1+\mathbf{B}_{N_Q}$. For the Hamming distance between the binary codes X_i with $i = M/2, .., M$ and the binary codes X_j with $i = 0, .., M$, one gets the same result but vice-versa, resulting in

$$\mathbf{B}_{N_Q+1} = \begin{pmatrix} \mathbf{B}_{N_Q} & 1 + \mathbf{B}_{N_Q} \\ 1 + \mathbf{B}_{N_Q} & \mathbf{B}_{N_Q} \end{pmatrix},$$

which is equivalent to (A.5) what was to be shown.

A.4.2 Hamming distance matrix for binary-reflected Gray coding of (2.91)

In the following, it is proven by mathematical induction that the $2^{N_Q} \times 2^{N_Q}$ Hamming distance matrix $\mathbf{B}_{N_Q}$ for binary-reflected Gray coding applying N_Q bits is given by Eq. (2.91).

Note that the translation from a binary value X_{bin} to the corresponding binary reflected Gray code X_{gray} is given by $X_{\text{gray}} = X_{\text{bin}} \oplus X_{\text{bin}/2}$ where $X_{\text{bin}/2}$ denotes the 1-bit shifted version of X_{bin} to the right and $\oplus$ denotes the exclusive OR (XOR) operation.

1) Basis with $N_Q = 2$.
   Applying two bits, there are four different Gray codes x_i with $i = 1, .., 4$ leading to $x_1 = [0, 0]$, $x_2 = [0, 1]$, $x_3 = [1, 1]$ and $x_4 = [1, 0]$. From this, it follows that the Hamming distance matrix $\mathbf{B}_2$ is given by

$$\mathbf{B}_2 = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix}.$$

   Applying (2.91) with $N_Q = 2$ yields

$$\mathbf{B}_2 = \begin{pmatrix} \mathbf{B}_1 & 2 \cdot \mathbf{I}_{\mathbf{B},1} + \mathbf{B}_1 \\ 2 \cdot \mathbf{I}_{\mathbf{B},1} + \mathbf{B}_1 & \mathbf{B}_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix}$$

   what was to be shown.

2) Induction hypothesis.
   If

$$\mathbf{B}_{N_Q} = \begin{pmatrix} \mathbf{B}_{N_Q-1} & 2 \cdot \mathbf{I}_{\mathbf{B},N_Q-1} + \mathbf{B}_{N_Q-1} \\ 2 \cdot \mathbf{I}_{\mathbf{B},N_Q-1} + \mathbf{B}_{N_Q-1} & \mathbf{B}_{N_Q-1} \end{pmatrix}$$

   holds, then

$$\mathbf{B}_{N_Q+1} = \begin{pmatrix} \mathbf{B}_{N_Q} & 2 \cdot \mathbf{I}_{\mathbf{B},N_Q} + \mathbf{B}_{N_Q} \\ 2 \cdot \mathbf{I}_{\mathbf{B},N_Q} + \mathbf{B}_{N_Q} & \mathbf{B}_{N_Q} \end{pmatrix} \tag{A.7}$$

   also must hold for any N_Q.

3) Inductive step.
   At first, the following lemma is introduced.

   Lemma 1. *Inverting the first element b_1 of a binary sequence $s_1 = [b_1, b_2, \ldots, b_N]$ which results in $s_2 = [\bar{b}_1, b_2, \ldots, b_N]$, the first two elements of the Gray encoded sequence of s_2 referred to as S_2 are the inverses of the first two elements of the Gray encoded sequence of s_1 referred to as S_1 while the remaining $N-2$ elements $S_1(i)$ and $S_2(i)$ with $i = 3, \ldots, N$ are identical.*

   Proof. The Gray encoded sequence of $s_1 = [b_1, b_2, b_3, \ldots, b_N]$ is given by

$$S_1 = [b_1, b_2, b_3, \ldots, b_N] \oplus [0, b_1, b_2, \ldots, b_{N-1}] = [b_1 \oplus 0, b_2 \oplus b_1, b_3 \oplus b_2, \ldots, b_N \oplus b_{N-1}] \tag{A.8}$$

   while the Gray encoded sequence of $s_2 = [\bar{b}_1, b_2, \ldots, b_N]$ is given by

$$S_2 = [\bar{b}_1, b_2, b_3, \ldots, b_N] \oplus [0, \bar{b}_1, b_2, \ldots, b_{N-1}] = [\bar{b}_1 \oplus 0, b_2 \oplus \bar{b}_1, b_3 \oplus b_2, \ldots, b_N \oplus b_{N-1}]. \tag{A.9}$$

Comparing the first element of $S_1(1) = b_1 \oplus 0 = b_1$ with $S_2(1) = \bar{b}_1 \oplus 0 = \bar{b}_1$, one can see that $S_2(1)$ is the inverse of $S_1(1)$. For the second elements, one gets $S_1(2) = b_2 \oplus b_1 = b_2 \cdot \bar{b}_1 + \bar{b}_2 \cdot b_1$ and $S_2(2) = b_2 \oplus \bar{b}_1 = b_2 \cdot b_1 + \bar{b}_2 \cdot \bar{b}_1$. Inverting $S_2(2)$ and appyling De Morgan's laws leads to $\overline{S_2(2)} = (\bar{b}_2 + \bar{b}_1) \cdot (b_2 + b_1) = S_1(2)$. Finally, one can see that the remaining $N - 2$ elements $S_1(i)$ and $S_2(i)$ with $i = 3, .., N + 1$ are identical what was to be shown. $\square$

Using $N_Q + 1$ bits to Gray encode $M = 2^{N_Q+1}$ quantization levels and the corresponding binary sequences results in

$$
\begin{aligned}
X_{\mathrm{bin},0} &= \overbrace{[0, \overbrace{0, 0, 0, .., 0}^{N_Q}]}^{N_Q+1} &\Rightarrow\quad& \overbrace{[0, \overbrace{0, 0, 0, .., 0}^{N_Q}]}^{N_Q+1} = X_{\mathrm{gray},0} \\
X_{\mathrm{bin},1} &= [0, 0, 0, 0, .., 1] &\Rightarrow\quad& [0, 0, 0, 0, .., 1] = X_{\mathrm{gray},1} \\
& &\vdots\quad& \\
X_{\mathrm{bin},M/4-1} &= [0, 0, 1, 1, .., 1] &\Rightarrow\quad& [0, 0, 1, 0, .., 0] = X_{\mathrm{gray},M/4-1} \\
X_{\mathrm{bin},M/4} &= [0, 1, 0, 0, .., 0] &\Rightarrow\quad& [0, 1, 1, 0, .., 0] = X_{\mathrm{gray},M/4} \\
& &\vdots\quad& \\
X_{\mathrm{bin},M/2-1} &= [0, 1, 1, 1, .., 1] &\Rightarrow\quad& [0, 1, 0, 0, .., 0] = X_{\mathrm{gray},M/2-1} \\
X_{\mathrm{bin},M/2} &= [1, 0, 0, 0, .., 0] &\Rightarrow\quad& [1, 1, 0, 0, .., 0] = X_{\mathrm{gray},M/2} \\
& &\vdots\quad& \\
X_{\mathrm{bin},3M/4-1} &= [1, 0, 1, 1, .., 1] &\Rightarrow\quad& [1, 1, 1, 0, .., 0] = X_{\mathrm{gray},3M/4-1} \\
X_{\mathrm{bin},3M/4} &= [1, 1, 0, 0, .., 0] &\Rightarrow\quad& [1, 0, 1, 0, .., 0] = X_{\mathrm{gray},3M/4} \\
X_{\mathrm{bin},M-1} &= [1, 1, 1, 1, .., 1] &\Rightarrow\quad& [1, 0, 0, 0, .., 0] = X_{\mathrm{gray},M-1} \\
& .
\end{aligned}
\tag{A.10}
$$

Similar to section A.4.1, one can see that the Hamming distance between the Gray codes $X_{\mathrm{gray},i}$ with $i = 0, .., M/2 - 1$ do not change compared to the case of N_Q bits since there is only a 0 added at the beginning. Thus, the Hamming distance between the Gray codes $X_{\mathrm{gray},i}$ with $i = 0, .., M/2 - 1$ and the Gray codes $X_{\mathrm{gray},j}$ with $j = 0, .., M/2 - 1$ are expressed by $\mathbf{B}_{N_Q+1}(1, .., M/2; 1, .., M/2) = \mathbf{B}_{N_Q}$.
From Lemma 1 we know that inverting the first element of a binary sequence, the Gray codes of the original and the modified sequence only differ in the first two elements which are inverted while the remaining sequence of the Gray code stays the same. Hence, the Hamming distance between the Gray Codes $X_{\mathrm{gray},i}$ with $i = 1, .., M/4 - 1$ and the Gray codes $X_{\mathrm{gray},j}$ with $i = M/2, .., 3M/4 - 1$ is the same as the Hamming distance between Gray Codes $X_{\mathrm{gray},i}$ with $i = 0, .., M/4 - 1$ plus an additional Hamming distance

of 2 due to the two inverted elements, i.e., the Hamming distance is given by $\mathbf{B}_{N_Q+1}(1,..,M/4;M/2+1,..,3M/4) = 2+\mathbf{B}_{N_Q}(1,..,M/4;1,..,M/4)$. The same is true for the Hamming distance between the Gray codes $X_{\text{gray},i}$ with $i = M/4,..,M/2-1$ and the Gray codes $X_{\text{gray},j}$ with $i = 3M/4,..,M-1$, i.e., the Hamming distance is given by $\mathbf{B}_{N_Q+1}(M/4+1,..,M/2;3M/4+1,..,M) = 2+\mathbf{B}_{N_Q}(M/4+1,..,M/2;M/4+1,..,M/2)$.
The Hamming distance between Gray codes whose binary sequences differ in the first element *and* the second element of the binary sequence is the same as the Hamming distance between Gray codes whose binary sequences have an identical first element and a different second element since due to Lemma 1, the first two elements of the Gray code are inverted compared to the Gray code whose binary code sequence has the same first element. However, the Hamming distance remains the same. Hence, the Hamming distance between Gray codes $X_{\text{gray},i}$ with $i = 1,..,M/4-1$ and the Gray codes $X_{\text{gray},j}$ with $i = 3M/4,..,M-1$ is given by $\mathbf{B}_{N_Q+1}(1,..,M/4;3M/4+1,..,M) = \mathbf{B}_{N_Q}(1,..,M/4;M/4+1,..,M/2)$ and the Hamming distance between Gray codes $X_{\text{gray},i}$ with $i = M/4,..,M/2-1$ and the Gray codes $X_{\text{gray},j}$ with $i = M/2,..,3M/4-1$ is given by $\mathbf{B}_{N_Q+1}(M/4+1,..,M/2;M/2+1,..,3M/4) = \mathbf{B}_{N_Q}(M/4+1,..,M/2;1,..,M/4)$. For the Hamming distances between Gray codes $X_{\text{gray},i}$ with $i = M/2,..,M$ and Gray codes $X_{\text{gray},j}$ with $i = 0,..,M$ one gets the same result but vice-versa, resulting in (2.91) what was to be shown.

A.5 Derivation of N_{rS} of (3.246)

In the following, it is proven that the number N_{rS} of possible realisations of the modulation scheme vector $\mathbf{b}^{(u)} = [b_1^{(u)},..,b_L^{(u)}]$ with $b_{q-1}^{(u)} \leq b_q^{(u)}$ and $b_q^{(u)} \in \mathbb{N}\ \forall\ q = 1..,L$ representing the number of bits per data symbol corresponding to the applied modulation scheme in the q-th quantisation level is given by $N_{\text{rS}} = f(L,M)$ assuming there are M different modulation schemes available. To do so, the following lemma is formulated.

Lemma 2. *The number N_{x} of possible realisations of a vector $\mathbf{x}$ with length L whose elements $x_l \in \{1,2,..,M\}$ with $l = 1,..,L$ where $x_{l-1} \leq x_l$ is given by*

$$N_{\text{x}} = f(L,M), \tag{A.11}$$

where $f(L,M)$ is a recursive function given by

$$\begin{aligned} f(L,M) &= f(L-1,M) + f(L,M-1) \\ \textit{with} \;\; & f(1,M) = M \\ \textit{and} \;\; & f(L,1) = 1. \end{aligned} \tag{A.12}$$

Proof. The number $f(L,M)$ of possible realisations of a vector $\mathbf{x}$ with length L and M possible element values decomposes in two sets. In the first set, the first element x_1 of $\mathbf{x}$ equals 1, i.e., $x_1 = 1$. In the second set, the first element and, thus, all other elements are larger than 1, i.e., $x_l \geq 2 \; \forall \; l = 1, .., L$. If in the first set the first element is omitted, a vector $\tilde{\mathbf{x}}$ with length $L-1$ and M possible element values remains for which $f(L-1,M)$ possible realisations exist. In the second set, no element of the vector is equal to 1, i.e., a vector $\check{\mathbf{x}}$ with length L but only $M-1$ possible element values is left for which $f(L,M-1)$ possible realisations exist. In case that there is only one possible element value ($M=1$), only one possible realisation exists no matter L, i.e., $f(L,1) = 1$. In case that the length of vector is $L = 1$, there exists M possible realisations, i.e., $f(1,M) = M$. $\square$

Since, without loss of generality, it can be assumed that the modulation scheme with the lowest number of bits per symbol is represented by $b_q^{(u)} = 1$ while the next higher modulation scheme is represented by $b_q^{(u)} = 2$ and so on, Lemma 2 can be applied, i.e., the number N_{rS} of possible realisations of the modulation scheme vector is given by $N_{\mathrm{rS}} = f(L,M)$ what was to be shown.

List of Acronyms

3GPP	Third Generation Partnership Project
AF	Adaptive First
AWGN	Additive White Gaussian Noise
B-EFDMA	Block Equidistant Frequency Division Multiple Access
B-IFDMA	Block Interleaved Frequency Division Multiple Access
BER	Bit Error Rate
BS	Base Station
CDF	Cumulative Probability Density Function
CE	Channel Estimation
CLT	Central Limtit Theorem
CQI	Channel Quality Information
CSI	Channel State Information
CP	Cyclic Prefix
DFT	Discrete Fourier Transform
DL	Downlink
DL-PT	Downlink Pilot Transmission
ES	Exhaustive Search
FB	Feedback
FDD	Frequency Division Duplex
FDMA	Frequency Division Multiple Access
FFT	Fast Fourier Transform
FRS	Fair Resource Scheduling
FTS	Fair Throughput Scheduling
GI	Guard Interval

ICI Inter Carrier Interference

IDFT Inverse Discrete Fourier Transform

IFDMA Interleaved Frequency Division Multiple Access

ISI Inter Symbol Interference

LFDMA Localized Frequency Division Multiple Access

LOS Line-Of-Sight

LS Least Squares

LTE Long Term Evolution

M-PSK M-ary Phase Shift Keying

M-QAM M-ary Quadrature Amplitude Modulation

MIMO Multiple Input Multiple Output

MRC Maximum Ratio Combining

MS Mobile Station

NAF Non-Adaptive First

NLOS Non-Line-Of-Sight

OSTBC Orthogonal Space Time Block Coding

OSTBC-MRC Orthogonal Space Time Block Coding in combination with Maxium Ratio Combining

OFDM Orthogonal Frequency Division Multiplex

OFDMA Orthogonal Frequency Division Multiple Access

PACE Pilot Assisted Channel Estimation

PDF Probability Density Function

PFS Proportional Fair Scheduling

PRB Physical Resource Block

PSK Phase Shift Keying

PT	Pilot Transmission
QAM	Quadrature Amplitude Modulation
QPSK	Quaternatary Phase Shift Keying
QWPFS	Quantized Weighted Proportional Fair Scheduling
R-CSI	Receive Channel State Information
RedCom	Reduced Complexity
RX	Receive
SC-FDMA	Single Carrier Frequency Division Multiple Access
SINR	Signal-to-Interference plus Noise Ratio
SNR	Signal-to-Noise Ratio
STC	Space Time Coding
SF	Super Frame
SS	Signaling of Side Information
T-CSI	Transmit Channel State Information
TAS	Transmit Antenna Selection
TAS-FA	Transmit Antenna Selection Feedback All
TAS-FB	Transmit Antenna Selection Feedback Best
TAS-MRC	Transmit Antenna Selection in combination with Maximum Ratio Combining
TX	Transmission
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
TP	Throughput
UL	Uplink
UL-PT	Uplink Pilot Transmission

WiMAX Worlwide interoperability for Microwave Access

WINNER Wireless World Initiative New Radio

WPFS Weighted Proportional Fair Scheduling

XOR exclusive OR

List of Symbols

a	Normalisation factor
$\mathbf{b}^{(u)}$	Modulation scheme vector of user u
b_m	Number of bits per symbol corresponding to the m-th modulation scheme
b_{SS}	Number of bits per symbol for signaling
B	Bandwidth
B_{C}	Coherence bandwidth
$\mathbf{B_{N_Q}}$	Hamming distance matrix for N_Q bit coding
BER_{T}	Target bit error Rate
$\overline{BER}^{(u)}$	Average bit error rate of user u
$\widehat{BER}_m^{(u)}(\hat{\gamma})$	Actual bit error rate selecting the m-th modulation scheme based on the estimated SNR value $\hat{\gamma}$
c	Speed of light
d_u	Distance between user u and the base station
d_0	Minimum distance between any user and the base station
$d_{\mathrm{bin}}^{(u)}$	binary data of user u
$\hat{d}_{\mathrm{bin}}^{(u)}$	estimated binary data of user u
$d^{(u)}$	Data symbol of user u
d_p	Pilot symbol
D_u	resource demand of user u
$\mathbf{D}$	User resource demand vector
$\tilde{\mathbf{D}}$	Modified user resource demand vector
e	Base of the natural logarithm, also called Napier's constant
$E\{\cdot\}$	Expectation operator
$\mathbf{E}$	Error probability matrix
f_0	Carrier frequency
$f_{D,u}$	Doppler frequency of user u
$f(L, M)$	Number of the possible realisations of a vector with length L and M possible element values
$F^{(u)}(\hat{\gamma})$	CDF of measured SNR value $\hat{\gamma}$
$\mathbf{F_Q}$	Discrete Fourier Transform matrix
$\mathbf{F_Q}^{\mathrm{H}}$	Inverse Discrete Fourier Transform matrix
G	Number of user demand groups
$G_{\max}$	Maximum number of user demand groups

$H_u^{(i,j)}(n,k)$	Transfer function of the radio link between transmit antenna element i and receive antenna j of user u on resource unit n in time frame k
$\hat{H}_u(k)$	Estimated channel transfer factor of user u in time frame k
$\mathbf{I}$	Identity matrix
$\mathbf{I}_{\mathbf{B},N_Q}$	$2^{N_Q} \times 2^{N_Q}$ diagonal block unity matrix
$J_0(\cdot)$	0th-order Bessel function of the first kind
k	Time frame index
$\mathbf{l}_{\mathrm{hw}}^u$	Quantisation level index vector considering all users with a higher weighting factor as user u
$\mathbf{l}_{\mathrm{lw}}^u$	Quantisation level index vector considering all users with a lower weighting factor as user u
$\mathbf{l}_{\mathrm{sw}}^u$	Quantisation level index vector considering all users with the same weighting factor as user u
L	Number of quantisation levels
L_P	Pathloss
L_{SF}	Super frame length
m	Modulation scheme index
M	Number of modulation schemes
$M_{\mathrm{DLPT-STC}}$	Number of OFDMA symbols used for downlink pilot transmission applying Space Time Coding
$M_{\mathrm{DLPT-TAS}}$	Number of OFDMA symbols used for downlink pilot transmission applying Transmit Antenna Selection
M_{DLT}	Number of OFDMA symbols used for downlink data transmission
$M_{\mathrm{FB-STC}}$	Number of OFDMA symbols used for CQI feed back applying Space Time Coding
$M_{\mathrm{FB-TAS-FA}}$	Number of OFDMA symbols used for CQI feed back applying Transmit Antenna Selection with Feedback All
$M_{\mathrm{FB-TAS-FB}}$	Number of OFDMA symbols used for CQI feed back applying Transmit Antenna Selection with Feedback Best
M_P	Number of pilots per resource unit
$M_{P,\mathrm{CQI}}$	Number of pilots during CQI phase in the uplink
$M_{\mathrm{SS-STC}}$	Number of OFDMA symbols used for signaling applying Space Time Coding
$M_{\mathrm{SS-TAS}}$	Number of OFDMA symbols used for signaling applying Transmit Antenna Selection
M_T	Number of OFDMA symbols per time frame
M_{ULPT}	Number of OFDMA symbols used for uplink pilot transmission
M_{ULT}	Number of OFDMA symbols used for uplink data transmission

n	Resource unit index
n_T	Number of transmit antennas
n_R	Number of receive antennas
N	Number of subcarriers
N_0	One-sided noise power spectral density
$N_{\mathrm{O,ES}}$	Number of operations applying Exhaustive Search algorithm
$N_{\mathrm{O,RedCom}}$	Number of operations applying RedCom algorithm
$N_{\mathrm{O,RedCom2}}$	Number of operations applying RedCom2 algorithm
N_{pS}	Number of all possible realisations of the modulation scheme vector
N_Q	Number of quantisation bits
N_{rS}	Number of all reasonable realisations of the modulation scheme vector $\mathbf{b}^{(u)}$
N_{ru}	Number of resource units
N_{tuple}	Number of user demand group G-tuples
$p(\eta, \kappa)$	Number of partitions of η into κ summands
$p^{(u)}(\hat{\gamma})$	PDF of measured SNR value $\hat{\gamma}$
p_b	Feedback bit error rate
p_u	Weighting factor of user u
$\mathbf{p}$	Weighting vector
$\tilde{\mathbf{p}}$	Modified weighting vector
$\mathbf{p}'$	Extended weighting vector
P_q	Probability that SNR value lies in the q-th quantisation level
$P_{<q}$	Probability that SNR value lies in a quantisation level beneath the q-th level
$\tilde{P}_q$	Probability that SNR value is assumed to lie in the q-th quantisation level
$P_{<q}$	Probability that SNR value is assumed to lie in a quantisation level beneath the q-th level
$P^{(u)}(\mathbf{p})$	Channel access probability of user u as a function of the weighting vector $\mathbf{p}$
P_T	Transmit power
$P_{T,\mathrm{sub}}$	Transmit power per subcarrier
q	Quantisation level index
Q	DFT length
Q_{sub}	Number of subcarriers per frequency block
r_{n_T}	Data rate of Space Time Code with n_T transmit antennas

$r(\eta, i)$	returns the index of the i-th 1 in the multi-index η
R	Cell radius
$\bar{R}_{\mathrm{A}}^{(u)}$	Average data rate of the adaptive user u
$\bar{R}_{\mathrm{A,opt}}^{(u)}$	Maximum achievable average data rate of the adaptive user u applying optimised SNR thresholds
$\bar{R}_{\mathrm{A,eff,opt}}^{(u)}$	Maximum achievable effective average data rate of the adaptive user u applying optimised SNR thresholds
$\bar{R}_{\mathrm{min}}^{(u)}$	Minimum required data rate of user u
$\bar{R}_{\mathrm{N}}^{(u)}$	Average data rate of the non-adaptive user u
$\bar{R}_{\mathrm{N,opt}}^{(u)}$	Maximum achievable average data rate of the non-adaptive user u applying optimised SNR thresholds
$\bar{R}_{\mathrm{N,eff,opt}}^{(u)}$	Maximum achievable effective average data rate of the non-adaptive user u applying optimised SNR thresholds
$\bar{R}_{\mathrm{pureA,eff,opt}}^{(u)}$	Maximum achievable effective average data rate of the pure adaptive user u applying optimised SNR thresholds
$\bar{R}_{\mathrm{pureN,eff,opt}}^{(u)}$	Maximum achievable effective average data rate of the pure non-adaptive user u applying optimised SNR thresholds
$s^{(u)}$	Time domain OFDMA signal of user u
$\bar{R}_{\mathrm{sys}}$	Average system data rate
$\bar{R}_{\mathrm{sys,opt}}$	Maximum achievable average system data rate assuming optimized SNR threshold and user serving vectors
S	User satisfaction
T	Time delay
T_{C}	Coherence time
u	User index
U	Number of users
U_{A}	Number of adaptively served users
U_{NA}	Number of non-adaptively served users
$\bar{v}$	Average user velocity in cell
$\mathrm{Var}\{\cdot\}$	Variance
W_{A}	Number of resource units dedicated for adaptive users
W_{NA}	Number of resource units dedicated for non-adaptive users
$\mathbf{X}^{(u)}$	Allocation matrix of user u
${\mathbf{X}_{\mathrm{M}}}^{(u)}$	Modulation scheme matrix of user u
Z	Number of possible user demand vector realisations
α	Pathloss exponent
β_m	Modulation scheme exponent

γ	SNR value
$\hat{\gamma}$	Estimated SNR value
$\bar{\gamma}_u$	Average SNR of user u
$\gamma_{\mathrm{IDFT},u}(k)$	Resulting SNR of user u in time frame k after IDFT operation
$\gamma_u^{(i,j)}(n,k)$	SNR value of link between transmit antenna i and receive antenna j of user u on resource unit n in time frame k
$\gamma_u^{\mathrm{q}}(n,k)$	Quantized SNR value of the link of user u on resource unit n in time frame k
$\gamma_{\mathrm{th}}^{(u)}$	SNR threshold vector of user u
$\gamma_{\mathrm{th},l}^{(u)}$	l-th element of SNR threshold vector of user u
$\gamma_{\mathrm{th,opt}}^{(u)}$	optimized SNR threshold vector of user u
$\mathbf{\Gamma}$	Correlation coefficient vector
Δf	Subcarrier spacing
η	Multi-index with $\eta_j \in \{0,1\}\ \forall\ j = 1,..,U_{\mathrm{A}} - 1$
ϑ	User serving vector
κ_{UL}	Uplink factor
λ	Lagrange multiplier
Λ	Auxiliary variable
μ_u	Auxiliary variable for user u
$\mu_{\mathrm{CLT},u}$	Mean value of central limit theorem approximation for the post IDFT SNR of user u
ν	Multi-index with $\nu_j \in \{0,1,..,n_T \cdot n_R - 1\}\ \forall\ j = 1,..,v-1$
ρ_u	Correlation between the outdated channel and the actual channel of user u
$\sigma_{\mathrm{CLT},u}$	Variance of central limit therem approximation for the post IDFT SNR of user u
$\sigma_{E,u}^2$	Channel estimation error variance of user u
σ_n^2	Noise variance
σ_r^2	Auxiliary variable
σ_v^2	Variance of the x- and y-component of the user velocity
$\mathbf{\Sigma}$	Estimation error variance vector
$\tau_{\max}$	Maximum time delay of the channel
Υ	Auxiliary variable
$\mathcal{G}_i$	Demand group of index i
$\mathcal{Q}_{u,N_Q}(\cdot)$	Returns the quantisation level index of the argument considering the quantisation levels of user u with N_Q quantisation bits
$\mathcal{S}_{\mathrm{hw}}^{(u)}$	Set of users with a higher weighting factor as user u

$\mathcal{S}_{\mathrm{lw}}^{(u)}$	Set of users with a lower weighting factor as user u
$\mathcal{S}_{\mathrm{sw}}^{(u)}$	Set of users with the same weighting factor as user u
$\mathbb{N}$	Set of positive integer numbers
$\mathbb{Z}$	Set of integer numbers
$(\cdot)^{\mathrm{T}}$	Transpose of a vector or matrix
$(\cdot)^{\mathrm{H}}$	Conjugate transpose of a vector or matrix
$(\cdot)^{*}$	Conjugate of a scalar, vector, or matrix
$(\cdot)^{-1}$	Inverse of a square matrix
$\lvert \cdot \rvert$	Absolute value of a scalar

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